Application of Matroid Theory to Reliability Study of Coordinate Sensing Systems for Automatic Fault Diagnosis

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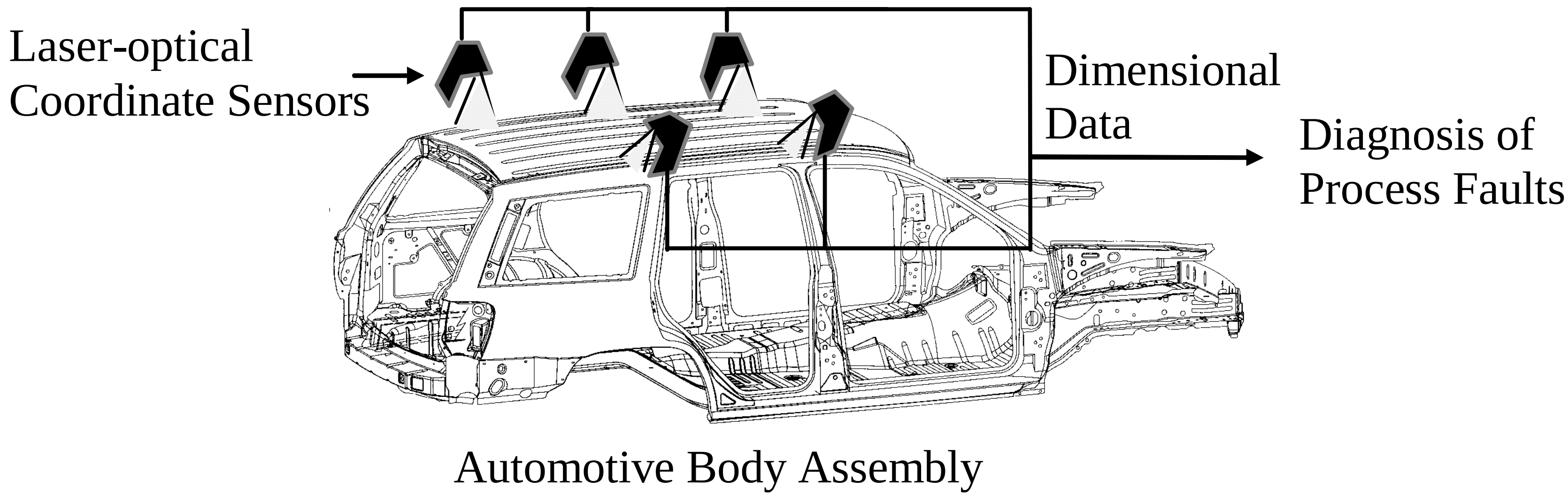
Example of a Coordinate Sensing System:

Optimal Coordinate Measurement Machine for Automotive Body Assembly



Recent Research Development for Coordinate Sensing Systems

- Automatic fault diagnosis
  - Single stage system: Ceglarek and Shi (1996), Apley and Shi (1998)
  - Multistage system: Jin and Shi (1999), Ding et al. (2002), Zhou et al. (2004)
- Sensing system design:
  - Single stage system: Khan et al. (1999),
  - Multistage system: Khan and Ceglarek (2000), Ding et al. (2003)
- Sensor failure has not been considered
- It is important to evaluate the risk resulted from sensor failures

Objective

To evaluate the reliability of a coordinate sensing system to maintain a desired diagnosability for the process faults of interests

Fault-Measurement Model

$$\mathbf{y} = \mathbf{\Gamma u} + \mathbf{v}$$

$\mathbf{\Gamma}$: Determined by kinematics analysis based on product and process design information (Jin and Shi, 1999; Zhou et al., 2003)

$\mathbf{v}$: Additive sensor noise

$\mathbf{y} \in \Re^{n \times 1}$ —— Measurements from *n* coordinate sensors

$\mathbf{u} \in \Re^{P \times 1}$ —— *P* unknown process faults

This model has been used for both single stage and multistage manufacturing processes

Diagnosability Condition

- We focus on mean faults, i.e., process faults manifested by mean shifts of $\mathbf{u}_i$
- Use the rank of $\mathbf{\Gamma}$ as the diagnosability index
- If the rank of $\mathbf{\Gamma}$ is k, P-k faults need to be known to identify each individual process fault (Zhou et al., 2003)
- Using the diagnosis algorithm by Apley (1998), the diagnosability condition for variance faults is the same as that for the mean faults

Failures of Individual Sensors and the Sensing System

- Each sensor has two possible states: working or failed. A failed sensor cannot provide any information on process faults.
- The reliability of sensor i is p_i, i=1, ..., n
- The coordinate sensing system is working if the matrix $\mathbf{\Gamma}$ with rows corresponding to the failed sensors removed maintains its original rank.

Introduction to Matroid

- consider the following axiom regarding a finite ground set *E* and a collection, *I*, of subsets of *E:*
  - $\emptyset \in I$
  - $H_1 \subseteq H_2 \in I \Rightarrow H_1 \in I$
  - *for every* $H \subseteq E$*, if* $H_1, H_2 \subseteq H$ *are maximal over subsets of H in I, then* $|H_1| = |H_2|$
- The pair (*E*, *I*) is called a *matroid* if it satisfy the above conditions.
- Members in *I* are called *independent subsets*.
- A maximal independent subset is a *base*.
- A minimal dependent subset is a *circuit*.
- Matroid theory was mainly applied in combinatorial optimization. Also applied for electrical systems and static analysis.

A Matroid Structure for Reliability of Coordinate Sensing Systems

For reliability of a coordinate sensing system,

- A minimal path is a base of $V(\mathbf{\Gamma}^T)$, the vectorial matroid defined on $\mathbf{\Gamma}^T$, whose independent subsets are linearly independent subsets of the columns of $\mathbf{\Gamma}^T$
- A minimal cut is a cocircuit of $V(\mathbf{\Gamma}^T)$, or the circuit of the dual matroid of $V(\mathbf{\Gamma}^T)$

Properties of Reliability Structure Based on Matroid Theory

- All minimal paths have the same cardinality, $k = rank(\mathbf{\Gamma})$. (but in general **not** a *k*-out-of-*n*:G system since **not** every subset of cardinality *k* is a minimal path)
- The subset of nonzero elements of the reduced row echelon form of $\mathbf{\Gamma}^T$, denoted as RREF($\mathbf{\Gamma}^T$), is corresponding to a minimal cut.

Evaluation of System Reliability

- In general, reliability computation for a coordinate sensing system is an NP-Complete problem
- If no minimal cut has cardinality larger than 2, the reliability of the coordinate sensing system is the same as the reliability of a system consisting of a number of subsystems connected in series with each subsystem to be either a single component or a 2-out-of-n_i:F system.

Enumeration of All Minimal Cuts

- Equivalent to enumerate all circuits of the dual matroid of $V(\mathbf{\Gamma}^T)$
- An algorithm is proposed by Boros et al. (2003) to enumerate circuits of any matroid
- An efficient implementation procedure of the algorithm by Boros et al. (2003) is developed to enumerate all minimal cuts based on evaluation of the reduced row echelon form of $\mathbf{\Gamma}^T$ with proper column permutations
- Computation time is polynomial of the number of minimal cuts and n

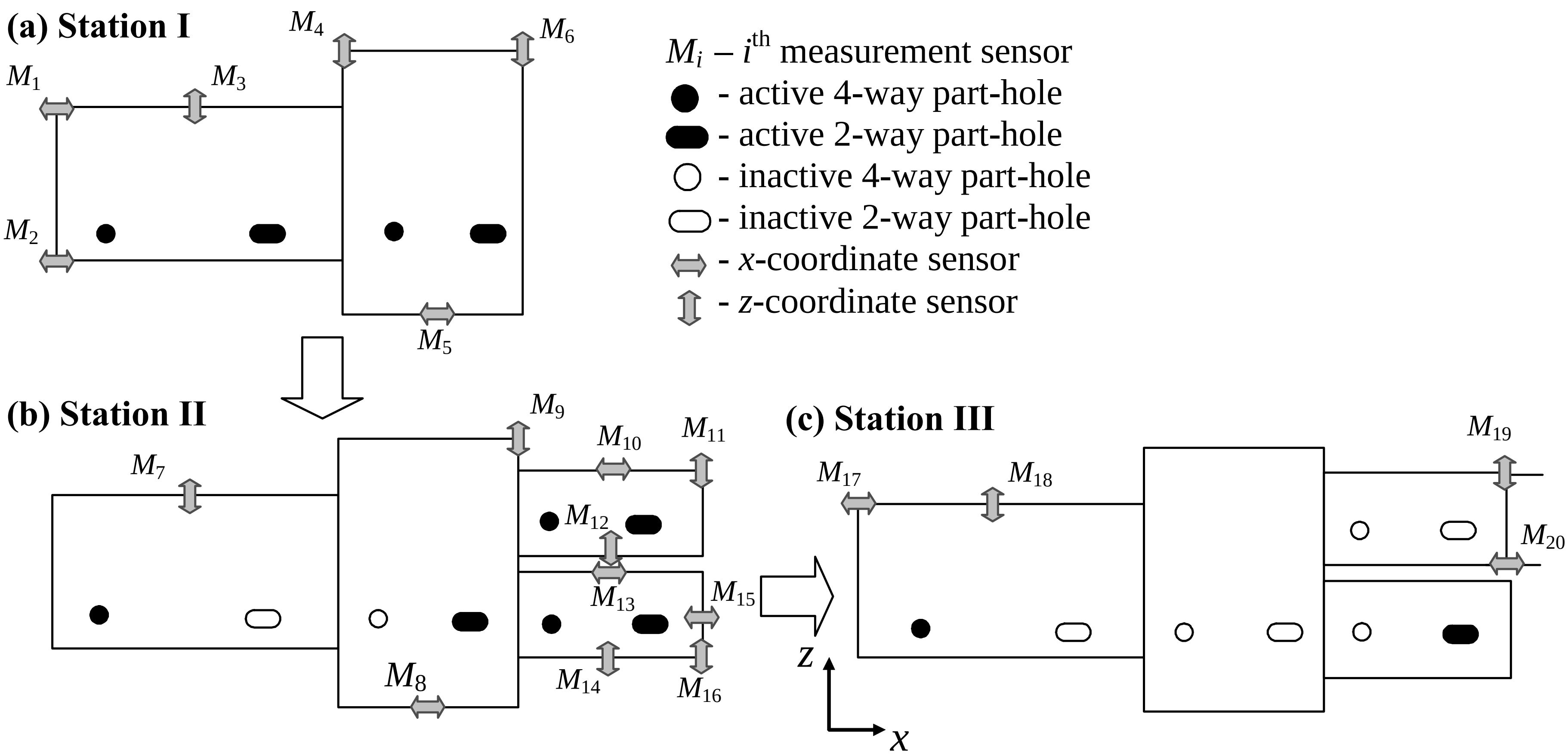
Efficient Calculation of a Lower Bound of System Reliability with *s*-Dependent Sensor Failures

- Assuming only associations among sensor failures, a lower bound of the sensing system reliability is

$$\max_{1 \le i \le r} \prod_{j \in MP_i} p_j$$

- Evaluation of this bound requires searching for the minimal path with the maximum product of component reliability
- Based on matroid theory, a greedy algorithm exists to find such a minimal path and thus evaluate the above lower bound easily

Case Study—Three Station Automotive Body Assembly



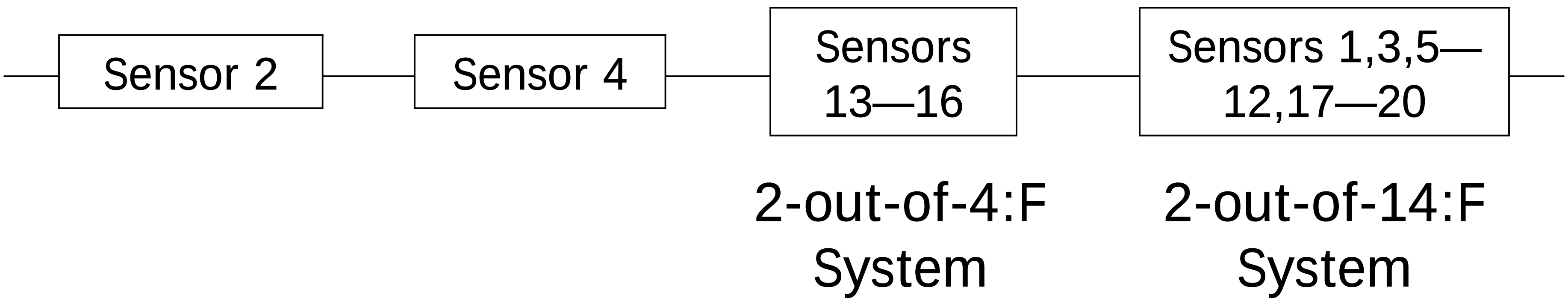
20 coordinate sensors; 18 potential process faults at the locators

Sensor Reliability

Sensors	1,2,3	4,5,6	7	8,9	10,11,12	13—16	17,18	19,20
p_i	0.99	0.95	0.98	0.97	0.96	0.95	0.98	0.98

To Identify All 18 Faults

- To identify all 18 process faults, 99 minimal cuts are obtained. The largest cardinality of any minimal cut is 2. The system structure is as follows:



- Assume sensor failures are *s*-independent, the system reliability can be easily evaluated as equal to 0.873

To Identify Faults in Station 1 Only

- Only fault 1—fault 6, i.e., the faults in station 1, are concerned.
- 11 sensors are relevant to diagnosis of these faults
- 81 minimal cuts are obtained with cardinality ranging from 2 to 6
- Lower bound of reliability, with possible *s*-dependency of sensor failures, is obtained using the greedy algorithm as equal to 0.895, which is corresponding to the following minimal path with a maximum product of component reliability: (M_1, M_2, M_3, M_8, M_9, M_{17})

Summary

- A reliability model for coordinate sensing systems is proposed considering catastrophic sensor failures
- A matroid structure for the reliability of coordinate sensing systems is discovered
- Matroid theory is utilized to reveal useful properties of reliability structure of coordinate sensing systems
- Algorithms are developed to efficiently enumerate minimal cuts of a coordinate sensing systems, evaluate system reliability exactly under special conditions, and evaluate reliability bound