# Bayesian Spatial Model for Form Error Assessment Using Coordinate Measurements

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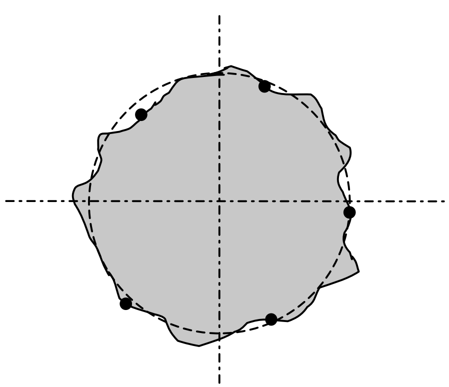
# Motivation

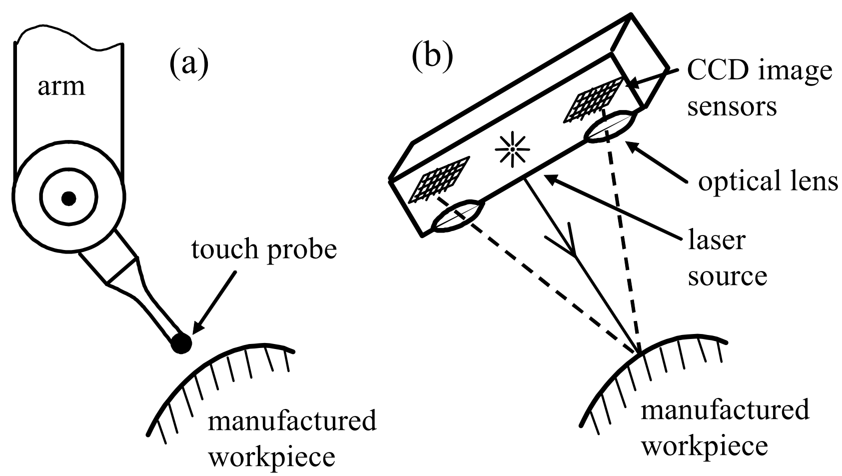
- Form error assessment
- Coordinate Measuring Machine (CMM) properties
  - CMM with mechanical touch probe: high-resolution but low-efficiency
  - CMM with optical scanner: low-resolution but high-efficiency
- How about using both machines to assess the same part?



Roundness form error

Actual part surface

Fitted nominal surface




Two different probes of CMMs'

Sources of figure: Liu, Q., Ding, Y., and Chen Y.,(2005) ,*IIE Transactions*, 37, 877-889.

# Motivated Research Work

**Bayesian Hierarchical Model for Integrating Multi-resolution Data**

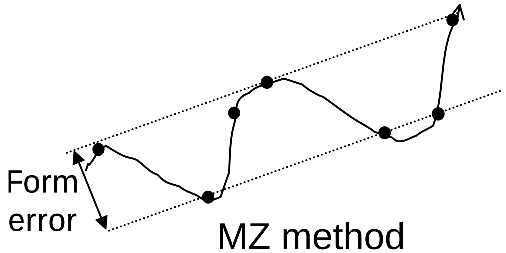
- Model for single-resolution data
- Linkage Model to connect multi-resolution data

- Low-resolution data model + linkage model
- High-resolution data model + linkage model

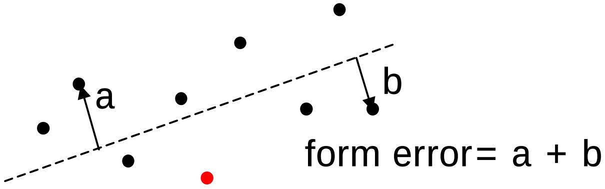
Bayesian spatial model for form error assessment using coordinate measurements

# Traditional Methods

- Minimum zone (MZ) method
- Orthogonal least squares (OLS) method
- Variants of Minimum zone and OLS methods




MZ method




OLS method

- Usually underestimate the form errors
- Do not consider the estimation uncertainty
- Not account for systematic form error

Dowling, Griffin, Tsui and Zhou, 1997, "Statistics Issues in Geometric Feature Inspection Using Coordinate Measuring Machines," Technometrics, Vol. 39, 3-17.

# Inspirited Spatial Model by Computer Experiments (CE)

- Analogy between CE and CMM measuring

Inputs $\mathbf{n}_i$ → Computer Simulation → Responses $q_i$

meta-model: $$q_i = \mathbf{f}(\mathbf{n}_i)^T \boldsymbol{\beta} + \eta + \varepsilon$$

Inputs: points to measure $\mathbf{n}_i$ → Manufactured parts and CMM measuring → Respnses: CMM Measurement $\mathbf{q}_i$

CMM data model: $$\mathbf{q}_i = \mathbf{f}(\mathbf{n}_i, \boldsymbol{\beta}) + \boldsymbol{\eta} + \boldsymbol{\varepsilon}$$

- $\boldsymbol{\eta}$ spatially-correlated systematic error; $\boldsymbol{\varepsilon}$ random error
- In computer experiment, true response surface unknown and approximated with linear regression model $\mathbf{f}(\mathbf{n}_i)^T \boldsymbol{\beta}$
- Generally in CMM data model, $\mathbf{f}(.,.)$ known from geometry design and usually nonlinear function

# Existing work on Modeling Systematic Form Error

- Sequentially added deterministic components
  - Polynomial components (e.g. Yeh, Ni and Hu 1994)
  - Fourier components (e.g. Desta, Feng and OuYang 2003)
  - B-spline components (e.g.Yang and Menq 1993)

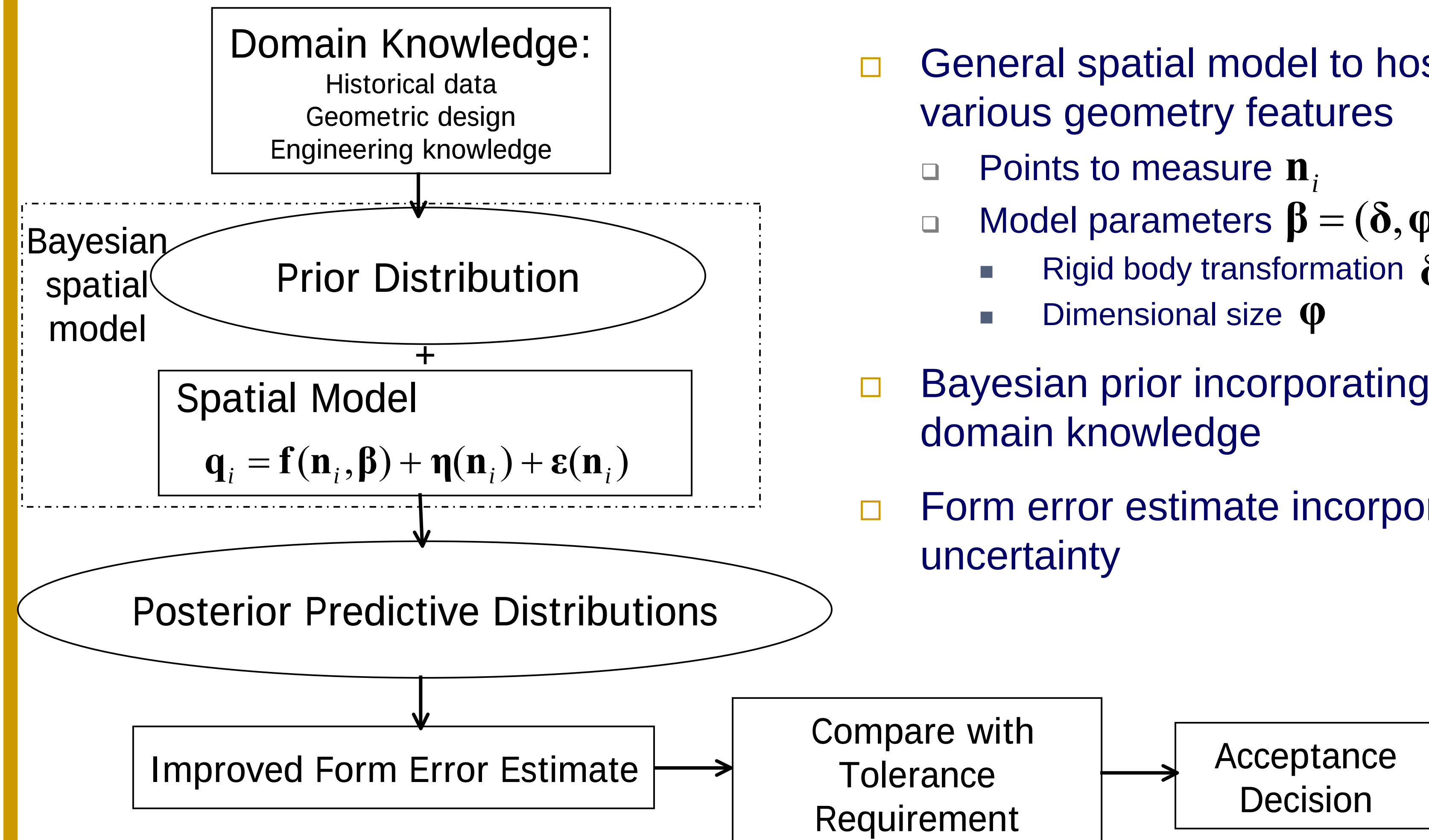
- Stochastic modeling with spatially correlated component
  - Yang and Jackman 2000;Dowling, Griffin and Zhou 1993

    $z_i = f(x_i, y_i) + \eta(x_i, y_i) + \varepsilon(x_i, y_i)$ solve with non-bayesian kriging

  - Proposed Bayesian spatial model

    $(x_i, y_i, z_i)^T = \mathbf{f}(\mathbf{n}_i, \boldsymbol{\beta}) + \boldsymbol{\eta}(\mathbf{n}_i) + \boldsymbol{\varepsilon}(\mathbf{n}_i)$ solve with Bayesian method

# Proposed Method




- General spatial model to host various geometry features
  - Points to measure $\mathbf{n}_i$
  - Model parameters $\boldsymbol{\beta} = (\boldsymbol{\delta}, \boldsymbol{\varphi})$
    - Rigid body transformation $\boldsymbol{\delta}$
    - Dimensional size $\boldsymbol{\varphi}$
- Bayesian prior incorporating domain knowledge
- Form error estimate incorporated uncertainty

# Spatial Model for Form Error

- Multi-variate spatial Model

$$\mathbf{q}_i = (x_i, y_i, z_i)^T = \mathbf{f}(\mathbf{n}_i, \boldsymbol{\delta}, \boldsymbol{\varphi}) + \boldsymbol{\eta}(\mathbf{n}_i) + \boldsymbol{\varepsilon}(\mathbf{n}_i)$$

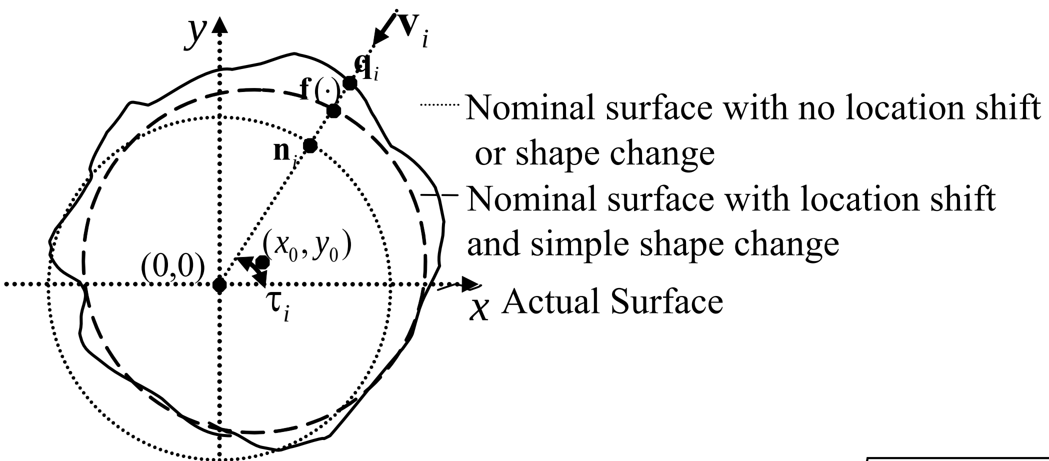
- Hulting (1997) : project $\mathbf{q}_i$ along the approaching direction of CMM probe, denoted by $\mathbf{v}_i$.

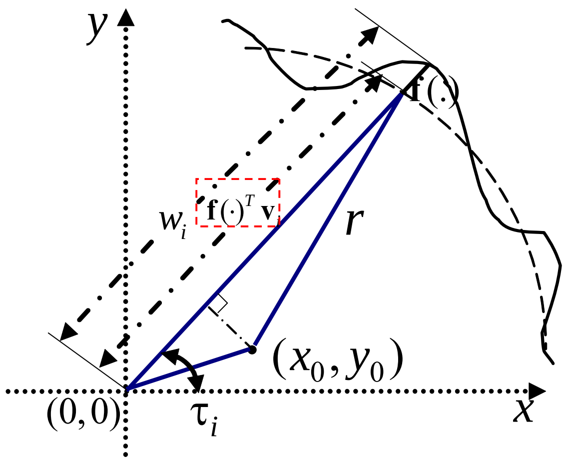
- Univariate spatial model

$$z_i = \mathbf{q}_i^T \mathbf{v}_i = \mathbf{f}(\mathbf{n}_i, \boldsymbol{\delta}, \boldsymbol{\varphi})^T \mathbf{v}_i + \eta(\mathbf{n}_i) + \varepsilon(\mathbf{n}_i)$$

# Determine $\mathbf{f}(\cdot)^T \mathbf{v}_i$ : roundness feature

- Decide approach direction $\mathbf{v}_i$ and the approaching line
- Solve equation systems for the intersection point of the approaching line and the nominal surface with location shift and simple size change



$$-x_i \cos\tau_i - y_i \sin\tau_i = -x_0 \cos\tau_i - y_0 \sin\tau_i - \sqrt{r^2 - (y_0 \cos\tau_i - x_0 \sin\tau_i)^2} + \eta(\mathbf{n}_i) + \varepsilon(\mathbf{n}_i)$$


Xia, Ding and Wang, "Bayesian Spatial Model for Form Error Assessment Using Coordinate Measurements ", submitted to Technometrics.
Hulting, 1997, "Discussion: Statistics Issues in Geometric Feature Inspection Using Coordinate Measuring Machines," Technometrics, Vol. 39, 18-20.

# Noise Structure

- Random errors $\varepsilon(\cdot) \sim \text{white noise}(0, \sigma_\varepsilon^2)$
- Systematic errors $\eta(\cdot) \sim \text{Gaussian process}(0, \sigma_\eta^2 \rho_\upsilon)$

$$\text{cov}(\eta(\mathbf{n}_i), \eta(\mathbf{n}_j)) = \sigma_\eta^2 \rho_{\boldsymbol{\upsilon}}(\| \mathbf{n}_i - \mathbf{n}_j \|)$$

$$\rho_{\boldsymbol{\upsilon}}(\mathbf{n}_i, \mathbf{n}_j) = \exp\{-(\upsilon_1 \| \mathbf{n}_i - \mathbf{n}_j \|)^{\upsilon_2}\}$$

smoothness parameter

scale parameter

- $\upsilon_1$ control the correlation decay rate when distance increase
- $\upsilon_2$ decide the differentiability of sample path; fix $\upsilon_2$=1 for this application

Thomas, Best, Lunn, Arnold and Spiegelhalter (2003), *Winbugs User Manual*, Version 1.4, http://www.mrc-bsu.cam.uk/bugs

# Choosing Prior Distributions

$$p(\boldsymbol{\delta},\boldsymbol{\varphi},\sigma_\eta^2,\sigma_\varepsilon^2,\upsilon_1) = p(\boldsymbol{\delta},\boldsymbol{\varphi})\,p(\sigma_\eta^2)\,p(\sigma_\varepsilon^2)\,p(\upsilon_1)$$

where

$(\boldsymbol{\delta},\boldsymbol{\varphi}) \sim Normal(\boldsymbol{\mu},\mathbf{P})$ rigid body transformation and dimensional parameters

$\sigma_\eta^2 \sim Unif(0,c)$ variance of systematic errors

$\sigma_\varepsilon^2 \sim Unif(0,c)$ variance of random errors

$\upsilon_1 \sim Unif(a_\upsilon,b_\upsilon)$ scale parameter for correlation function

- A simple example of using domain knowledge to build prior distributions

Domain Knowledge ⇨ Typical tolerance limits for turning: $\pm 0.05mm$ ⇨ $\sigma = 0.05/3$ ⇨ $c = (2\sigma)^2 = 0.001$

# Bayesian Inference

- Posterior distribution

$$p(\boldsymbol{\delta},\boldsymbol{\varphi},\sigma_\eta^2,\sigma_\varepsilon^2,\upsilon_1 \mid \mathbf{z})$$
$$\propto p(\boldsymbol{\delta},\boldsymbol{\varphi},\sigma_\eta^2,\sigma_\varepsilon^2,\upsilon_1) \mid \mathbf{V} \mid^{-1/2} \exp\{-\frac{1}{2}(\mathbf{z}-\mathbf{g}(\boldsymbol{\delta},\boldsymbol{\varphi}))^T \mathbf{V}^{-1}(\mathbf{z}-\mathbf{g}(\boldsymbol{\delta},\boldsymbol{\varphi}))\}$$

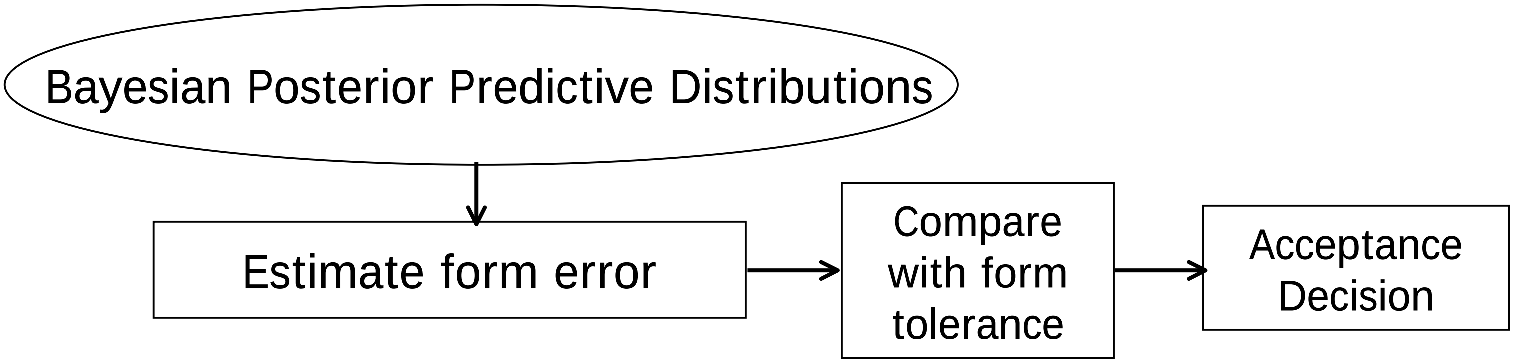
where $\mathbf{z}=(w_1,\ldots,w_n)$, $\mathbf{V}=\sigma_\eta^2\boldsymbol{\rho}_{\upsilon_1}+\sigma_\varepsilon^2\mathbf{I}$, $\mathbf{g}(\boldsymbol{\delta},\boldsymbol{\varphi})=(\mathbf{f}(\mathbf{n}_i,\mathbf{v}_i,\boldsymbol{\delta},\boldsymbol{\varphi})^T\,\mathbf{v}_i)$

- Posterior predictive distribution

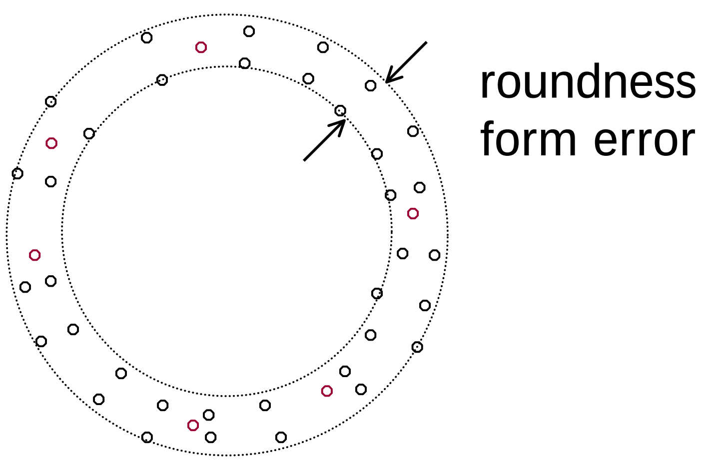
$$p(z(\mathbf{n}_0)\mid\mathbf{z}) =$$
$$\int_{\boldsymbol{\delta},\boldsymbol{\varphi},\sigma_\eta^2,\sigma_\varepsilon^2,\upsilon_1} p(z(\mathbf{n}_0)\mid\mathbf{z},\boldsymbol{\delta},\boldsymbol{\varphi},\sigma_\eta^2,\sigma_\varepsilon^2,\upsilon_1)p(\boldsymbol{\delta},\boldsymbol{\varphi},\sigma_\eta^2,\sigma_\varepsilon^2,\upsilon_1\mid\mathbf{z})d\boldsymbol{\delta}\,\mathrm{d}\boldsymbol{\varphi}\,d\sigma_\eta^2\,d\sigma_\varepsilon^2\,d\upsilon_1$$

  - $\mathbf{n}_0$ is the location to predict.
  - Markov Chain Monte Carlo algorithm for solutions

# Probabilistic Decision

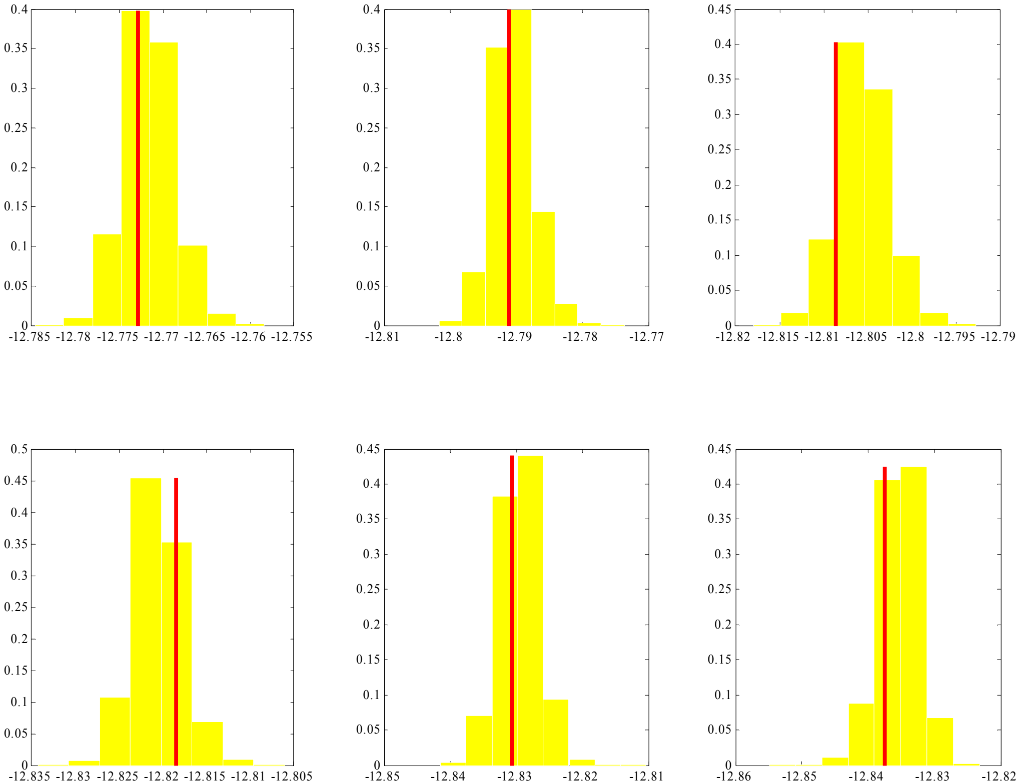



- Example: roundness form error assessment



# Model Verification:
## Posterior Predictive Checking

- Test model's consistency with data
- Use all other data to predict "left-out" data
- good fit if the data looks plausible in the posterior predictive distribution from the model



**Posterior Predictive Checking for Roundness Feature (red true data vs. yellow prediction histogram)**

Gelman, Carlin, Stein and Rubin (2003), Bayesian Data Analysis, 2nd edition, Chapman &Hall/CRC, Boca Raton, Florida

# Comparison with Traditional Methods

- Compare with MZ method and OLS method
  - using both simulated CMM data and real CMM data
- Comparison scenarios for simulated data
  - Two features: straightness and roundness
  - Three process conditions for each feature
  - Sample size 8-40
  - Comparison criterion: form error estimate/true form error
- Comparison scenarios for real data
  - Two features: straightness and roundness
  - One process conditions for each feature
  - Sample size 30

# Simulated Data-Straightness Cases

- Generator equation for straightness features

$$y = \phi + \psi x - \frac{64}{L^6} R(x^3 (L - x)^3) + A\sin(\frac{2\pi}{\lambda} x) + \varepsilon$$

rigid body transformation | Surface deflection | systematic variation | Random noises

**Simulation scenarios for roundness features (unit: mm)**

| | Process characteristics | $\phi$ | $\psi$ | $A$ | $\lambda$ | $R$ | $\sigma_\varepsilon$ |
|---|---|---|---|---|---|---|---|
| Case I | Sine wave dominates (face milling) | .04 | .02 | .04 | 10 | .01 | .01 |
| Case II | Deflection dominates (turning) | .05 | .01 | .005 | 4 | .025 | .005 |
| Case III | Pure random errors (grinding) | .06 | -.01 | 0 | N/A | 0 | .002 |

Dowling, Griffin, Tsui and Zhou, 1995, "A Comparison of the Orthogonal Least Squares and Minimum Enclosing Methods for Form Error Estimation," Manufacturing Review, Vol. 8, 120-138.

# Simulated Data-Roundness Case

- Generator equation for roundness features

$$x = x_0 + (r + A\cos(2\pi/\lambda\tau) + \varepsilon)\cos\tau$$

$$y = y_0 + (r + A\cos(2\pi/\lambda\tau) + \varepsilon)\sin\tau$$

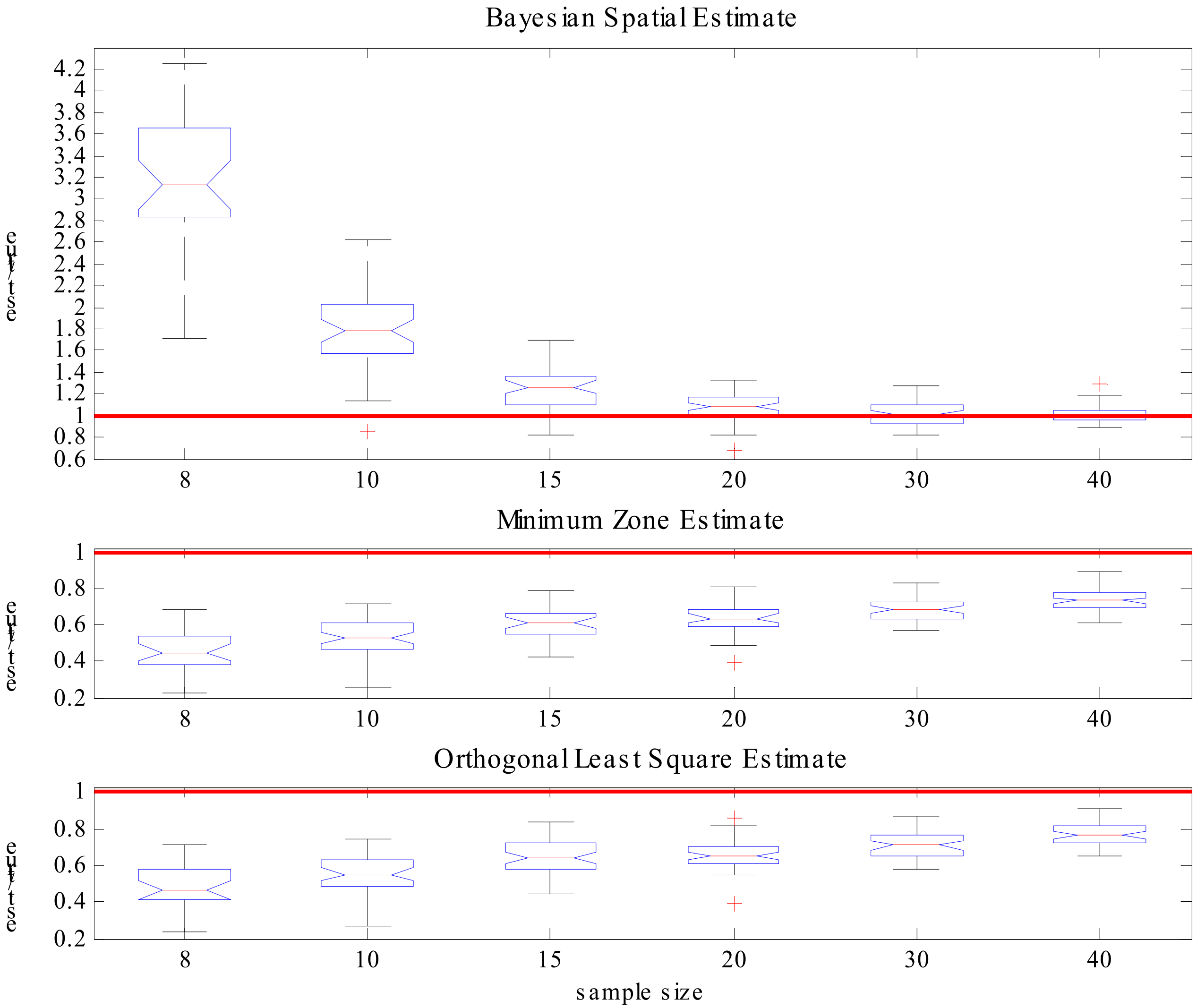
rigid body transformation | radius | systematic variation | random noises

**Simulation scenarios for roundness features (unit: mm)**

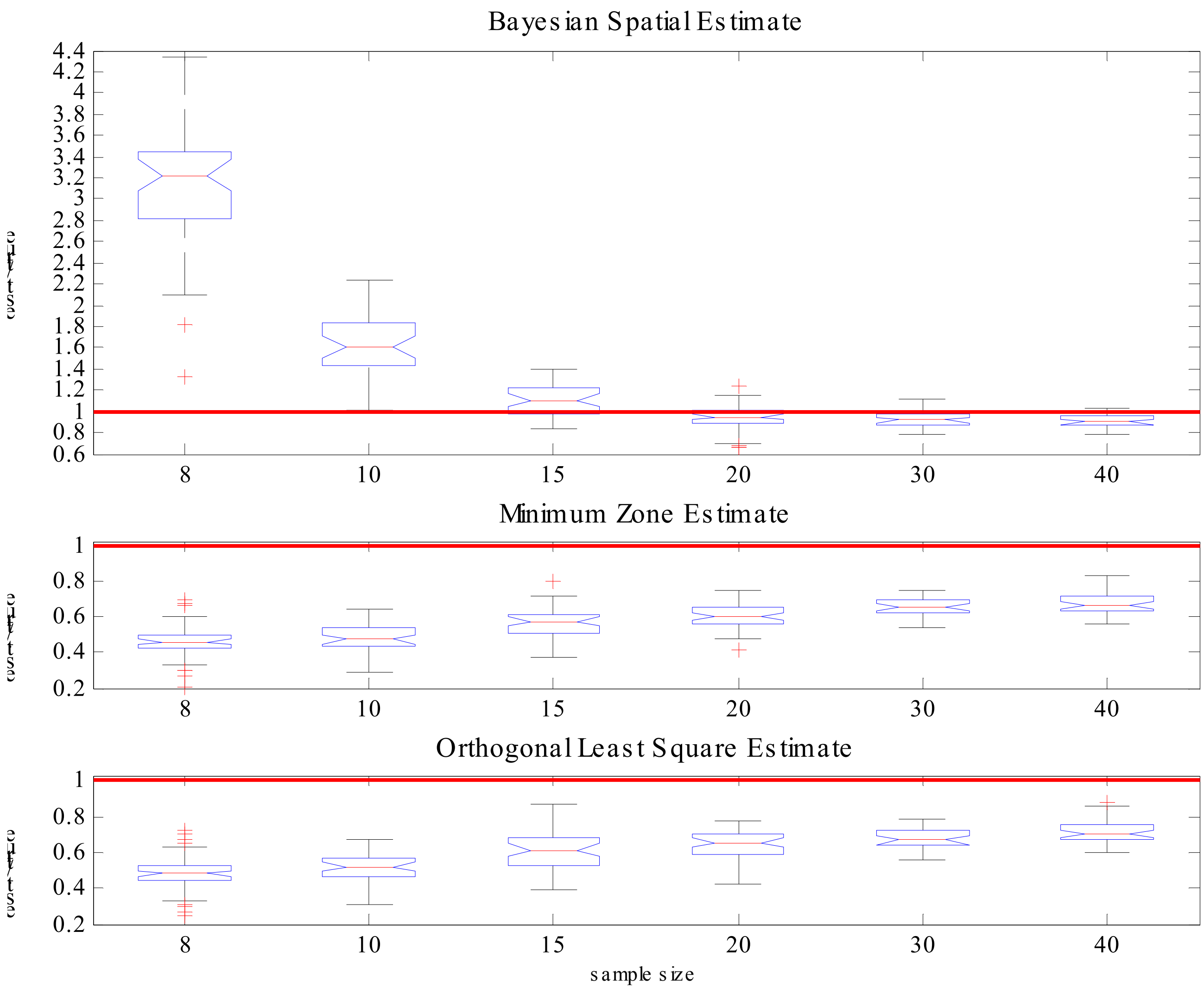
| | Process characteristics | $x_0$ | $y_0$ | $A$ | $\lambda$ | $r$ | $\sigma_\varepsilon$ |
|---|---|---|---|---|---|---|---|
| Case I | Systematic errors dominate (turning) | .2 | .02 | .03 | 2/3π | 25 | .006 |
| Case II | Systematic errors and radius change (turning) | .03 | .2 | .01 | 2/3π | 25.03 | .01 |
| Case III | Pure random errors (turning) | .2 | .02 | 0 | N/A | 25 | .017 |

Desta, Feng and OuYang, 2003, "Characterization of general systematic form errors for cicular features," International Journal of Machine Tools & Manufacture, Vol. 43, 1069-1078.

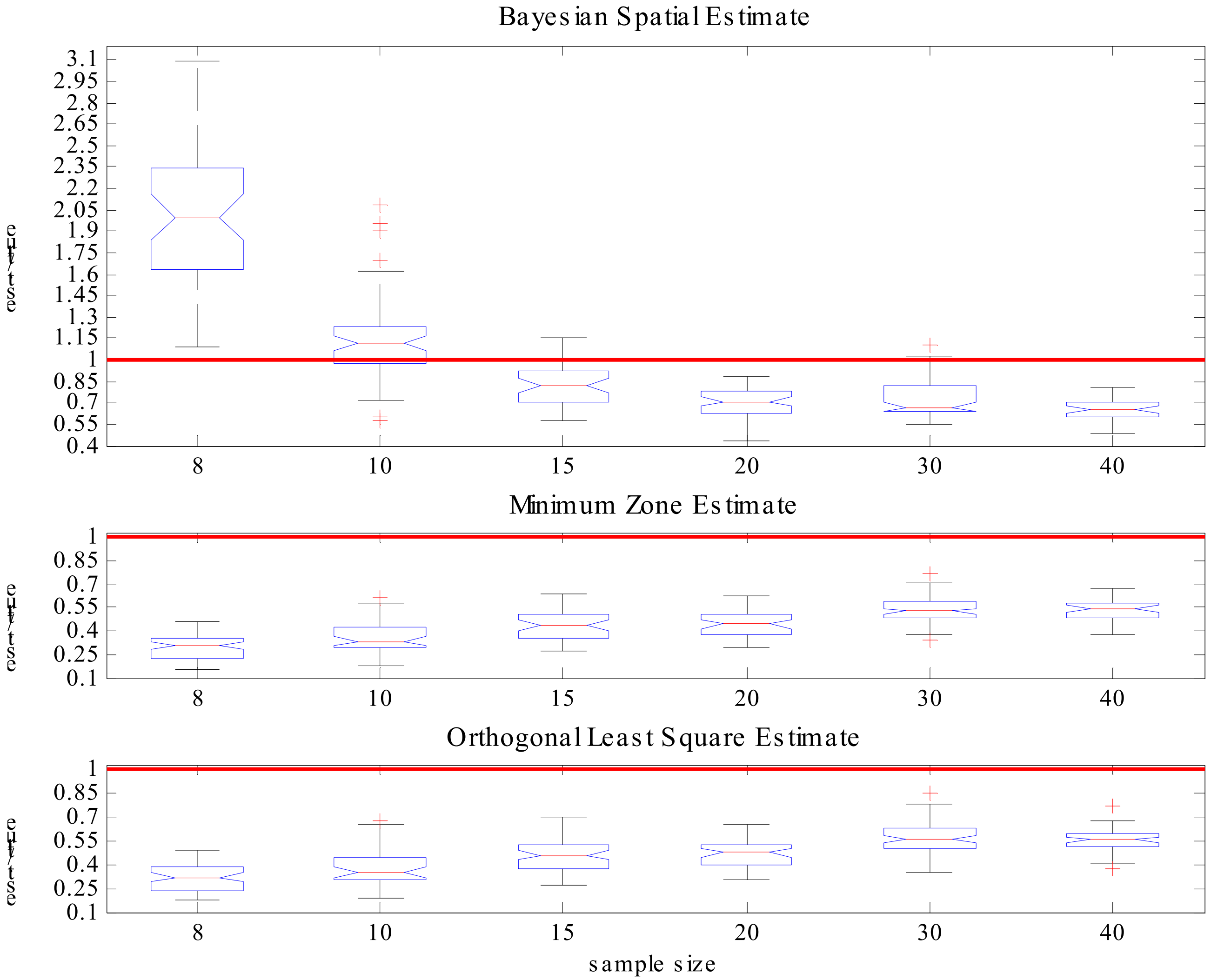
# Straightness Case I



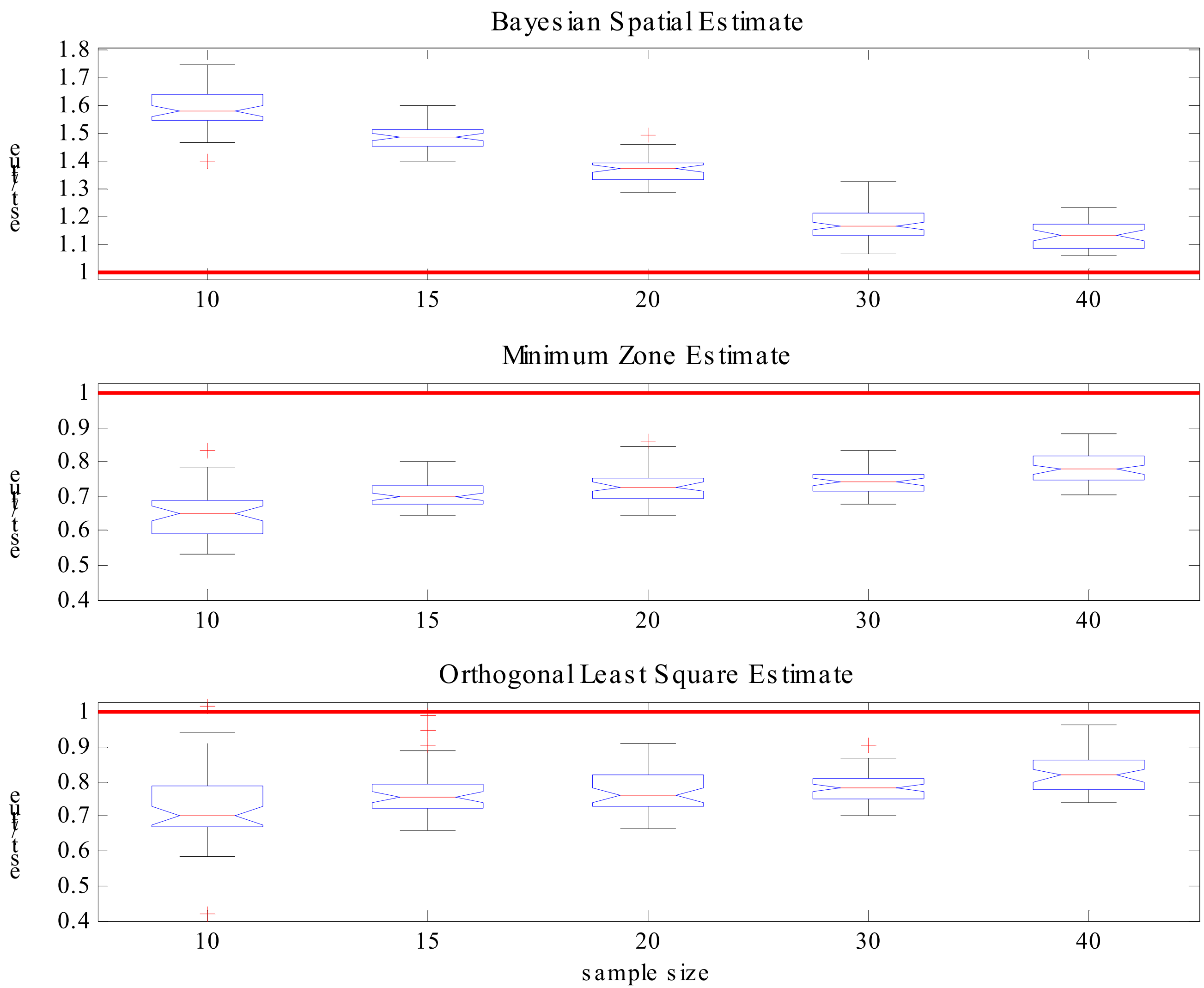
# Straightness Case II



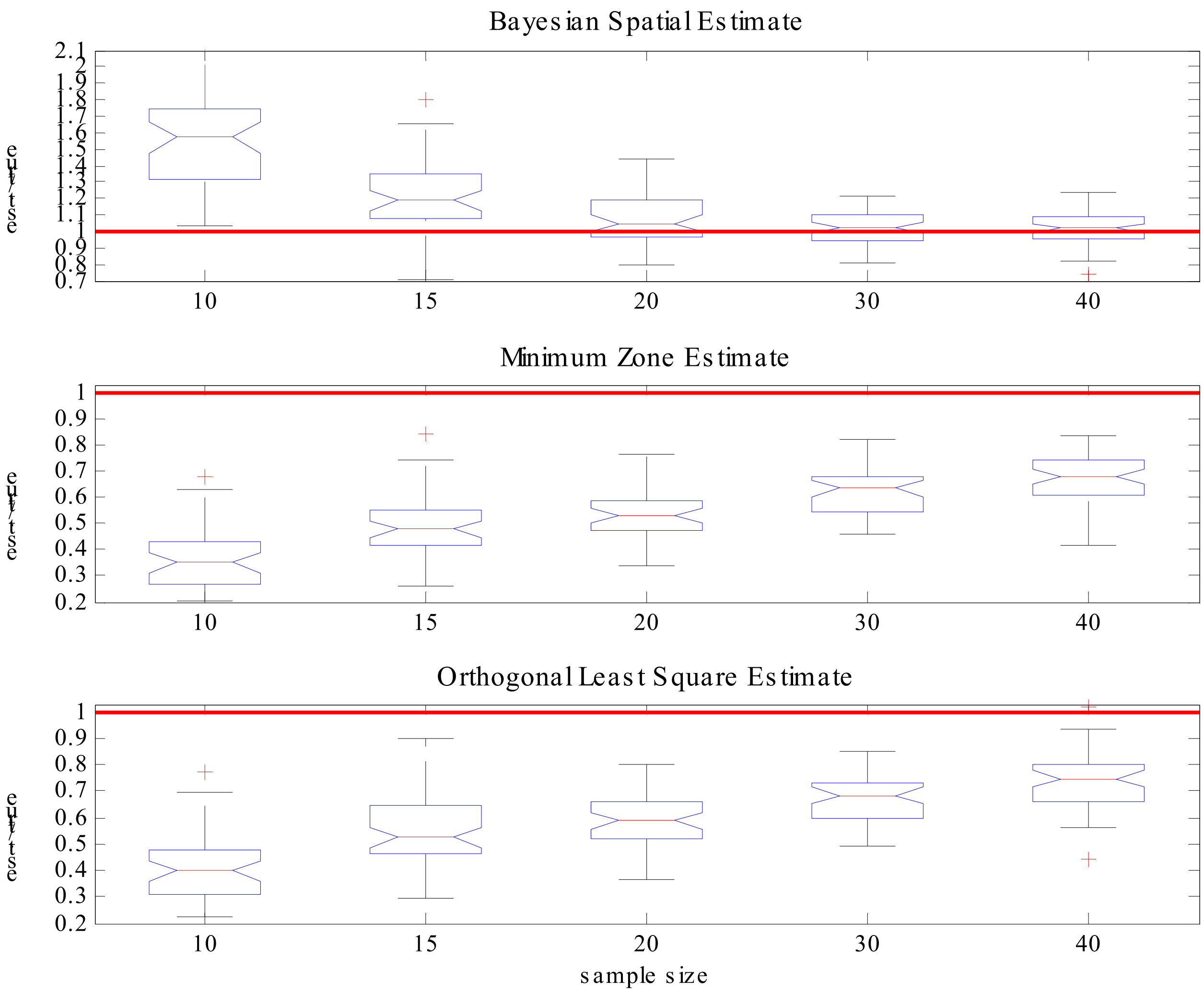
# Straightness Case III



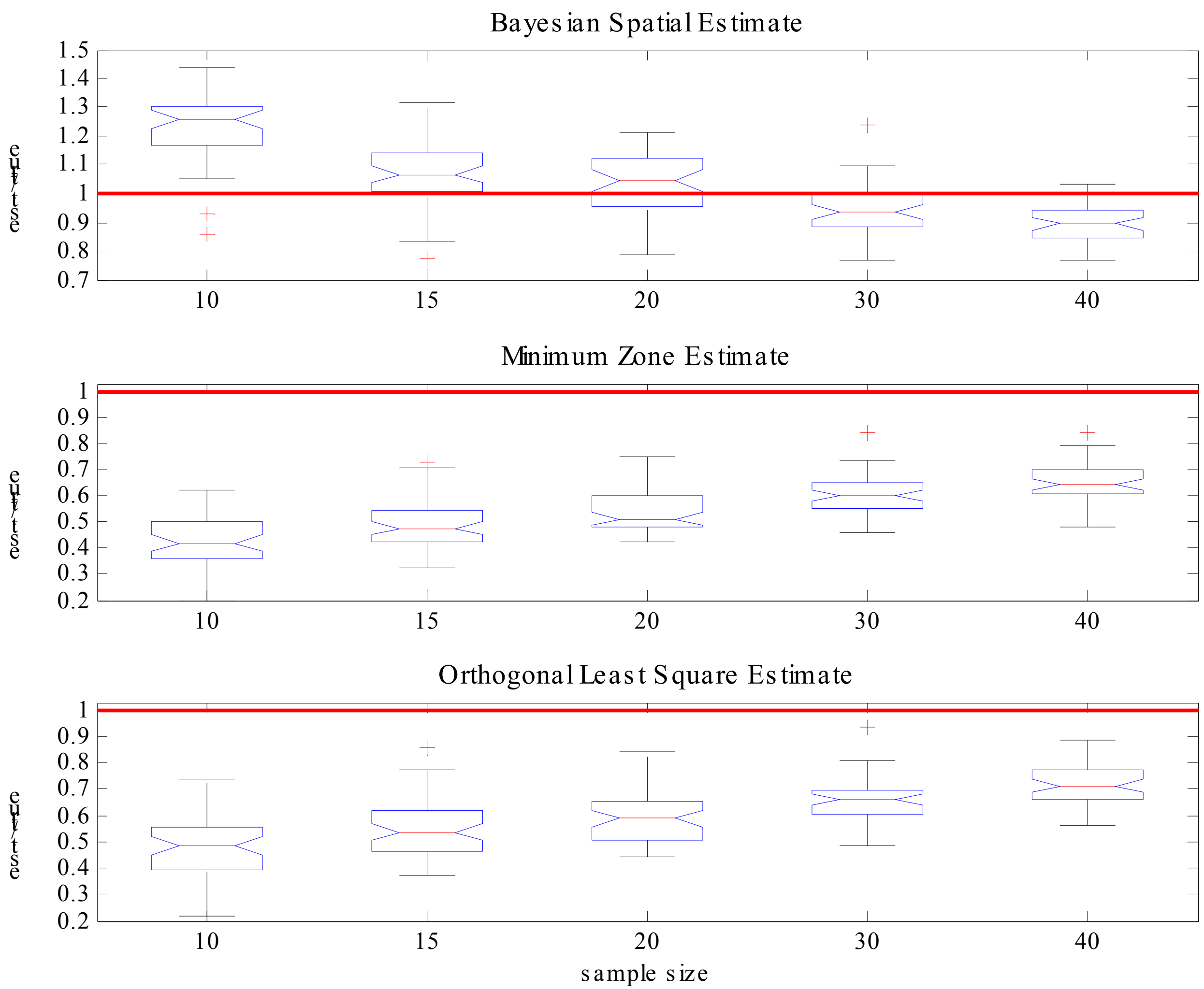
# Roundness Case I



# Roundness Case II



# Roundness Case III



# Real Data

Form error estimation for the straightness feature

| | $\hat{h}$ (mm) | $\hat{\sigma}_\varepsilon$ (mm) | $\hat{\phi}$ (mm) | $\hat{\psi}$ (mm) |
|---|---|---|---|---|
| Bayesian spatial method | .0482 | .0034 | .0140 | -.8866 |
| MZ method | .0364 | N/A | .0140 | -.8866 |
| OLS method | .0412 | .0099 | .0221 | -.8867 |

Form error estimation for the roundness feature

| | $\hat{h}$ (mm) | $\hat{\sigma}_\varepsilon$ (mm) | $\hat{r}$ (mm) | $\hat{x}_0$ (mm) | $\hat{y}_0$ (mm) |
|---|---|---|---|---|---|
| Bayesian spatial method | .0127 | .0016 | 12.7541 | -.00004 | .0025 |
| MZ method | .0094 | N/A | 12.7555 | .00040 | -.0035 |
| OLS method | .0109 | .0023 | 12.7540 | -.00004 | -.0026 |

# Conclusion and Future Work

- Developed Bayesian spatial model for form error assessment using coordinate measurement
- Demonstrated consistently improved form error estimate