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Efficient Factor Screening for Simulation Experiments

Hua Shen and Hong Wan,

School of Industrial Engineering, Purdue University

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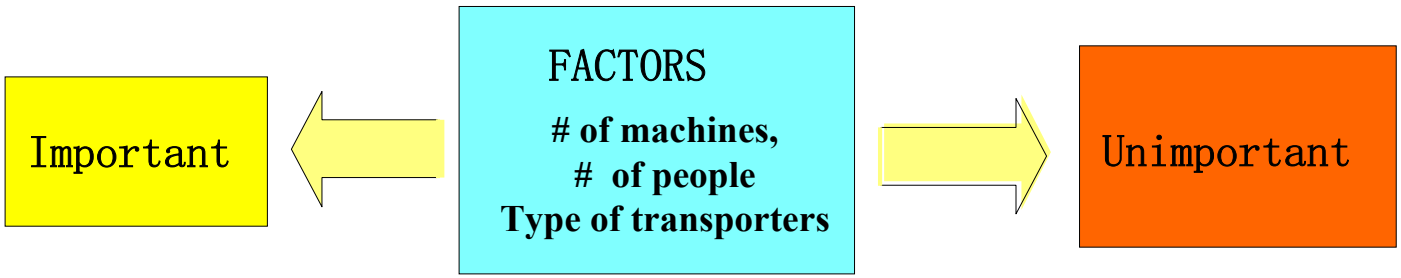
Screening Experiments

- **Screening Experiments**: *investigate the controllable factors to eliminate the unimportant ones*



- **A Good Screening Experiment**: *identify the important factors correctly with an economical number of replications*

Simulation *vs.* Physical Experiments

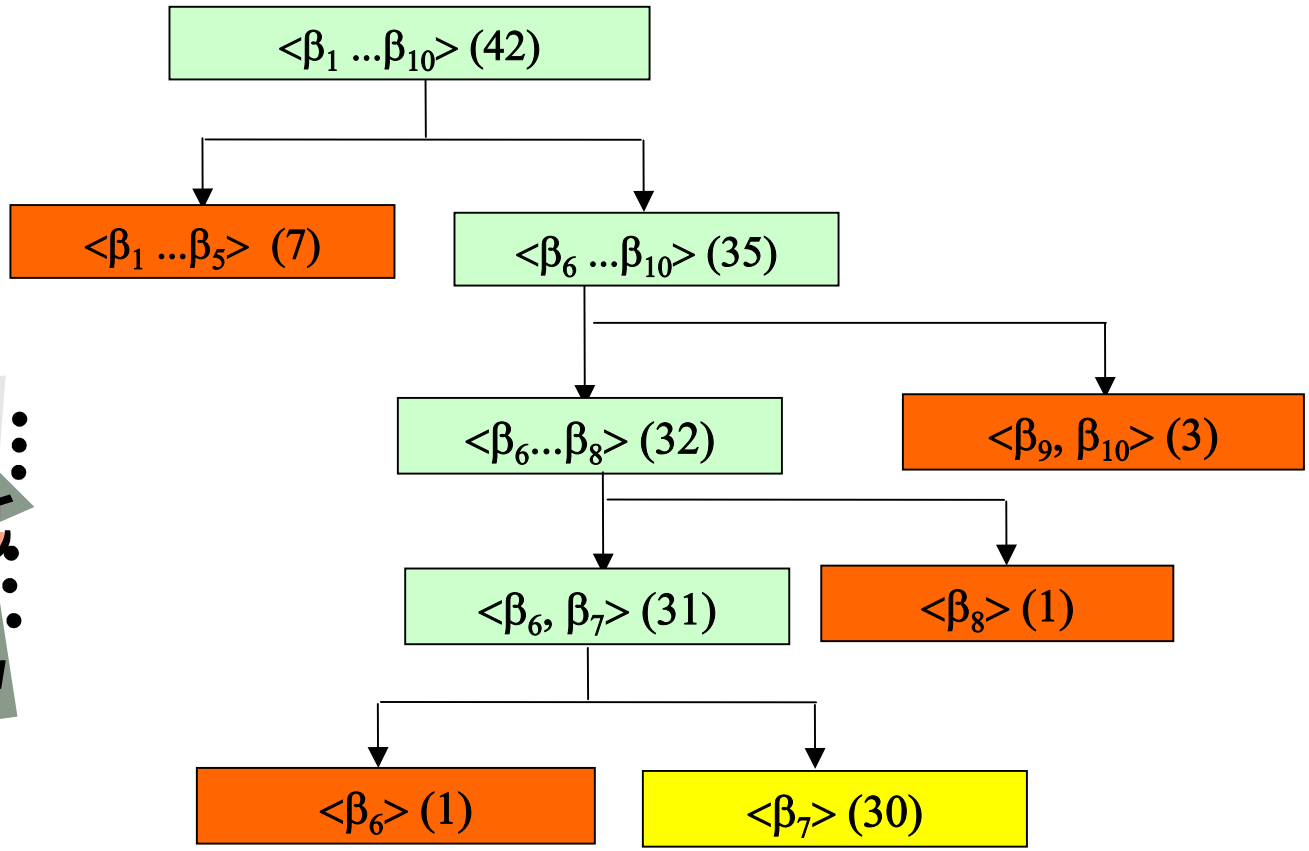
Factors: Decision variables related to staffing, capacity, operating rules, etc.

Simulation: Discrete-event stochastic simulation

	Simulation Experiments	***Physical Experiments***
Number of factors	Large	Small
Switching between settings	Cheap	Expensive
Variance reduction	Allow Common Random Numbers (CRN)	N/A
Emphasis	Precision-driven	Budget-driven

Sequential Bifurcation

Threshold of Importance: 8 $\beta=\{1, 1, 2, 2, 1, 1, 30, 1, 2, 1\}$



Bettonvil, B., and J. P. C. Kleijnen. 1997. *European Journal of Operational Research* 96 (1): 180-194

Model Assumption

Main Effect or Two-factor interaction Model:

Suppose there are K factors of interest with effect coefficients $\beta = \{\beta_1, \beta_2, \ldots, \beta_K\}$. The output of interest from a simulation replication is denoted by Y, and Y is represented as:

$$Y = \beta_0 + \sum_{i=1}^{K} \beta_i x_i + \left(\sum_{i=1}^{K-1} \sum_{j=i}^{K} \beta_{ij} x_i x_j \right) + \varepsilon$$

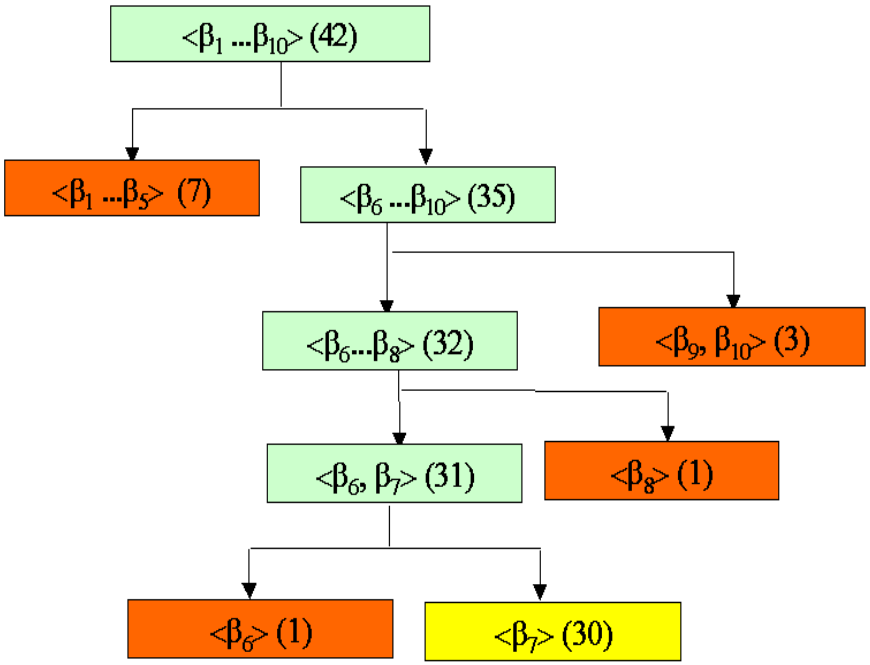
Normal Error $\varepsilon \sim N(0, \sigma^2(x))$

Unequal variance

Common random numbers applicable

Known directions of main effects

Challenges



- **Model**: Can we compare factor effects fairly?
- **Error**: Can we guarantee the correctness of the results?
- **Robustness**: Can we handle unequal variances and CRN?

Controlled Sequential Bifurcation (CSB) compares factor effects relative to given thresholds, accounts for differing costs (if desired) and controls experimental error (assuming no interactions).

CSB-X: Eliminates the bias from any two-factor interactions.

CSB and CSB-X: Algorithm

Initialization: Create an empty LIFO queue for groups. Add the group $\langle \beta_1, \beta_2, \ldots \beta_K \rangle$ to the LIFO queue.

While queue is not empty, do

Remove: remove a group from the queue

Test the sum of the effects in the group:

Unimportant: classify all factors in the group as unimportant.

Important (size=1): Classify the factor as important.

Important (size >1): Split the group into two subgroups such that all factors in the first subgroup have a smaller index than those in the second subgroup. Add each subgroup to the LIFO queue.

End Test

End While

Test Variable for CSB and CSB-X

An observation at level k: $x_i(\pm k) = \begin{cases} \pm 1, & i = 1, 2, \ldots, k \\ 0, & i = k+1, k+2, \ldots, K \end{cases}$

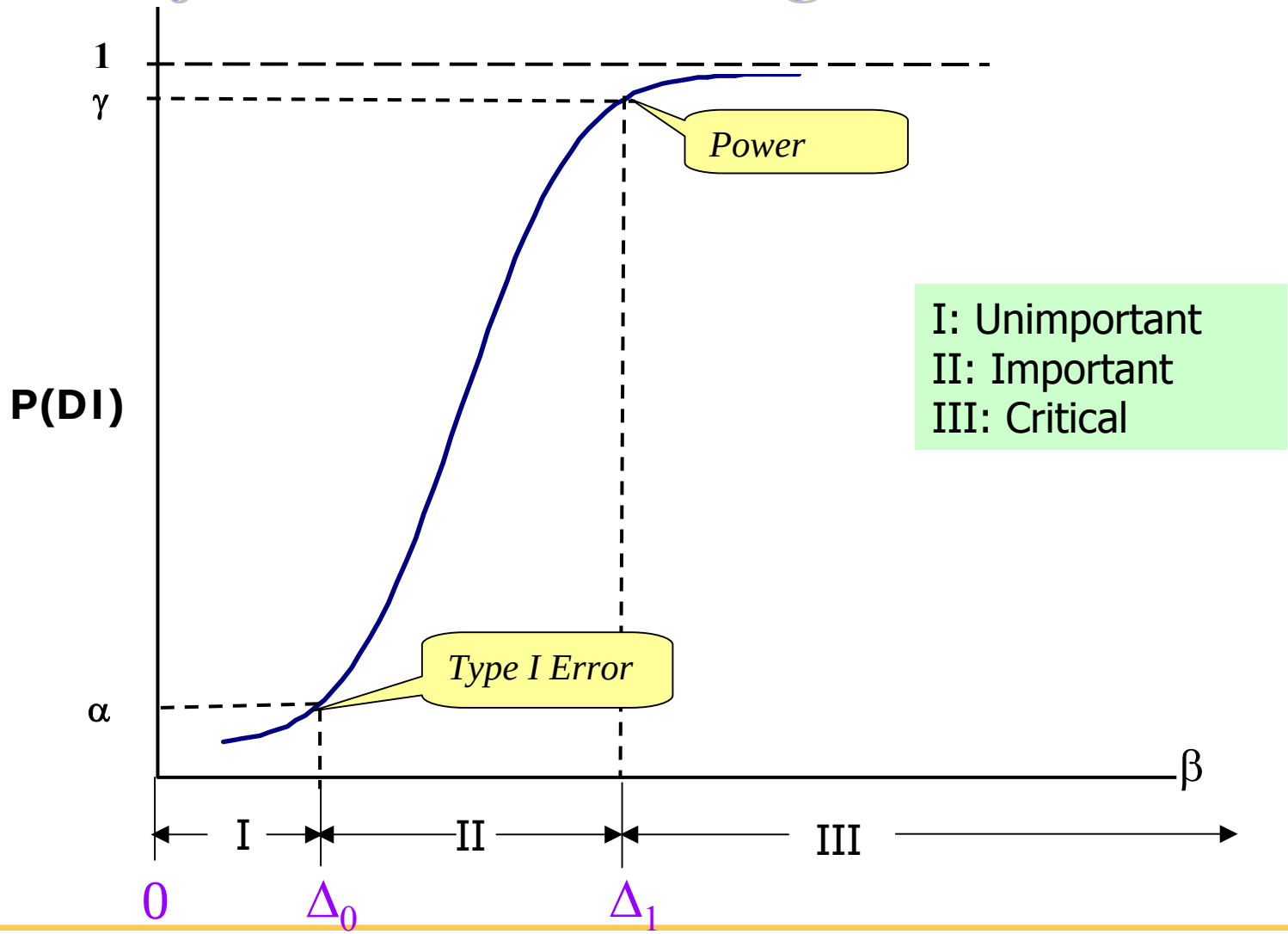
$$E[Y(k)] = \beta_0 + \sum_{i=1}^{k} \beta_i + \left(\sum_{i=1}^{K-1} \sum_{j=i}^{K} \beta_{ij}\right)$$

$$E[Y(-k)] = \beta_0 - \sum_{i=1}^{k} \beta_i + \left(\sum_{i=1}^{K-1} \sum_{j=i}^{K} \beta_{ij}\right)$$

CSB: $D(k_1, k_2) = Y(k_2) - Y(k_1)$

CSB-X: $D(k_1, k_2) = \dfrac{[Y(k_2) - Y(-k_2)] - [Y(k_1) - Y(-k_1)]}{2}$

Objective of Screening Procedure



Desired Test Performance

If the main effects or two-factor interaction model holds with normally distributed error, then:

$$\Pr\{Declare < \beta_{k_{l+1}} \ldots \beta_{k_m} > important \mid \sum_{i=k_l+1}^{k_m} \beta_i \leq \Delta_0\} \leqslant \alpha \qquad (1)$$

$$\Pr\{Declare < \beta_{k_{l+1}} \ldots \beta_{k_m} > important \mid \sum_{i=k_l+1}^{k_m} \beta_i \geq \Delta_1\} \geqslant \gamma \qquad (2)$$

CSB and CSB-X: Performance

- **Theorem 1:** If any test gives the guarantees (1) and (2), then CSB and CSB-X guarantee that:

$$\Pr\{\textit{Declare factor } i \textit{ important} \mid \beta_i < \Delta_0\} \leqslant \alpha$$

for each factor *i* individually.

- **Theorem 2:** If any test gives the guarantees (1) and (2), then CSB and CSB-X guarantee that:

$$\Pr\{\textit{Declare} < \beta_{k_{l+1}} \ldots \beta_{k_m} > \textit{important} \mid \sum_{i=k_l+1}^{k_m} \beta_i \geq \Delta_1 \} \geqslant \gamma$$

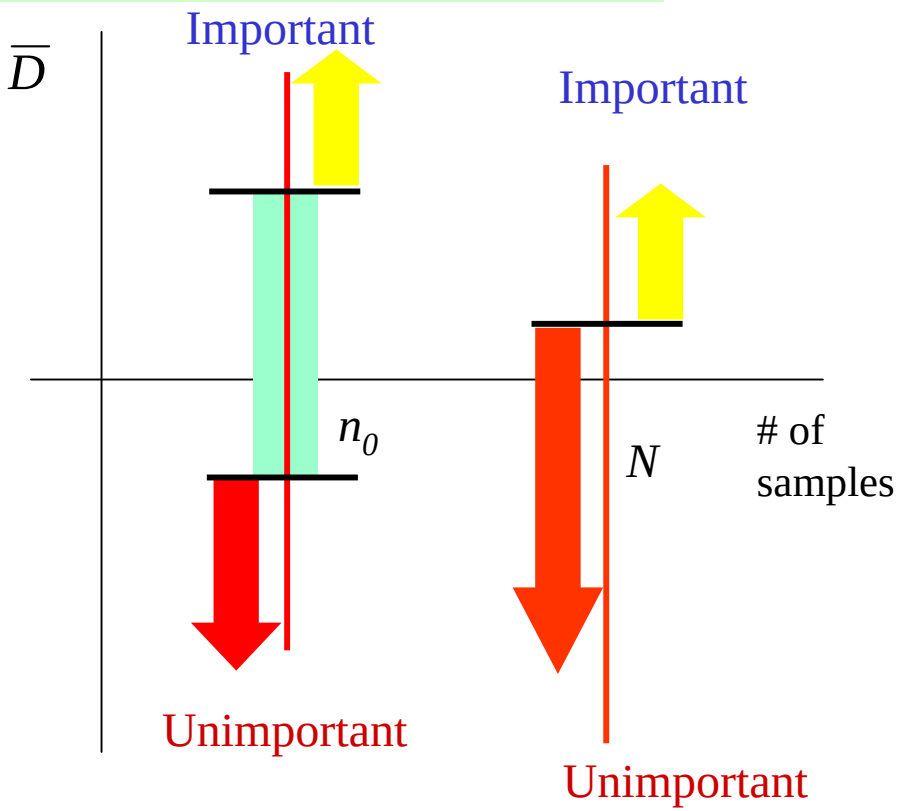
for each group $< \beta_{k_{l+1}} \ldots \beta_{k_m} >$ tested.

Qualified Test: Two-Stage Procedure

Hypothesis: $H_0 : \sum_{i=k_l+1}^{k_m} \beta_i \le \Delta_0 \quad vs. \quad H_1 : \sum_{i=k_l+1}^{k_m} \beta_i > \Delta_0$

Stage I: Build a two-sided confidence interval for the estimate of the group effect from n_0 replications. Make classification if possible; else go to Stage II.

Stage II: Get N-n_0 replications at each level, and make a lower confidence bound for the estimate of the group effect . Make classification.



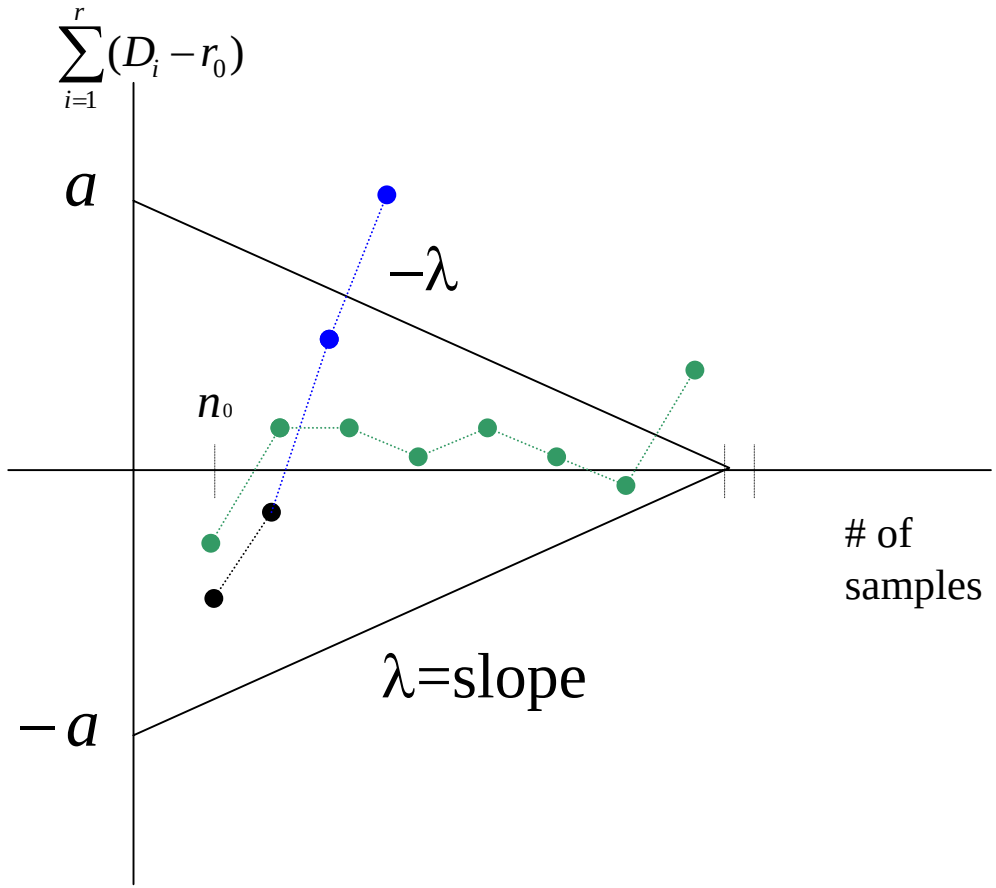
Qualified Test: Fully Sequential Test

Suppose $D_1, D_2, \ldots D_n \ldots$ are i.i.d. $N(\mu, \sigma^2)$

Then the partial sum process $\sum_{i=1}^{r} (D_i - r_0)$ is like Brownian motion with drift $\mu - r_0$.

We are able to control the probability of the partial sum leaving on either side.

Contribution: We are able to handle the cases $\alpha \neq 1-\gamma$.



S.-H. Kim. 2005. Comparison with a Standard via Fully Sequential Procedure, ACM TOMACS, 15(2):1-20 .

CSB vs. Fractional Factorial Design

- Evaluate the efficiency of CSB relative to a minimal FFD assuming equal variance & no power control
- CSB provides more inference, for about the same or fewer replications

Scenario: K=500, σ=1, β = 0 or 5 $\Delta_0 = 2$, $\Delta_1 = 4$, $\alpha = 1-\gamma = 0.05$	**CSB Reps**	**FFD Reps**
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10} important	148	512
{1, 51, 101, 151,…,451} important	573	512

CSB and CSB-X: Limitations

- May not be effective in the presence of higher-order interactions .
- Cannot screen interactions.
- Assume known direction of main effects.
- Not efficient if the percentage of significant factors is large.

FF-CSB

- Add a pre-screening stage (a saturated factorial design currently) to separate positive and negative effects.
- CSB will be implemented on each sorted group individually.
- CSB stage is typically more efficient due to sorting even all factors are positive/negative (can save as much as ½ of simulation runs).
- The initial stage add a overhead..

CSFD: Assumptions

- Response Model with L factors of interest

$$Y = \beta_0 + \sum_{i=1}^{L} \beta_i x_i + \sum_{i=1}^{L-1} \sum_{j=i+1}^{L} \beta_{ij} x_i x_j + \ldots + \beta_{12\ldots L} x_1 x_2 \cdots x_L + \varepsilon$$

- Normal error with unequal variance

$$\varepsilon \sim N(0, \sigma^2(x))$$

where $x = (x_1, x_2, \ldots, x_L)$.

- Does not assume known directions of effects.

CSFD: the Fractional Factorial Design

- Selecting an “unit” factorial design is the first step of CSFD.
- The design determines the resolution of the screening design.
- Will be sequentially implemented.
- Recommended designs available (Wu and Hamada 2000)

CSFD: Random Observations

- Generated in batches.
- Each batch is called one replication.
- One replication contains one observation at each design point.
- One replication provides one estimate for *each* desired effect.

CSFD: Performance

With any qualified hypothesis testing procedure, CSFD guarantee that for each individual effect *i*

$$\Pr\{\textit{Declare effect i important} \mid \beta_i \leqslant \Delta_0\} \leqslant a$$

$$\Pr\{\textit{Declare effect i important} \mid \beta_i \geqslant \Delta_1\} \geqslant \gamma$$

Empirical Evaluation

	Small Variance		*Large variance*			*Small Variance*		*Large variance*	
Effect	***CSB-X***	***CSFD***	***CSB-X***	***CSFD***	***Effect***	***CSB-X***	***CSFD***	***CSB-X***	***CSFD***
$\beta_1 = 2$	0.000	0.000	0.006	0.000	$\beta_9 = 4.64$	**0.000**	1.000	**0.000**	1.000
$\beta_2 = 2$	0.000	0.000	0.009	0.000	$\beta_{10} = 5$	1.000	1.000	1.000	1.000
$\beta_3 = 2$	**1.000**	0.000	**0.987**	0.000	$\beta_{12} = 1.75$	N/A	0.000	N/A	0.000
$\beta_4 = 2.44$	0.004	0.000	0.083	0.008	$\beta_{46} = -2.5$	N/A	0.000	N/A	0.005
$\beta_5 = 2.88$	0.324	0.000	0.373	0.273	$\beta_{58} = 3.9$	N/A	1.000	N/A	1.000
$\beta_6 = 3.22$	0.855	1.000	0.789	0.918	$\beta_{123} = 1.9$	N/A	0.000	N/A	0.000
$\beta_7 = 3.76$	0.985	1.000	0.971	1.000	$\beta_{789} = -4.5$	N/A	1.000	N/A	1.000
$\beta_8 = 4.2$	0.998	1.000	0.993	1.000	***# of runs***	**279**	**2048**	**11632**	**3515**

CSFD Limitation

- The sample size in each replication of CSFD increases exponentially as the number of factors increases.
- It also increases exponentially in terms of the resolution of the selected factorial design.
- Each replication repeat the whole unit design.

CSB vs. CSFD

- A complementary relationship

	Strength	*Weakness*
CSB	▪ Can classify a group of unimportant effects together ▪ More Efficient if the percentage of important effects is small ▪ Can handle relatively more factors	▪ Can not handle interactions ▪ Not efficient if the percentage of important factors is large ▪ Assume known directions
CSFD	▪ Can handle interactions ▪ Insensitive to the percentage of important factors ▪ Do not assume known directions	▪ Has to classify effects one by one ▪ Simulation efforts increase exponentially in terms of the number of factors

The Hybrid Method

- Apply CSFD to potentially "important" effects and/or important interactions
- Apply CSB to screen "unimportant" effects, positive ones and negative ones separately
- Would work well for experiments with large number of factors and little prior information
- Require an initial pre-screening

The Hybrid Method (cont'd)

- Phase 1: Run pre-screening test
  - To determine an proper response model
  - To estimate the directions of effects
  - To classify some effects if possible
- Phase 2: With the information obtained
  - Assign effects into three different queues: IMP, POS and NEG
  - Apply CSB on queues POS and NEG, and CSFD on queue IMP

Table 6: Simulation Runs Required for Cases with Main-Effects Model

Cases	Variance Factor	CSFD	FF-CSB	Hybrid
200-Factor	$m = 0.01$	512	392	336 (34.4%)
	$m = 0.1$	559	798	371 (33.6%)
	$m = 0.3$	792	2817	475 (40.0%)
	$m = 1.0$	1807	22523	1475 (18.4%)
500-Factor	$m = 0.01$	1024	1379	656 (35.9%)
	$m = 0.1$	1304	4500	734 (43.7%)
	$m = 0.3$	1678	26986	998 (40.5%)
	$m = 1.0$	4169	275101	4064 (2.5%)

Research Issues

- Collaboration of three screening procedures: pre-screening, CSB and CSFD
  - How to assign effects into queues to maximum the efficiency. For example, a possible rule is
    - Assigning all factors with estimated effect coefficient greater than $\Delta/2$ to queue IMP.
    - Of the rest effects, assigning those with positive signs to queue POS, those with negative signs to queue NEG.

Research Issues (cont'd)

- Error control of the hybrid method
  - Phase 1 pre-screening is not supposed to accurately estimate β so the error is not controlled and will pass to the next step.

  A bound possible?
- Efficient pre-screening in large-scale cases
  - A saturate fractional factorial design
  - Space-filling designs like Latin Hypercube Sampling.
- Efficient screening of interactions

Table 9: Mis-Assignments of Important Factors with $m = 1.0$ (Main-Effects Mode

Cases	Prescreening Sample Size	Factors	
		Critical	Important but not Critical
200-factor	1	0.002 (0.04%)	0.088 (1.76%)
	2	0.000 (0.00%)	0.013 (0.26%)
	3	0.000 (0.00%)	0.002 (0.04%)
500-factor	1	0.005 (0.04%)	0.395 (3.29%)
	2	0.000 (0.00%)	0.099 (0.83%)
	3	0.000 (0.00%)	0.026 (0.22%)

Thanks!

Questions?