# Experimental Designs for ADTs and Robust Product-Reliability Optimization

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# Introduction

- In searching the best combination of controllable manufacturing variables product reliability is as important as quality characteristic, especially for electronic or semiconductor devices.
- Accelerated degradation test (ADT) is needed to assess reliability of new devices.
- Typical experimental designs for accelerated life or degradation tests assume that products are from the same manufacturing condition.

# Research Scope

A. Experimental Design for Collecting Data to Estimate Model Parameters

Manufacturing Controllable Variables and Noise Factors

#Product-Replicates at Each Process Condition

Product Quality and Reliability Evaluation

Accelerated Degradation Reliability Test:
(1) #Stress Levels and Their Levels
(2) #Specimens at Each Stress
(3) Measurement Time for Each Specimen's Testing

Combined Quality and Reliability DOE

B. Robust Parameter Design

Meet Quality and Reliability Targets.

# Q-R Robust Parameter Design

This presentation proposes a framework of

1) designing accelerated degradation tests to minimize variance of percentile lifetime estimate, and
2) selecting manufacturing controllable variables that lead to
    a) longest product percentile lifetime,
    b) minimized variance of lifetime distribution,
    ( c) achieved quality requirements.)

# Literature Review

1. Tseng, S.-T., Hamada, M., and Chiao, C.-H. (1995), "Using Degradation Data to Improve Fluorescent Lamp Reliability," *Journal of Quality Technology.*

2. Joseph, V. R., and Yu, I-T. (2004), "Reliability Improvement Experiments with Degradation Data," *IEEE Transactions on Reliability.*

These papers studied quality and reliability improvement methods without considering data collection issues.

# Literature Review

3. Boulanger, M., and Escobar, L. A. (1994), "Experiment Design for a Class of Accelerated Degradation Testes," *Technometrics*

4. Yu, H., and Chiao, C.-H. (2002), "An Optimal Designed Degradation Experiment for Reliability Improvement," *IEEE Transactions on Reliability*

These papers explore experimental designs for collecting degradation data.

# Our Research

1. Quality variables include controllable manufacturing (*X*) and noise factors (*O*).

- Cross-array designs (Wu and Hamada (2000), page 445, *New York : J. Wiley.*) are used.
- Our focus is on the decision of number of replicates in each experimental run and combination of manufacturing variables leading to

  a) longest product percentile lifetime, and

  b) minimized variance of lifetime distribution.

# ADT Decision Variables

2. Products from different runs have distinct lifetime distributions. ADTs are used to collect data for estimating these distributions. In degradation testing, decision variables include

   a) stress levels,

   b) proportion of sample size for each stress level, and

   c) measurement time.

# Random-Coefficient Degradation Model

$$y_{i,j,s}(t) = \phi(\beta_{i,j,s}, \alpha_{i,j,s}, t) + \sigma_B B_t + e_t$$

- $y_{i,j,s}(t)$ is the degradation measurement at time *t.* The subscripts *i, j, s* are for experimental run, product replicate and stress, respectively.
- $\phi$ is a continuous monotone function. $\beta$ and $\alpha$ are random coefficients modeling product characteristics.
- $B_t$ is the Brownian motion representing product reliability's stochastic variations
- $e_t \sim N(0, \sigma_e^2)$ represents measurement error.

# Integrated Quality and Reliability Robust Parameter Design

- $X_1, \cdots, X_m$ are the manufacturing variables and $O_1, \cdots, O_p$ are the noise variables.
- The combined cross-array has *r* runs.
- Their level-combinations for the *i*th run are $C_{i1}, \cdots, C_{im}$ and $U_{i1}, \cdots, U_{ip}$, respectively.
- Let $t_{i,j,1}, \cdots, t_{i,j,l_s}$ be the measurement time for product *j* from run *i*, tested under stress level $s_i$ (*i* = 1, 2, 3).

# Notation

| | Sample Size for each stress level | Measurement time | Random Parameter | Distribution of random parameters |
|---|---|---|---|---|
| Run $i$<br><br>Replicate<br>Size= $N_{r,i}$ | $s_1$,<br>$N_{r,i} \times p_1$ | $t_{i,j,1}, \cdots t_{i,j,l_1}$ | $\alpha_{i,j,1},\ \beta_{i,j,1}$ | $N\left(\begin{pmatrix} \mu_{\alpha,i,1} \\ \mu_{\beta,i,1} \end{pmatrix}, \Sigma_i\right)$ |
| | $s_2$,<br>$N_{r,i} \times p_2$ | $t_{i,j,1}, \cdots t_{i,j,l_2}$ | $\alpha_{i,j,2}, \beta_{i,j,2}$ | $N\left(\begin{pmatrix} \mu_{\alpha,i,2} \\ \mu_{\beta,i,2} \end{pmatrix}, \Sigma_i\right)$ |
| | $s_3$,<br>$N_{r,i} \times p_3$ | $t_{i,j,1}, \cdots t_{i,j,l_3}$ | $\alpha_{i,j,3}, \beta_{i,j,3}$ | $N\left(\begin{pmatrix} \mu_{\alpha,i,3} \\ \mu_{\beta,i,3} \end{pmatrix}, \Sigma_i\right)$ |

# Two-Step Estimation Method

1. Obtain WLSEs $\hat{\alpha}_{i,j,s}, \hat{\beta}_{i,j,s}$ for each degradation path.
2. Explore functional relationships between WLSEs, manufacturing variables, noise factors and stress levels.
3. Extrapolate the modeled relationships to estimate the distribution parameters of the random coefficients under the use condition.

# Replication Size for Each Run

For each run in the cross-array obtain the variance of mean parameter estimate under use condition. Intuitively, the number of replications is proportional to the size of the variance. For example,

a) Let $Var(\hat{\mu}_{\alpha,i,0})$ be the variance of $\hat{\mu}_{\alpha,i,0}(X,O,S_0)$

b) Replication size for each run is

$$N_r \times \frac{Var(\hat{\mu}_{\alpha,i,0})}{\sum_{j=1}^{r} Var(\hat{\mu}_{\alpha,j,0})},$$

where $N_r$ is decided from economic considerations.

# Sample Size for Each Stress Level

To simplify illustration, consider a log-linear degradation function

$$\log y_{i,j,s}(t) = \log \beta_{i,j,s} + \alpha_{i,j,s} t + \sigma_B B_t + e_t$$

where $\begin{pmatrix} \alpha_{i,j,s} \\ \log \beta_{i,j,s} \end{pmatrix} \sim N\left( \begin{pmatrix} \mu_{\alpha,i,s} \\ \mu_{\beta,i,s} \end{pmatrix}, \Sigma_i \right)$ is independent of $B_t \sim N(0,t)$ and $e_t \sim N(0, \sigma_e^2)$.

# Steps for Deciding Sample Size

1. Obtain estimate of 10th percentile lifetime,

$$\hat{t}_{i,0.1}(\mu_{i,0},\Sigma_i)=\hat{t}_{i,0.1}(X,O,S_0,\Sigma_i)$$

2. Derive its variance,

$$Var(\hat{t}_{i,0.1})=\left(\frac{\partial h}{\partial \hat{t}_{i,0.1}}\right)^{-2}\times\frac{\partial h}{\partial \hat{\mu}'_{i,o}}\times Cov(\hat{\mu}_{i,0})\times\frac{\partial h}{\partial \hat{\mu}_{i,o}}=g(\mu_{i,0},\Sigma_i,D)$$

where $h(t_{i,0};\mu_{i,0},\Sigma_i)$ is the function for solving the percentile lifetime and *D* is the model matrix involving (*X*, *O*, *S*) in the two-step estimation.

# Minimized Variance of Estimating the Quadratic Coefficient

- Linear acceleration relationship implies that only two stress levels are needed.
- Add a middle stress level to check whether a quadratic relationship exists.
- Choose the proportion of sample size for each stress level by using a constrained optimization method:

a) Minimized variance of quadratic coefficient's estimate under the constraint that

b) Bounded increase of variance of percentile lifetime estimate from adding a middle stress level.

# Robust Parameter Design

1. Choose the level-combination of controllable manufacturing variables to maximize percentile lifetime, and

2. Choose the level-combination of other manufacturing variables (called adjustment factors) to minimize variance of lifetime distribution for making products less sensitive to noise factors.

3. Our approach: Variance ≈ 25%-tile – 10%-tile.

# Conclusion

Our research involves

a) experimental designs for data collection to explore model relationships and estimate model parameters, and

b) optimize the level-combination of manufacturing variables to achieve high product-reliability and low noise-sensitivity.

# Future Work

1) Consider robust parameter design problems for both *quality and reliability* performance variables.

2) Explore negotiation models to balance the quality and reliability requirements.

3) Research combined designs for collecting quality and reliability data.

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*Thank you!*