Bayesian Approach to Optimal Release Policy of Software System

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Contents

Abstract

In this paper, we propose a new software reliability growth model which is the mixture of two exponential reliability growth models, one of which has the reliability growth and the other one does not have the reliability growth after the software is released upon completion of testing phase. The mixture of two such models is characterized by a weighted factor, which is the proportion of reliability growth part within the model. Firstly, this paper discusses an optimal software release problem with regard to the expected total software cost incurred during the warranty period under the proposed software reliability growth model, which generalizes Kimura, Toyota and Yamada's(1999) model with consideration of the weighted factor. The second main purpose of this paper is to apply the Bayesian approach to the optimal software release policy by assuming the prior distributions for the unknown parameters contained in the proposed software reliability growth model. Some numerical examples are presented for the purpose of comparing the optimal software release policies depending on the choice of parameters by the non-Bayesian and Bayesian methods.

2. Software reliability growth model

- **The software fault detection phenomenon**

$$\Pr\{N(t)=n\}=\frac{e^{-m(t)}\{m(t)\}^n}{n!}, \qquad n=0,1,2,\ldots,$$

- **The mean value function**

$$m(t)=E[N(t)]=\int_0^t h(\tau)\,d\tau,$$

- **The Exponential Software Reliability Growth intensity function**

$$h(t)=\alpha\beta\, e^{-\beta t}, \quad \alpha>0, \quad \beta>0 \qquad (1)$$

The new intensity function with a weighted factor p

$$h_p(t) = h(t)I_{[0\le t<T]} + \{ph(t) + (1-p)h(T)\}I_{[T\le t\le T+T_w]}$$
$$= \alpha\beta e^{-\beta t}I_{[0\le t<T]} + \{p\alpha\beta e^{-\beta t} + (1-p)\alpha\beta e^{-\beta T}\}I_{[T\le t\le T+T_w]}$$

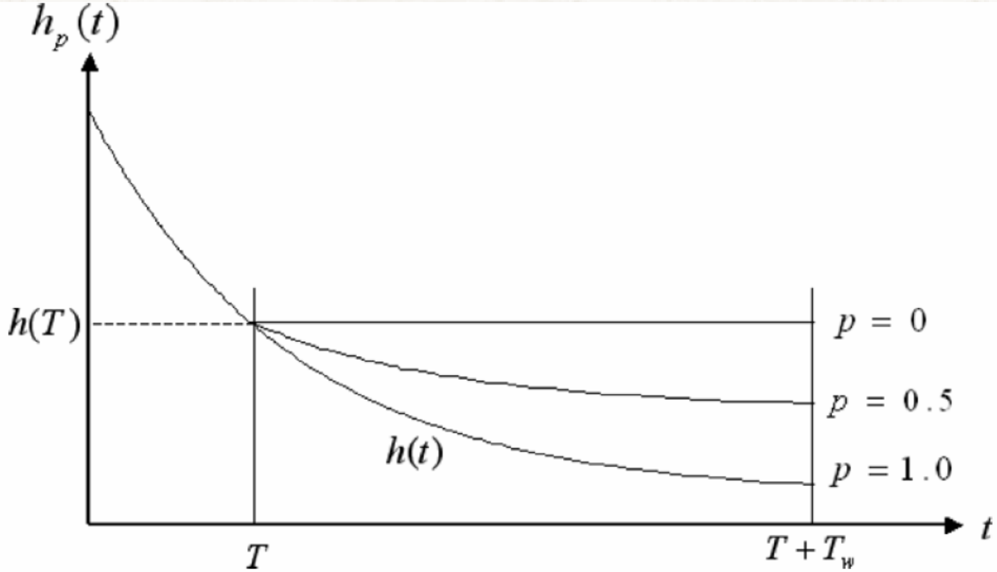

Fig. 1 Intensity function $h_p(t)$ for $p = 0$, 0.5 and 1.0.

3. Optimal software release time

- **Notations**

c_0	**set-up cost for testing**
c_t	**testing cost per unit time**
c_w	**maintenance cost per fault during the warranty period**
T	**software release time**
T^A	**optimal software release time**
T^B	**Bayesian optimal software release time with Poisson prior**
T^C	**Bayesian optimal software release time with binomial prior**
T_w	**warranty period**
γ	**discount rate of the cost**

- **The total expected software cost**

$$C_1(T) = c_0 + c_t \int_0^T e^{-\gamma t} dt + c_w \int_T^{T+T_w} \left\{ p\alpha\beta e^{-\beta t} + (1-p)\alpha\beta e^{-\beta T} \right\} e^{-\gamma t} dt. \quad (3)$$

<The optimal release policy with non-Bayesian method>

$$h(T)=\frac{c_t}{c_w\{p(1-e^{-(\beta+\gamma)T_w})+(1-p)((\beta+\gamma)/\gamma)(1-e^{-\gamma T_w})\}}\equiv\frac{c_t}{c_w\,k(p)},\qquad (4)$$

where $h(T)=\alpha\beta\,e^{-\beta T}$

$$T_1=\frac{1}{\beta}\ln\left[\frac{\alpha\beta\,c_w\,k(p)}{c_t}\right]\qquad (5)$$

Note that $d^2C_1(T)/dT^2\big|_{T=T_1}>0$ and $h(T)$ is a decreasing function of T for $T\geq 0$. Thus, if $h(0)>c_t/\{c_w\,k(p)\}$, $C_1(T)$ achieves its minimum at $T^A=T_1$, which is finite and uniquely determined. If $h(0)\leq c_t/\{c_w\,k(p)\}$, then $dC_1(T)/dT>0$ for all $T\geq 0$ and hence $C_1(T)$ is a monotonically increasing function of T. Thus, $T^A=0$ in this case.

(A1) If $h(0)>c_t/\{c_w\,k(p)\}$, **the optimal release time is** $T^A=T_1$.

(A2) If $h(0)\leq c_t/\{c_w\,k(p)\}$, **the optimal release time is** $T^A=0$.

4. Bayesian method for optimal software release policy

- **Prior distribution for** β

$$P_l = \Pr(\beta = \beta_l) = \int_{\beta_l - \delta_\beta/2}^{\beta_l + \delta_\beta/2} g_1(u)\, du,$$

where $\beta_l = \beta_L + \delta_\beta (2l - 1)/2$, $\delta_\beta = (\beta_U - \beta_L)/m_\beta$, $l = 1,2,\cdots,m_\beta$, $\sum_{l=1}^{m_\beta} P_l = 1$,

$$g_1(u) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \frac{(u-\beta_L)^{a-1}(\beta_U - u)^{b-1}}{(\beta_U - \beta_L)^{a+b-1}}, \quad 0 \le \beta_L \le u \le \beta_U, \quad a > 0, \quad b > 0.$$

- **Prior distribution for** p

$$P_j = \Pr(p = p_j) = \int_{p_j - \delta_p/2}^{p_j + \delta_p/2} g_2(v)\, dv,$$

where $P_j = \delta_p (2j - 1)/2$, $\delta_p = 1/m_p$, $\mathrm{j} = 1,2,\cdots,m_p$, $\sum_{j=1}^{m_p} P_j = 1$,

$$g_2(v) = \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)} v^{c-1}(1-v)^{d-1}, \quad 0 < v < 1, \quad c > 0, \quad d > 0.$$

Prior distribution for α : Poisson, Binomial

1. Poisson Prior

$$\Pr(\alpha = n) = \frac{e^{-\lambda}\lambda^{n}}{n\,!}, \qquad n = 0,1,2,\ldots$$

where λ represents the expected number of initial faults.

The total expected software cost

$$C_2(T) = E_{\alpha,\beta,p}\, C_1(T)$$

$$= E_{\alpha,\beta,p}\left[c_0 + c_t \int_0^T e^{-\gamma t} dt + c_w \int_T^{T+T_w} \{ p\alpha\beta\, e^{-\beta t} + (1-p)\alpha\beta\, e^{-\beta T} \} e^{-\gamma t} dt \right]$$

$$= c_0 + c_t \int_0^T e^{-\gamma t} dt + c_w \lambda \sum_{l=1}^{m_\beta} \sum_{j=1}^{m_p} \frac{\beta_l}{\beta_l + \gamma} e^{-(\beta_l+\gamma)T} \left\{ p_j (1 - e^{-(\beta_l+\gamma)T_w}) + (1-p_j)\left(\frac{\beta_l+\gamma}{\gamma}\right)(1 - e^{-\gamma T_w}) \right\} P_l P_j$$

<The optimal release policy with Bayesian method-Poisson>

$$\sum_{l=1}^{m_\beta}\sum_{j=1}^{m_p} \beta_l \, e^{-\beta_l T} H_{lj}(p_j) P_l \, P_j = \frac{c_t}{c_w \lambda}, \qquad (13)$$

where $$H_{lj}(p_j) = p_j\left(1 - e^{-(\beta_l+\gamma)T_w}\right) + (1-p_j)\left((\beta_l+\gamma)/\gamma\right)(1 - e^{-\gamma T_w})$$

Let $g(T)$ denote the left-hand side of (13). Then, it can be readily shown that $g(T)$ is a monotonically decreasing function of T, $\lim_{T\to 0} g(T) > 0$ and $\lim_{T\to\infty} g(T) = 0$. Thus, when $\sum_{l=1}^{m_\beta}\sum_{j=1}^{m_p} \beta_l H_{lj}(p_j) P_l P_j > \frac{c_t}{c_w \lambda}$, there exists a finite unique solution for T satisfying the equation (13), call it T_2. Since $d^2C_2(T)/dT^2 \big|_{T=T_2} > 0$, $C_2(T)$ is a convex function with respect to T and thus, $C_2(T)$ achieves its minimum at $T^B = T_2$. If $\sum_{l=1}^{m_\beta}\sum_{j=1}^{m_p} \beta_l H_{lj}(p_j) P_l P_j \le \frac{c_t}{c_w \lambda}$, then $dC_2(T)/dT > 0$ for all $T \ge 0$ and hence $C_2(T)$ is a monotonically increasing function of T. Clearly, $T^B = 0$ in this case.

(B1) If $\sum_{l=1}^{m_\beta}\sum_{j=1}^{m_p} \beta_l H_{lj}(p_j) P_l P_j > \frac{c_t}{c_w \lambda}$, **the optimal release time is** $T^B = T_2$.

(B2) If $\sum_{l=1}^{m_\beta}\sum_{j=1}^{m_p} \beta_l H_{lj}(p_j) P_l P_j \le \frac{c_t}{c_w \lambda}$, **the optimal release time is** $T^B = 0$.

2. Binomial Prior for α

$$\alpha \sim b(N,q) = \Pr(\alpha = n) = \binom{N}{n} q^n (1-q)^{N-n}, \qquad n = 0, 1, 2, \cdots, N,$$

where N is the total number of executable codes contained in the software system at the beginning of testing phase and $q\ (0<q<1)$ is the probability that each code contains exactly one fault.

- **The total expected software cost**

$$C_3(T) = E_{\alpha,\beta,p}\, C_1(T) = c_0 + c_t \int_0^T e^{-\gamma t} dt$$

$$+ c_w N q \sum_{l=1}^{m_\beta} \sum_{j=1}^{m_p} \frac{\beta_l}{\beta_l + \gamma} e^{-(\beta_l+\gamma)T} \left\{ p_j (1 - e^{-(\beta_l+\gamma)T_w}) + (1-p_j)\left(\frac{\beta_l+\gamma}{\gamma}\right)(1 - e^{-\gamma T_w}) \right\} P_l P_j.$$

<The optimal release policy with Bayesian method-Binomial>

$$\sum_{l=1}^{m_\beta}\sum_{j=1}^{m_p} \beta_l\, e^{-\beta_l T} H_{l\,j}(p_j) P_l\, P_j = \frac{c_t}{c_w\, N\, q}, \qquad (17)$$

where $H_{l\,j}(p_j) = p_j\left(1-e^{-(\beta_l+\gamma)T_w}\right) + (1-p_j)\left(\left(\beta_l+\gamma\right)/\gamma\right)(1-e^{-\gamma T_w}).$

(C1) If $\sum_{l=1}^{m_\beta}\sum_{j=1}^{m_p} \beta_l H_{l\,j}(p_j) P_l P_j > \frac{c_t}{c_w\, Nq}$, **the optimal release time is** $T^C = T_3.$

(C2) If $\sum_{l=1}^{m_\beta}\sum_{j=1}^{m_p} \beta_l H_{l\,j}(p_j) P_l P_j \le \frac{c_t}{c_w\, Nq}$, **the optimal release time is** $T^C = 0.$

5. Numerical Examples

Table 1 Optimal release time T^A(the first entry) and total expected cost $C_1(T^A)$(the second entry) for non-Bayesian case when $\alpha = 1000$, $\beta = 0.05$.

T_w	c_t = 1		3		5		10		30	
					$p = 0$					
1	78.6	1093.7	56.7	1220.8	46.4	1320.5	32.6	1510.3	10.6	1898.4
5	110.8	1122.4	88.8	1308.8	78.6	1468.5	64.7	1810.5	42.8	2819.1
10	124.6	1134.5	102.6	1345.7	92.4	1530.7	78.5	1936.6	56.6	3205.7
20	138.4	1146.3	116.4	1382.0	106.2	1591.8	92.5	2060.5	70.3	3585.6
50	156.4	1161.5	134.4	1428.7	124.2	1670.5	110.3	2220.2	88.4	4075.4
					$p = 0.5$					
1	78.4	1093.5	56.4	1220.1	46.2	1319.3	32.3	1508.0	10.4	1891.2
5	109.6	1121.4	87.6	1305.6	77.4	1463.1	63.5	1799.6	41.6	2785.7
10	122.3	1132.5	100.4	1339.7	90.2	1520.6	76.3	1916.2	54.3	3143.1
20	134.3	1142.8	112.3	1371.3	102.1	1573.9	88.2	2024.2	66.3	3474.3
50	148.8	1155.2	126.9	1409.2	116.6	1637.7	102.8	2153.6	80.8	3871.2
					$p = 1.0$					
1	78.1	1093.3	56.2	1219.4	45.9	1318.1	32.1	1505.6	10.1	1883.9
5	108.3	1120.3	86.4	1302.2	76.1	1457.4	62.3	1788.0	40.3	2750.1
10	119.8	1130.3	97.8	1332.9	87.6	1509.3	73.8	1893.1	51.8	3072.4
20	129.2	1138.4	107.2	1357.9	97.0	1551.3	83.2	1978.4	61.2	3333.9
50	136.5	1144.7	114.6	1377.2	104.3	1583.7	90.5	2044.2	68.5	3535.6

Table 2 Optimal release time T^B(the first entry) and total expected cost $C_2(T^B)$(the second entry) for Poisson prior of α with $(a, b) = (2, 2)$, $(c, d) = (1, 1)$ when $\lambda = 500$, $\lambda = 1000$ and $\lambda = 1500$.

	c_t									
T_w	1		3		5		10		30	
					$\lambda = 500$					
1	47.9	1064.4	33.5	1141.2	27.8	1201.1	20.8	1318.8	10.8	1612.6
5	75.2	1097.5	54.8	1219.1	47.0	1317.5	37.6	1521.3	25.0	2111.5
10	89.9	1115.2	66.0	1260.2	56.8	1378.3	46.0	1625.2	31.7	2357.3
20	106.7	1135.1	78.5	1306.1	67.7	1446.0	55.1	1739.8	38.9	2623.8
50	131.9	1164.6	97.1	1373.9	84.0	1545.8	68.7	1908.3	49.3	3013.3
					$\lambda = 1000$					
1	58.8	1077.6	42.2	1172.6	35.7	1248.4	27.8	1402.2	16.9	1822.9
5	91.1	1116.2	67.0	1263.2	57.8	1383.4	47.0	1635.0	32.6	2385.7
10	108.8	1137.0	80.3	1311.5	69.5	1454.9	56.8	1756.7	40.3	2670.4
20	129.0	1160.4	95.4	1365.8	82.6	1534.8	67.7	1892.0	48.7	2982.8
50	159.6	1195.3	117.9	1446.1	102.2	1653.0	84.0	2091.5	60.9	3441.2
					$\lambda = 1500$					
1	66.0	1086.3	47.9	1193.2	40.8	1279.3	32.3	1456.0	20.8	1956.4
5	101.7	1128.6	75.2	1292.4	65.1	1426.8	53.1	1709.7	37.6	2563.9
10	121.4	1151.3	89.9	1345.4	78.0	1505.4	64.0	1843.4	46.0	2875.5
20	144.0	1177.0	106.7	1405.1	92.6	1593.5	76.2	1992.6	55.1	3219.3
50	178.3	1215.5	131.9	1493.7	114.5	1723.9	94.3	2212.9	68.7	3724.7

Table 3 Optimal release time T^C(the first entry) and total expected cost $C_3(T^C)$(the second entry) with $\alpha \sim b\,(10000, q)$, $(a, b) = (2, 2)$, $(c, d) = (1, 1)$ when $q = 0.025$, $q = 0.05$ and $q = 0.1$.

T_w	c_t = 1		3		5		10		30	
					$q = 0.025$					
1	38.5	1053.0	25.9	1113.8	20.8	1159.4	14.4	1244.4	5.2	1420.5
5	61.7	1081.4	44.4	1181.0	37.6	1260.7	29.4	1422.5	18.1	1869.2
10	74.1	1096.4	53.8	1216.1	46.0	1312.6	36.6	1511.7	24.1	2084.0
20	88.1	1113.3	64.2	1254.9	55.1	1369.9	44.4	1609.1	30.3	2313.4
50	108.8	1138.3	79.7	1312.1	68.7	1454.1	55.9	1751.8	39.4	2647.1
					$q = 0.05$					
1	47.9	1064.4	33.5	1141.2	27.8	1201.1	20.8	1318.8	10.8	1612.6
5	75.2	1097.5	54.8	1219.1	47.0	1317.5	37.6	1521.3	25.0	2111.5
10	89.9	1115.2	66.0	1260.2	56.8	1378.3	46.0	1625.2	31.7	2357.3
20	106.7	1135.1	78.5	1306.1	67.7	1446.0	55.1	1739.8	38.9	2623.8
50	131.9	1164.6	97.1	1373.9	84.0	1545.8	68.7	1908.3	49.3	3013.3
					$q = 0.1$					
1	58.8	1077.6	42.2	1172.6	35.7	1248.4	27.8	1402.2	16.9	1822.9
5	91.1	1116.2	67.0	1263.2	57.8	1383.4	47.0	1635.0	32.6	2385.7
10	108.8	1137.0	80.3	1311.5	69.5	1454.9	56.8	1756.7	40.3	2670.4
20	129.0	1160.4	95.4	1365.8	82.6	1534.8	67.7	1892.0	48.7	2982.8
50	159.6	1195.3	117.9	1446.1	102.2	1653.0	84.0	2091.5	60.9	3441.2

Table 4 Optimal release time T^C(the first entry) and total expected cost $C_3(T^C)$(the second entry) with β ~discrete Beta(a, b), $\alpha \sim b$ $(10000, 0.05)$, $(c, d) = (2, 1)$ when $(a, b) = (2, 2)$, $(a, b) = (2, 3)$ and $(a, b) = (3, 2)$.

T_w	c_t = 1		3		5		10		30	
	$a = 2,\ b = 2$									
1	47.8	1064.3	33.5	1141.0	27.7	1200.7	20.7	1318.0	10.8	1610.2
5	74.8	1097.1	54.4	1217.9	46.6	1315.6	37.2	1517.4	24.6	2099.8
10	89.1	1114.4	65.2	1258.0	56.0	1374.6	45.2	1617.7	30.9	2334.9
20	105.1	1133.7	76.9	1301.9	66.2	1439.0	53.7	1725.7	37.4	2581.9
50	128.3	1161.5	93.7	1364.5	80.7	1530.3	65.6	1878.0	46.5	2927.1
	$a = 2,\ b = 3$									
1	52.4	1071.9	35.3	1154.2	28.6	1216.5	20.3	1334.4	8.8	1600.1
5	84.8	1110.8	60.5	1245.9	51.1	1353.6	39.9	1572.4	25.1	2181.2
10	102.3	1131.4	73.6	1293.9	62.7	1424.5	49.7	1693.3	32.7	2465.3
20	121.8	1154.3	88.0	1346.7	75.2	1502.2	60.1	1824.3	40.7	2766.5
50	150.4	1187.7	108.8	1422.4	93.1	1612.8	74.8	2009.4	51.7	3186.5
	$a = 3,\ b = 2$									
1	43.4	1055.3	31.9	1127.0	27.1	1184.7	21.0	1301.5	12.0	1611.5
5	63.0	1077.9	48.2	1183.1	42.2	1271.0	34.8	1457.3	24.2	2018.0
10	72.5	1088.9	55.9	1209.9	49.2	1311.6	40.9	1529.1	29.5	2197.8
20	82.5	1100.5	63.7	1237.6	56.2	1353.3	47.1	1601.9	34.5	2376.4
50	96.0	1116.1	74.3	1275.0	65.7	1409.5	55.4	1699.9	41.5	2617.2

Table 5 Optimal release time T^C(the first entry) and total expected cost $C_3(T^C)$(the second entry) with p ~discrete Beta(c,d), $\alpha \sim b$ (10000, 0.05), $(a,b) = (2,2)$ when $(c,d) = (1,2)$, $(c,d) = (2,1)$ and $(c,d) = (2,2)$.

T_w	c_t = 1		3		5		10		30	
					$c=1,\ d=2$					
1	48.0	1064.5	33.6	1141.5	27.9	1201.5	20.8	1319.6	10.9	1615.0
5	75.6	1097.8	55.2	1220.2	47.4	1319.4	38.0	1525.0	25.4	2122.8
10	90.7	1115.8	66.7	1262.3	57.6	1381.9	46.7	1632.2	32.4	2378.4
20	108.2	1136.4	79.9	1310.0	69.1	1452.6	56.5	1752.7	40.1	2661.7
50	135.2	1167.4	100.1	1382.2	86.9	1559.4	71.4	1934.7	51.7	3086.9
					$c=2,\ d=1$					
1	47.8	1064.3	33.5	1141.0	27.7	1200.7	20.7	1318.0	10.8	1610.1
5	74.8	1097.1	54.4	1217.9	46.6	1315.6	37.2	1517.4	24.6	2099.8
10	89.1	1114.4	65.2	1258.0	56.0	1374.6	45.2	1617.7	30.9	2334.9
20	105.1	1133.7	76.9	1301.9	66.2	1439.0	53.7	1725.7	37.4	2581.9
50	128.3	1161.5	93.7	1364.5	80.7	1530.3	65.6	1879.0	46.5	2927.1
					$c=2,\ d=2$					
1	47.9	1064.4	33.5	1141.2	27.8	1201.1	20.8	1318.8	10.8	1612.6
5	75.2	1097.5	54.8	1219.1	47.0	1317.5	37.6	1521.3	25.0	2111.5
10	89.9	1115.2	66.0	1260.2	56.8	1378.3	46.0	1625.2	31.7	2357.3
20	106.7	1135.1	78.5	1306.1	67.7	1446.0	55.1	1739.8	38.9	2623.8
50	131.9	1164.6	97.1	1373.9	84.0	1545.8	68.7	1908.3	49.3	3013.3

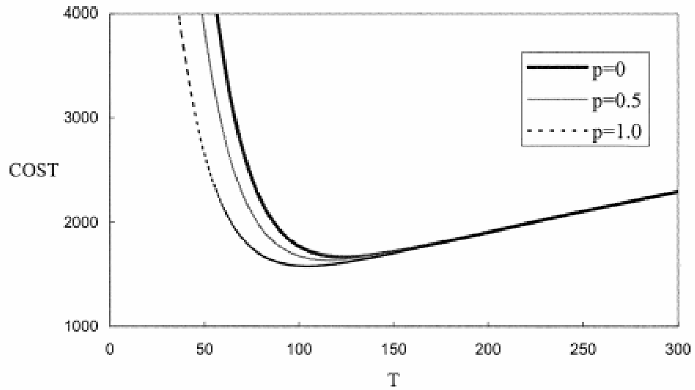

Fig. 2 Total expected software cost $C_1(T)$ with $c_t = 5$, $T_w = 50$, $\alpha = 1000$, $\beta = 0.05$ when $p = 0$, $p = 0.5$ and $p = 1.0$.

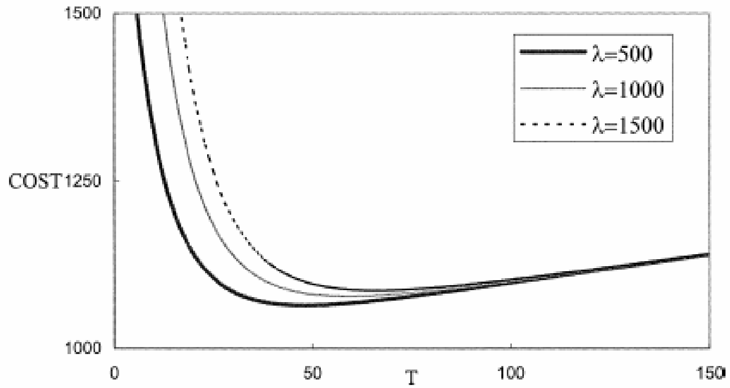

Fig. 3 Total expected software cost $C_2(T)$ with $c_t = 1$, $T_w = 1$, $(a, b) = (2, 2)$, $(c, d) = (1, 1)$ when $\lambda = 500$, $\lambda = 1000$ and $\lambda = 1500$.

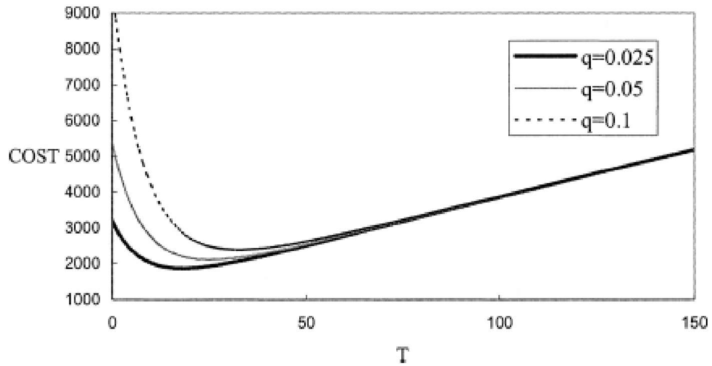

Fig. 4 Total expected software cost $C_3(T)$ with $c_t = 30$, $T_w = 5$, $(a, b) = (2, 2)$, $(c, d) = (1, 1)$ when $q = 0.025$, $q = 0.05$ and $q = 0.1$.

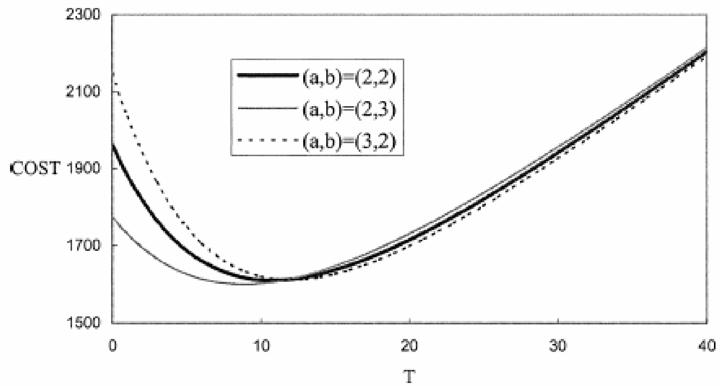

Fig. 5 Total expected software cost $C_3(T)$ with $c_t = 30$, $T_w = 1$, $\alpha \sim b$ $(10000, 0.05)$, $(c, d) = (2, 1)$ when $(a, b) = (2, 2)$, $(a, b) = (2, 3)$ and $(a, b) = (3, 2)$.

6. Concluding remarks

- This paper has two main purposes. Firstly, we propose a new software reliability growth model for which the reliability growth occurs with probability and the reliability growth does not occur with probability during the warranty period. By minimizing the total expected software cost, we determine the optimal software release time at which the testing phase ends and study the sensitivity analysis of parameters including on the software release time. Secondly, we consider three parameters of interests, initial number of faults in the system, fault detection rate per fault and the weighted factor, to be random and adopt the Bayesian approach to determine the optimal software release time under the proposed reliability growth model. The Bayesian approach may have an advantage over the non-Bayesian method when the parameters are unknown or difficult to estimate.
- In this paper, we assume the discrete beta priors for the fault detection rate and the weighted factor and the Poisson and binomial priors for the initial number of faults in the system. For those priors, the effects of prior parameters on the optimal software release policies have been discussed in details numerically and it shows that the more faults the software system has initially, the longer testing phase for the software we should have before it is released to the user.
- In the future study, it is desirable to collect and utilize the software failure data to update the priors of parameters to solve the software release problem, which could be more practical in real situations.

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