Six Sigma
Rolled Throughput Yield

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Scenario:
You are touring a production process and you see numerous steps with inspection followed by scrap or rework.

At several such steps, you ask, “What is the yield?”

Answer at step 1: 95%.
Answer at step 2: 90%.
...
Answer at final test: 99%.

Then you ask, “What is the overall yield?”
Answer: 99%
Repeat the question:

You finally ask,

“What percent of product goes straight through the process without problems?”

Answer: ____________________

If only 10% of units go straight through the process without being scrapped or reworked, then 90% of units cost more than they should and may deliver less value to customers than the “straight through” 10%

How to Evaluate the “Goodness” or Level of Problems to Allocate $ for Improvement?

One easy measure:

**Rolled Throughput Yield (RTY):**

\[ p = p_1 p_2 \cdots p_n = \prod_{i=1}^{n} p_i \]

\[ p_i = \text{yield at step } i \]
January 26

\[ p_2 = 100\% \]

\[ p_1 = 97.5\% \text{ from January 24} \]

\[ p_3 = 97.9\% \text{ for the week} \]

\[ p = p_1 \cdot p_2 \cdot p_3 = 95.4\% \]
Rolled Throughput Yield

= product of the yields at all steps in the process

We can compute a number

■ Does it have a purpose?
Net Yield in Chemical Engineering

The percent of the primary input that is converted to output.

- **This**
  - selects one $p_i$ as most important,
  - ignores others
  - is appropriate in some cases
  - is NOT sensible generally

**Interpretation:**

Japanese: *A Dictionary of Total Quality Control Terms*

“non-adjusted rate,” “go-through rate”:

Percent of product that passes straight through the production process without delays due to quality problems.
**Rework** (no scrap): 

Sequential

\[ p_1 \rightarrow p_2 \rightarrow \ldots \rightarrow p_n \]

Parallel

\[ p_1 \leftarrow p_2 \rightarrow \vdots \rightarrow p_n \rightarrow \text{perfect assembly} \]

If the probability of problems at one step is *independent* of problems at other steps, then

\[ \text{RTY} = \text{percent of product without problems} \]

**Scrap in a Sequential Process**

\[ p_1 \rightarrow p_2 \rightarrow \ldots \rightarrow p_n \]

The percent of product that finishes step \( i \)

\[ = p_i \times \text{the percent of product that finished step} \ (i - 1). \]

Thus,

\[ \text{RTY} = \text{percent of product entering the process that actually finishes}. \]
Scrap in a Sequential Process

\[ p_2 = \text{percent of product that finishes step 2 given it was good leaving step 1} \]

So, the percent of product starts that finish step 2

\[ = p_2 p_1 \]

Thus,

RTY = percent of product entering the process that actually finishes

Scrap in a Parallel Process

What is the “right” number for “yield”?

… it depends … .
A Reasonable Index

- For a process with scrap in parallel steps, the product of the yields is
  - a useful index of process quality,
  - without an interpretation as a % of some obvious count of units

Scrap and Rework

- Production problems
  - Increase labor and material costs
  - May cause scheduling and inventory difficulties
  - May contribute to reliability problems

- Scrap and rework
  - The cost of scrap may be more obvious and more easily quantified
  - Many costs associated with rework may be more subtle and harder to quantify

- For simplicity, we ignore the distinction between scrap and rework
Process Changes?

**Add inspections**
- If you catch EARLIER problems that would have been caught later: RTY is essentially unchanged
- If you catch problems previously overlooked: the new RTY is a better reflection of process health than without the added inspections

**If you change the way you think about the process**
- You don’t need to discard RTY history

Defects and Defectives

**Harry and Schroeder (2000) and Breyfogle (1999)** use the Poisson assumption to translate between
- the proportion of units with defects ($p$), and
- defects per unit (DPU, $u$)

\[
p = \exp(-u) \\
u = [-\ln(p)]
\]

Rolled Throughput Yield for a General Process

\[ p = \prod_{i=1}^{n} p_i = \text{a reasonable index of delays} \]
\[ \text{and productivity losses in the process.} \]

- Use this for
  - scrap and rework
  - series or parallel steps
  - defects or defectives

- Translate freely, if approximately, to DPU via

\[ u = [-\ln(p)] = \sum_{j=1}^{n} u_j \]

Rolled Throughput Yield

- Defined this way, RTY
  - is simple, easy to remember, easy to compute
  - provides a reasonable first cut analysis in many cases

- In many cases, RTY provides an obvious answer that would not be obvious without it

- In other cases, a Pareto using RTY may eliminate some options, narrowing the scope for more careful studies of potential problem areas
RTY in Process Control

- We discuss here
  - Variability
  - Statistical control limits
  - Example

Process Variability

\( \hat{p}_{j,t} = \) observed yield at step \( j \) on day \( t \).

\( p_j = \) the average yield at step \( j \) assuming the process is stable,

\( \hat{p}_t = \) the observed Rolled Throughput Yield for day \( t \)

\[
\hat{p}_t = \prod_{j} \hat{p}_{j,t}
\]

or

\[
\hat{u}_t = \sum_{j} \hat{u}_{j,t}
\]

\[
\text{var}(\hat{u}_{j,t}) = \begin{cases} 
    \frac{u_j}{n_{j,t}} & \text{if counting defects} \\
    \frac{(1 - p_j)(p_j n_{j,t})}{\text{with overdispersion}} & \text{if counting defectives}
\end{cases}
\]

\[
\text{var}(\hat{u}_{j,t})
\]
**Statistical Control Limits**

\[
\text{var}(\hat{u}_{j,t}) = \begin{cases} 
    \frac{u_j}{n_{j,t}} & \text{if counting defects} \\
    (1 - p_j)p_j n_{j,t} & \text{if counting defectives with overdispersion}
\end{cases}
\]

\[
\text{var}(\hat{u}_i) = \sum_{j} \text{var}(\hat{u}_{j,t})
\]

\[
\text{UCL}(\hat{u}_i) = u + 3\sqrt{\text{var}(\hat{u}_i)}
\]

\[
\text{LCL}(\hat{u}_i) = u - 3\sqrt{\text{var}(\hat{u}_i)}
\]

\[
\text{UCL}(\hat{\rho}_i) = \exp[-\text{LCL}(\hat{u}_i)]
\]

\[
\text{LCL}(\hat{\rho}_i) = \exp[-\text{UCL}(\hat{u}_i)]
\]

---

**Defects per Unit (DPU)**

\[0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \]

\[0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \]

\[1/21 \quad 1/26 \quad 1/31 \quad 2/5 \quad 2/10 \quad 2/15 \quad 2/20 \quad 2/25 \]

**Rolled Throughput Yield (RTY)**

\[0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \]

\[0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \]

\[1/21 \quad 1/26 \quad 1/31 \quad 2/5 \quad 2/10 \quad 2/15 \quad 2/20 \quad 2/25 \]

**Process Changes made take effect**
## A Forecasted Pareto

<table>
<thead>
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<th>Rolled Throughput Yield (last 3 months)</th>
<th>Sales Forecast for next year (K$)</th>
<th>Estimated Annual Savings (K$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>2</td>
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<tr>
<td>B</td>
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<td>3,300</td>
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<td>C</td>
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Rolled Sales Estimated Throughput Forecast Annual Product Yield for next Savings (last 3 year = $0.2 \times (1-\text{[2]}) \times [3]$)

### Evaluation

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- **This analysis suggests that product B problems may merit some attention**
- **A brief review of product C may also be appropriate**
Conclusions

*Rolled throughput yield*

- or the complementary DPU Poisson rate
- provides a simple way to summarize
  - defects and defectives
  - scrap and rework
  - series and parallel steps
  - for process control and project selection
