# Local Vote Decision Fusion for Target Detection in Wireless Sensor Networks.

Natallia Katenka

Joint work with Liza Levina and George Michailidis

Department of Statistics, University of Michigan - Ann Arbor

http://www.stat.lsa.umich.edu

# Introduction and Problem Formulation

# Wireless Sensor Network (WSN)

- Large number of small, inexpensive, low-power sensors
- Can be deployed in a regular pattern (e.g. grid) or at random (“smart dust”)
- Sensors communicate with other sensors and remote center
- Sensors collect information about the surrounding environment
- Sensors have some data processing capabilities
- Fusion center processes data from sensors and makes a global (and more precise) situational assessment

# Applications of WSN

- Environmental monitoring (endangered species, agriculture)
- Security (surveillance, intruder detection)

**Constraints**

- Low power: minimize data processing and communications costs
- Communication to center expensive ⇒ distributed algorithms

**Major design issues**

- Coverage ⇒ theory of coverage processes
- Connectivity ⇒ random graphs theory
- Localization (use of GPS, anchor nodes) ⇒ MDS, etc.
- Reliability

# Target Detection

- Detect and characterize targets in the monitored area
- Assume communication issues are resolved separately (connectivity, transmission protocols, etc)
- Assume sensor locations are known or can be estimated accurately
- Here focus on detection of a single target

# Problem Formulation

- N sensors deployed over monitored region R (WLOG unit square)
- A target at location $v$ emits signal with amplitude $S_0$
- Sensor $i$ located at $s_i$ gets energy readings

$$E_i = S_i + \epsilon_i = S_0 c(\|v - s_i\|, \eta) + \epsilon_i,$$

where $c(\cdot)$ is a monotone signal decay function

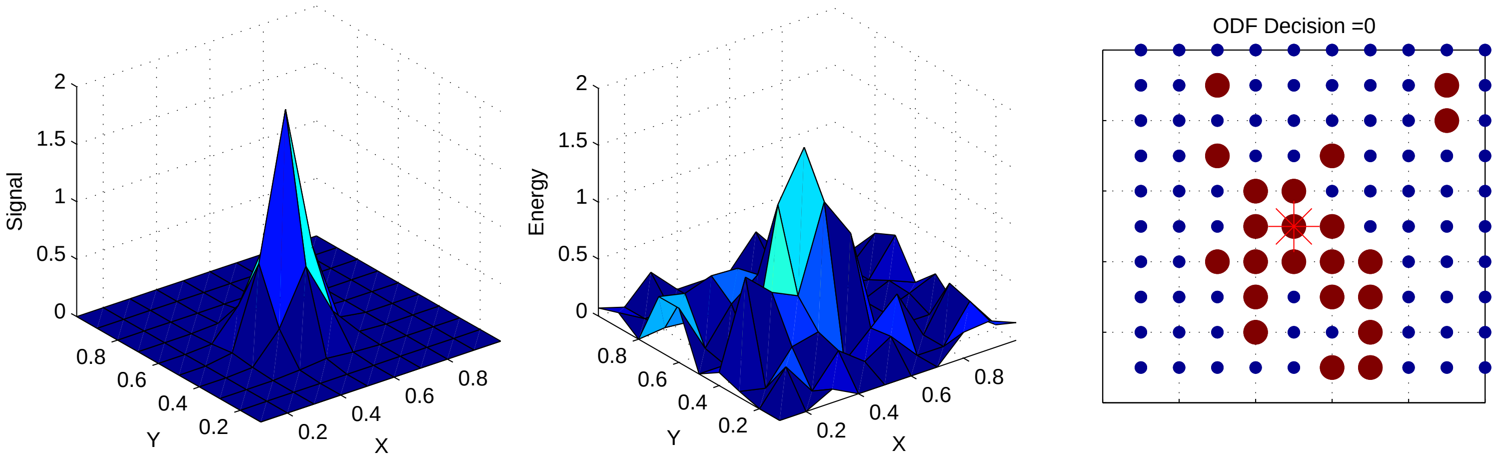
- $\epsilon_i$ is random noise; assume $\epsilon_i$ are i.i.d.
- Each sensor makes a decision

$$d_i = \mathbf{1}(E_i \geq \tau)$$

- The sensor's false alarm probability $\alpha$ is assumed known (prior information on noise or hardware); $v$, $S_0$, $\eta$ unknown

# Example: exponentially decaying signal model

$$S_i(v) = S_0 \exp(-||s_i - v||^2/\eta^2), \ \epsilon_i \sim \mathcal{N}(0, \sigma^2)$$



# Decision Fusion: Algorithms and Analysis

# Decision Fusion

- Only positive decisions $d_i$ are transmitted to fusion center
- Low communication cost
- More robust to noise

**Ordinary Decision Fusion (ODF):**

Fusion center makes the final decision $\mathbf{1}(\sum_{i=1}^{N} d_i \geq T)$

- Key issue: must choose $T$ to control system-wide false alarm $F$
- Decisions $d_i$ are i.i.d., so

$$F = \sum_{i=T}^{N} \binom{N}{i} \alpha^i (1-\alpha)^{N-i} \approx 1 - \Phi\left(\frac{T - N\alpha}{\sqrt{N\alpha(1-\alpha)}}\right)$$

# Decision fusion: long history

Early decision fusion algorithms: radar array applications

- Studied replacing $\sum_{i=1}^{N} d_i$ with $\sum_{i=1}^{N} w_i d_i$
- Classical decision theory can be applied if false alarm and detection probabilities are known for all sensors - unrealistic for WSN

Modern algorithms for WSN:

- ad hoc modifications of $d_i$ by various “quality” measures (e.g., level of neighborhood agreement) improve detection
- no performance guarantees (cannot choose $T$ to guarantee $F$)

# Local Vote Decision Fusion (LVDF)

1. Sensor $i$ makes an initial decision $d_i$ and communicates it to all sensors in its neighborhood $U(i)$.
2. Given the set of decisions $\{d_j : j \in U(i)\}$, sensor $i$ adjusts its initial decision according to a majority vote:

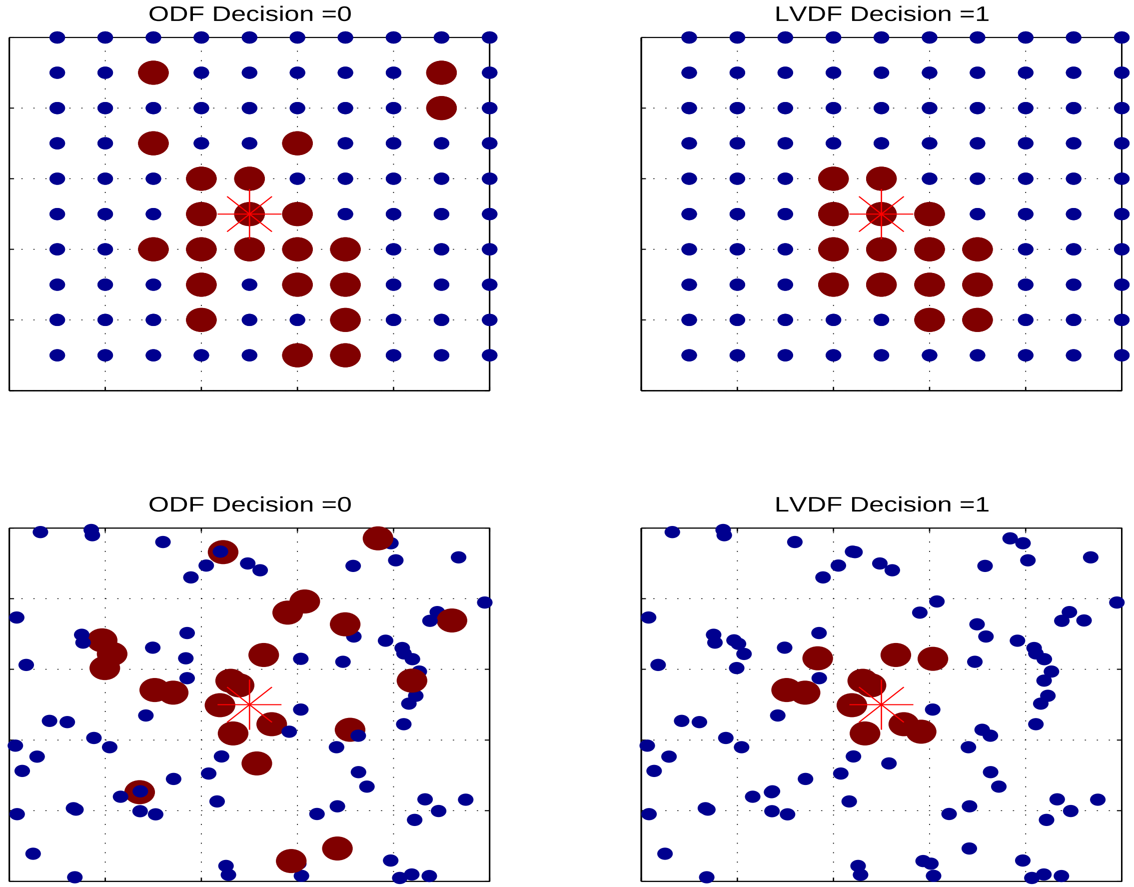
   $$Z_i = \mathbf{1}(\sum_{j \in U(i)} d_j > M_i/2),$$

   where $M_i = |U(i)|$ is the size of the neighborhood.
3. The fusion center makes the final decision

   $$\mathbf{1}(\sum_{i=1}^{N} Z_i \geq T_\ell).$$

In practice, sensors only transmit positive decisions in steps 1 and 3.

# ODF vs LVDF



ODF Decision =0
LVDF Decision =1
ODF Decision =0
LVDF Decision =1

# LVDF: threshold selection

Goal: a central limit theorem for the modified decisions $Z_i$

- $Z_i$ are binary, non-identically distributed, and weakly dependent
- Regular grids: adapted from CLT for non-stationary $\alpha$-mixing random fields (Guyon, 1995).
- Random deployment: adapted from CLT for functionals of marked Poisson and binomial point processes (Penrose & Yukich, 2001, 2002).
- Some care needed in taking limits as $N \to \infty$, region $R$ remains fixed.

# A normal approximation for the false alarm

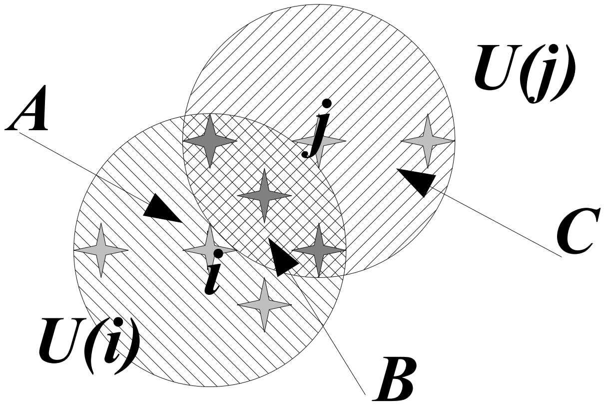
Let $n_{ij} = |U(i) \cap U(j)|$. $Z_i$ and $Z_j$ are dependent iff $n_{ij} > 0$.

$$F = P\left(\sum_{i=1}^{N} Z_i \geq T_\ell\right) \approx 1-\Phi\left(\frac{T_l - \sum_{i=1}^{N}\mu_i}{\sqrt{\sum_{i=1}^{N}\sigma_i^2 + \sum_{i\neq j, n_{ij}>0}\text{Cov}(Z_i, Z_j)}}\right).$$

- Mean and variance are easy (binomial):

$$\begin{aligned}\mu_i &= E(Z_i) = P(Z_i = 1) = \sum_{k=[M_i/2]+1}^{M_i} \binom{M_i}{k} \alpha^k (1-\alpha)^{M_i-k},\\ \sigma_i^2 &= \text{Var}(Z_i) = \mu_i(1-\mu_i),\end{aligned}$$

# Calculating Covariance



$A$: # positives in $U(i) - U(j)$
$B$: # positives in $U(i) \cap U(j)$
$C$: # positives in $U(j) - U(i)$
$A$, $B$, $C$ are independent binomial random variables. Condition on $B$:

$$E(Z_i Z_j) = \sum_{k=0}^{n_{ij}} P(B = k)\, P(A > \frac{M_i}{2} - k)\, P(C > \frac{M_j}{2} - k).$$

# Comments on the approximation

- Quite accurate even for moderate $N$; worsens as $M$ gets larger
- Simplifies significantly for regular grids
- For random deployments, depends on locations but
  - depends on locations only through the distribution of $M_i$, $n_{ij}$
  - error essentially the same if approximation is computed from one arbitrary deployment of $N$ sensors
  - hence localization can be avoided

# Performance Evaluation

# Performance Evaluation

The signal model is

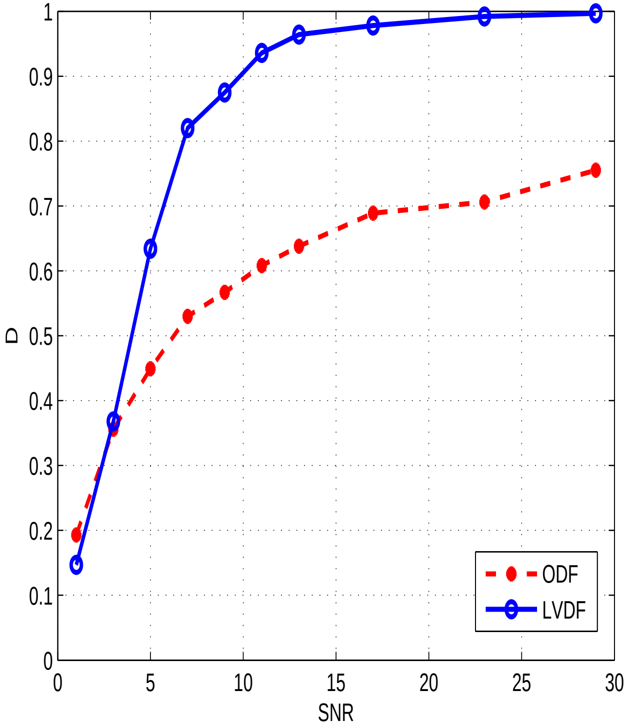
$$S_i = S_0 \exp(-||s_i - v||^2/\eta^2), \quad \epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

Same pattern for

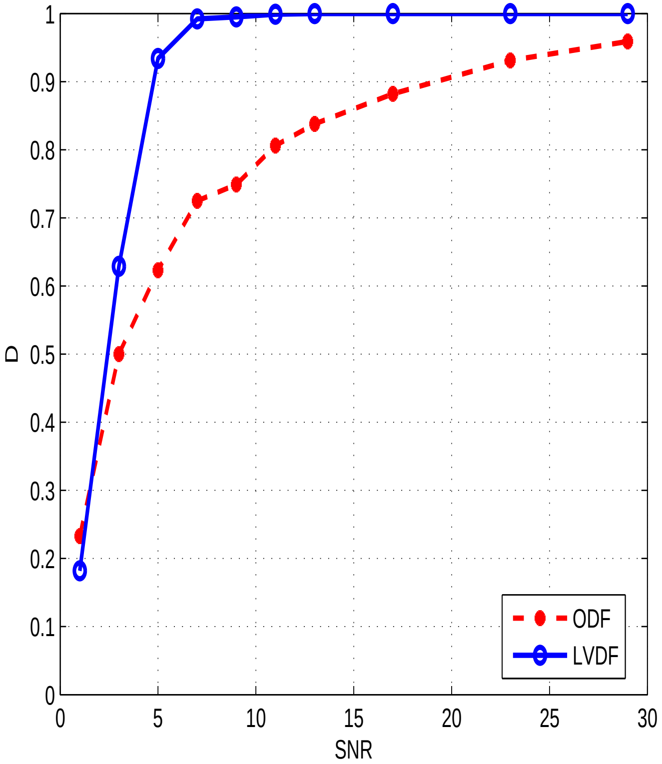
- regular grids and random deployments
- distance-based and nearest neighbors based neighborhoods
- a range of $N$, $\alpha$, $F$, $\eta$, SNR=$S_0/\sigma$.

# Probability of detection as a function of SNR

Random deployment, $N = 100$, $M = 9$, $\alpha = 0.2$, $S_0 = 2$, $F = 0.1$.



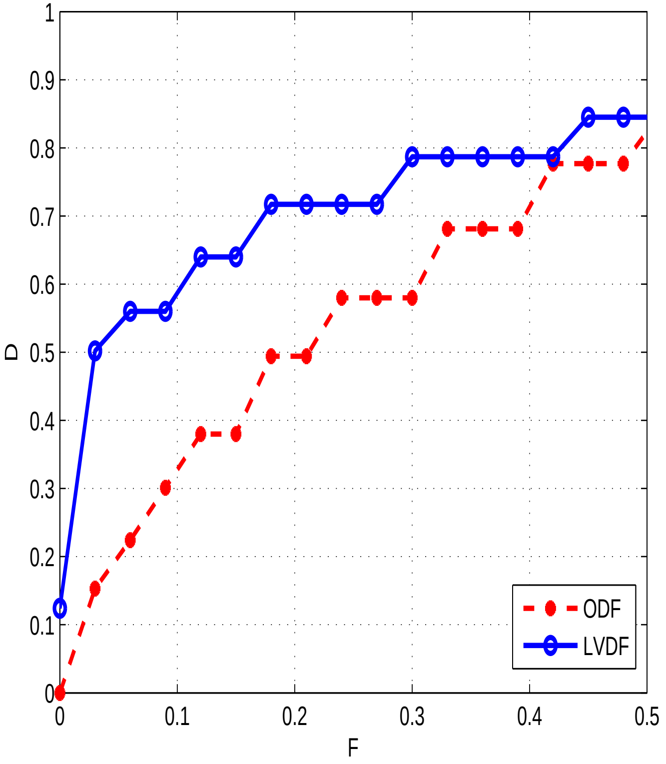
Smaller target ($\eta = 0.08$)



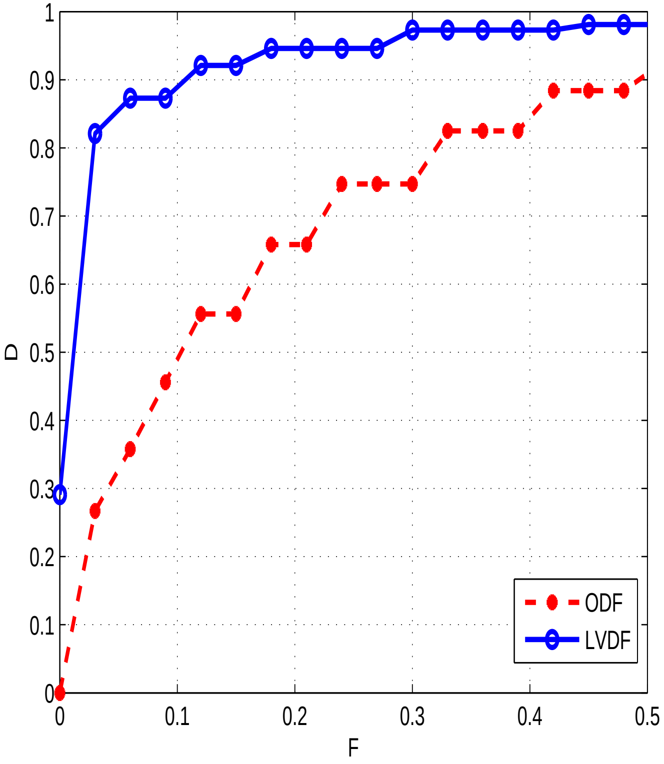
Larger target ($\eta = 0.1$)

# Probability of detection as a function of false alarm (ROC)

Random deployment, $N = 100$, $M = 9$, $\alpha = 0.2$, $S_0 = 2$, SNR=5.

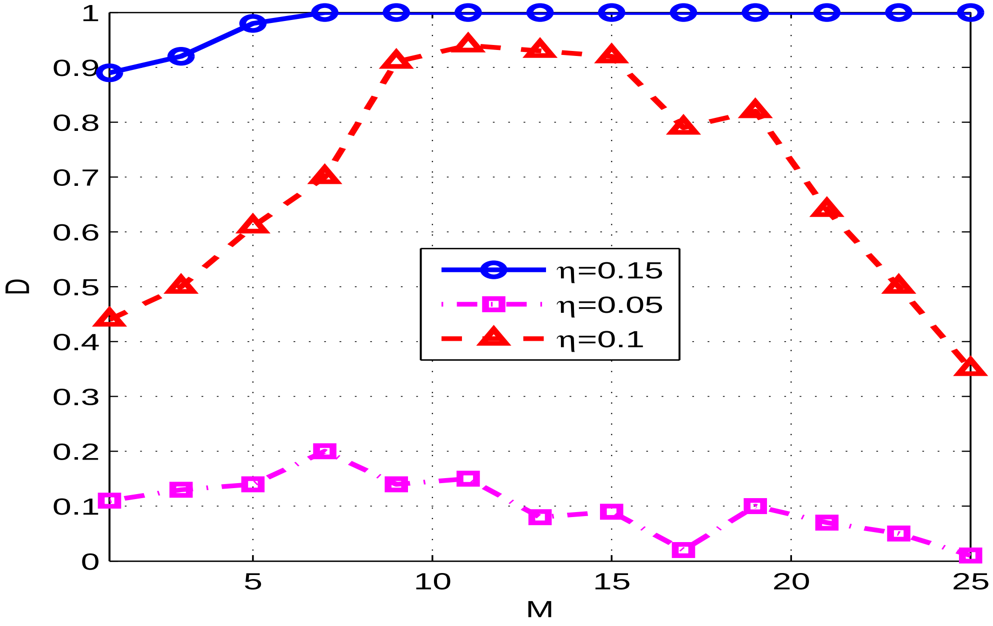


Smaller target ($\eta = 0.08$)



Larger target ($\eta = 0.1$)

# Choosing the size of the neighborhood



Choose neighborhood size similar to the size of the smallest target to be detected (subject to grid resolution)

# Sequential Decision Fusion (SDF)

- Decisions collected over time $t = 0, 1, \ldots$; target moves slowly relative to the rate of time sampling
- Fusion center stores corrected LVDF decisions $Z_i^t$
- Propose new decision algorithm: exponential moving average

$$\begin{aligned} Y_i^0 &= Z_i^0, \\ Y_i^t &= \lambda Z_i^t + (1-\lambda) Y_i^{t-1},\ t = 1, 2, \ldots \end{aligned}$$

- Networks decision at time $t$ is $\mathbf{1}\left(\sum_{i=1}^N Y_i^t \geq \tilde{T}_\ell\right)$.
- If noise is i.i.d. over time,

$$\tilde{T}_\ell = \sqrt{\frac{\lambda}{2-\lambda}} T_\ell$$

- Robust to moderate temporal correlations

# Performance evaluation for SDF

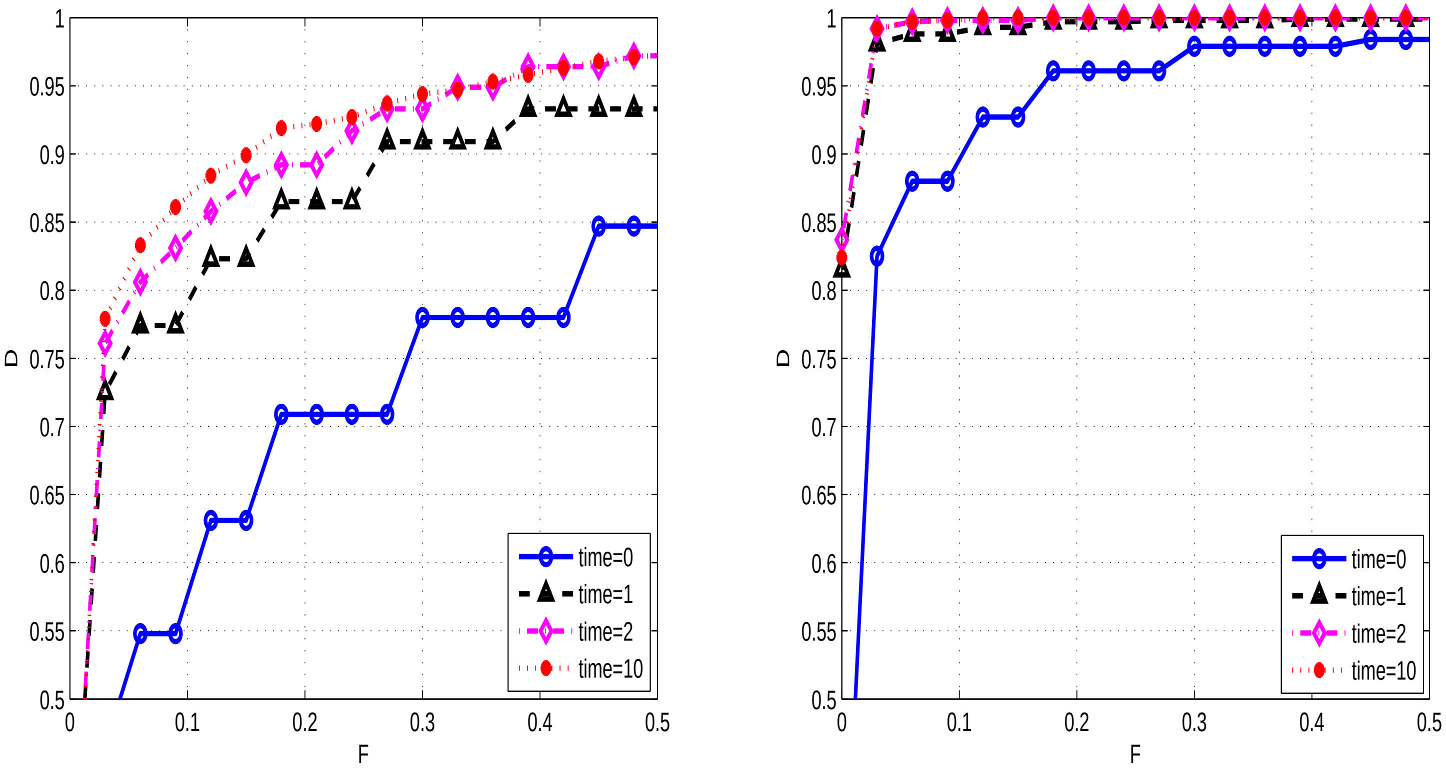
Introduction and Problem Formulation

Decision Fusion: Algorithms and Analysis

Performance Evaluation
- Performance Evaluation
- Probability of detection as a function of SNR
- Probability of detection as a function of false alarm (ROC)
- Choosing the size of the neighborhood
- Sequential Decision Fusion (SDF)
- Performance evaluation for SDF

Concluding remarks

- stationary target, $N = 100$ randomly deployed sensors



Smaller target ($\eta = 0.08$)

Larger target ($\eta = 0.1$)

# Concluding remarks

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**Local Vote Decision Fusion**

- increases detection probability and works well at low SNR
- particularly large gains for well-localized signals
- saves energy (fewer false positives sent to the fusion center)
- theoretical guarantees for the system false alarm

**Other interesting problems (in progress)**

- Target localization
- Estimation of target and environment parameters
- Tracking a target over time
- Multiple targets (detection and classification)