The Synergistic Roles of Optimal and Classical Designs for Quality Improvement in the 21st Century

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Message

1. Use of our tools (DOE vs no DOE, 6σ vs no 6σ, etc.,) can be the difference between survival and failure of an organization
2. Our issues:
    - *Optimal vs classical*
    - *Bayesian vs frequentist*
    - *Regular (FF) vs non-regular (PB)*
    - *Taguchi vs Box*
    - *Kiefer vs Box …*

   are trivial and irrelevant to the big picture
3. We need to promote/promulgate/evangelize the use of our tools

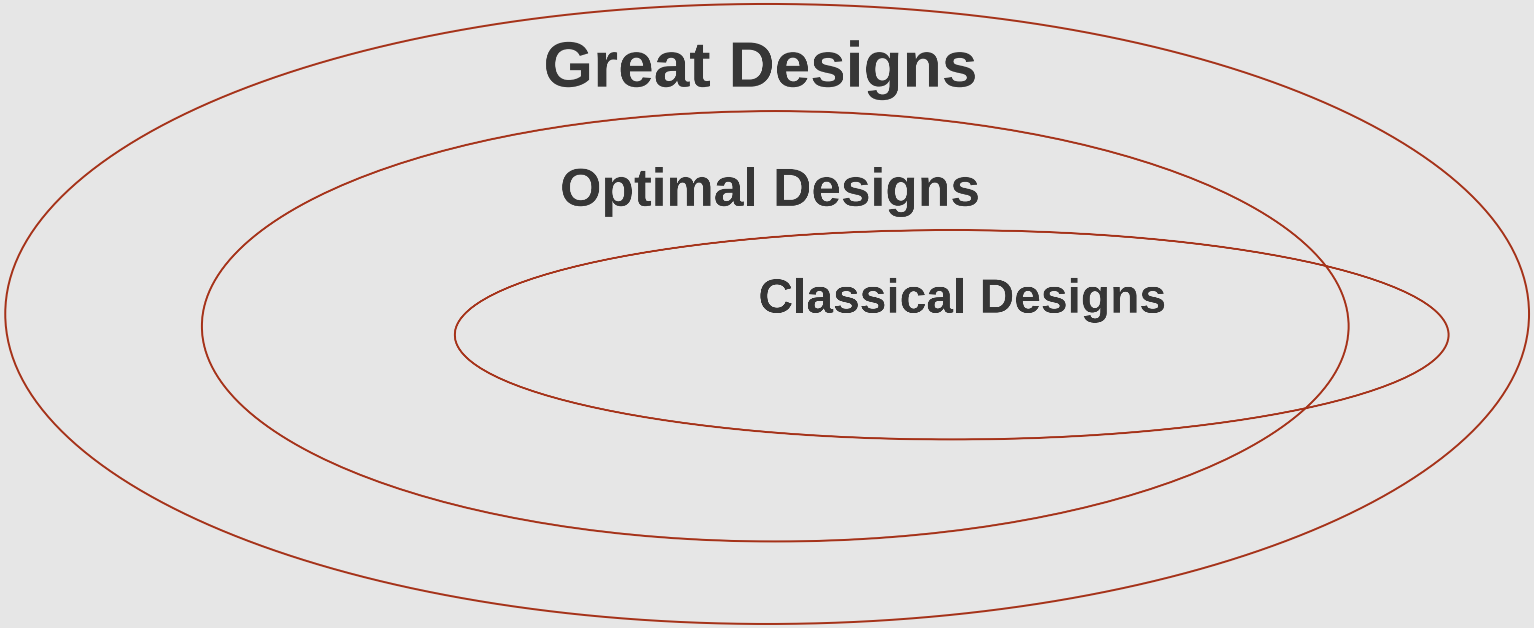
Optimal vs. Classical

- Actually, there is no “vs.”
- Classical designs are almost always D-Optimal for the model of interest
- The real “vs” is:

Design vs No Design

Optimal vs Classical

- So we have:



Great Designs

Optimal Designs

Classical Designs

Philosophy

- Classical designs are excellent, proven tools!
- But they don't always apply
- When classical designs are not applicable, what can we do for our clients?

Three Approaches

1. Force fit the problem to a tabled design
2. Hire an expert (e.g., Jim Lucas): a Ph.D. statistician with deep insights into the design problem
3. Get help from the computer to search for an excellent design

When sensible, classical approaches are appropriate, use them!

When they're not, optimal designs can be tailored to the design problem at hand

When are Classical Approaches not Applicable?

(Cook, Nachtsheim, Technometrics, 1990, “Response to Jim Lucas”)

1. Design for nonstandard sample sizes.
2. Design in the presence of irregular design spaces
3. Design for constrained mixture experiments
4. Design augmentation
5. Design for experiments having both qualitative and quantitative factors
6. Design for nonlinear regression models
7. Blocking in uneven settings
8. Design in the presence of fixed covariates (experimental unit heterogeneity)

When are Classical Approaches not Applicable?

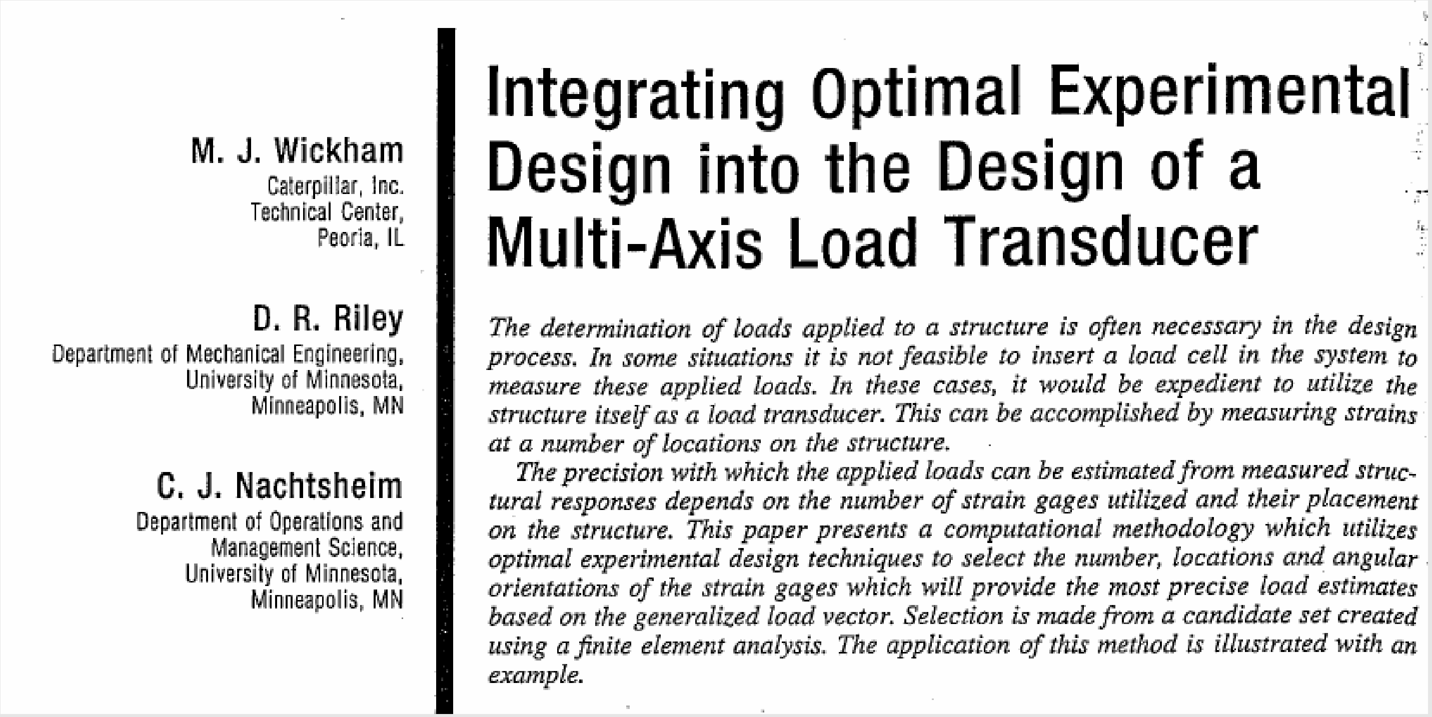
8. Designs for complicated response surface models with terms of varying orders
9. Designs for logistic regression and other exponential family models
10. Designs for computer experiments
11. Designs for model comparison
12. Designs for situations in which there is heteroscedasticity across the design space
13. Any problem wherein it is not obvious how to proceed from a classical view

When are Classical Approaches not Applicable? (Some new ones)

1. Above situations involving hard-to-change and easy-to-change factors (split-plots).
    - Lucas and coworkers
    - Bingham, Ye, and coworkers
    - Kowalski, Vining, and coworkers
    - Goos, Jones and coworkers
2. Designs for mixture and process variables
3. Designs for heterogenous sources of information (See Sally Keller-McNulty keynote, Hamada)
4. Designs for choice experiments (Kessels, Vandebroek, Goos, 2006, Kessels, Jones, Goos, Vandebroek, yesterday)

A Quick (Favorite) Example

Determining Gage Sites for Product Stress Testing



Integrating Optimal Experimental Design into the Design of a Multi-Axis Load Transducer

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The determination of loads applied to a structure is often necessary in the design process. In some situations it is not feasible to insert a load cell in the system to measure these applied loads. In these cases, it would be expedient to utilize the structure itself as a load transducer. This can be accomplished by measuring strains at a number of locations on the structure.

The precision with which the applied loads can be estimated from measured structural responses depends on the number of strain gages utilized and their placement on the structure. This paper presents a computational methodology which utilizes optimal experimental design techniques to select the number, locations and angular orientations of the strain gages which will provide the most precise load estimates based on the generalized load vector. Selection is made from a candidate set created using a finite element analysis. The application of this method is illustrated with an example.

The Product: A C-Spring

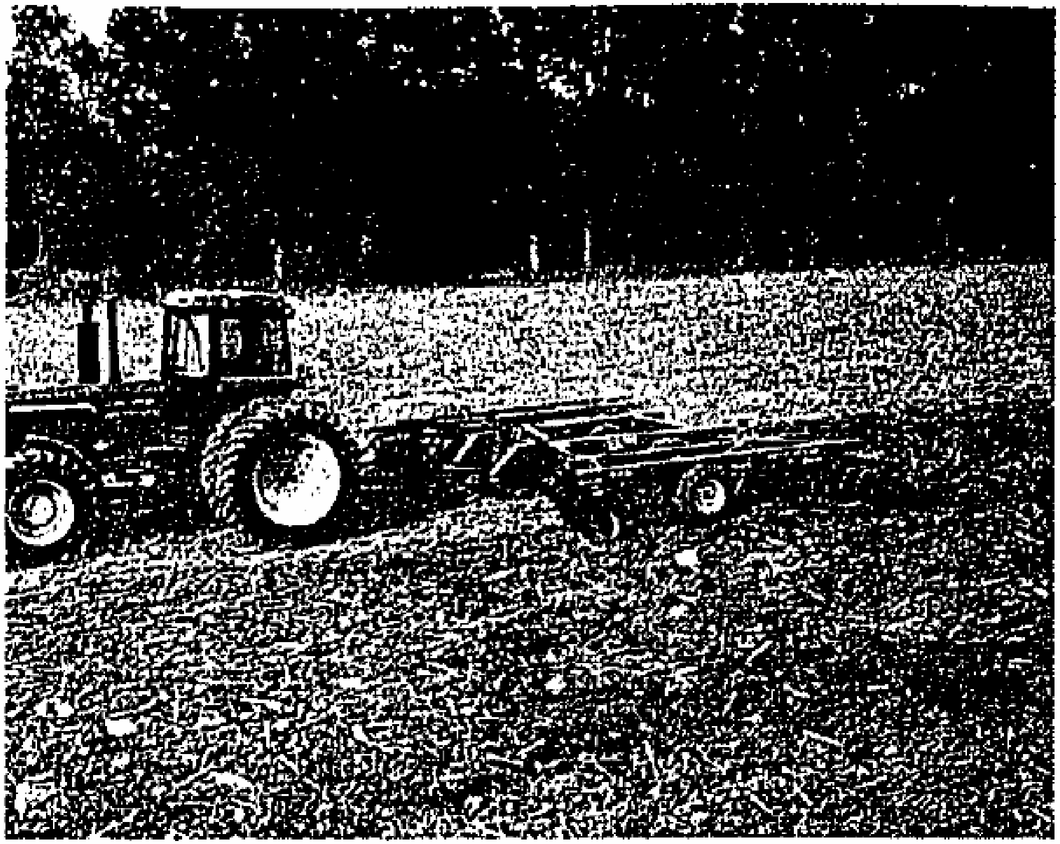


Fig. 1(*a*) Disk with C-Springs (Photograph courtesy of Deere & Co.)

The Product: A C-Spring

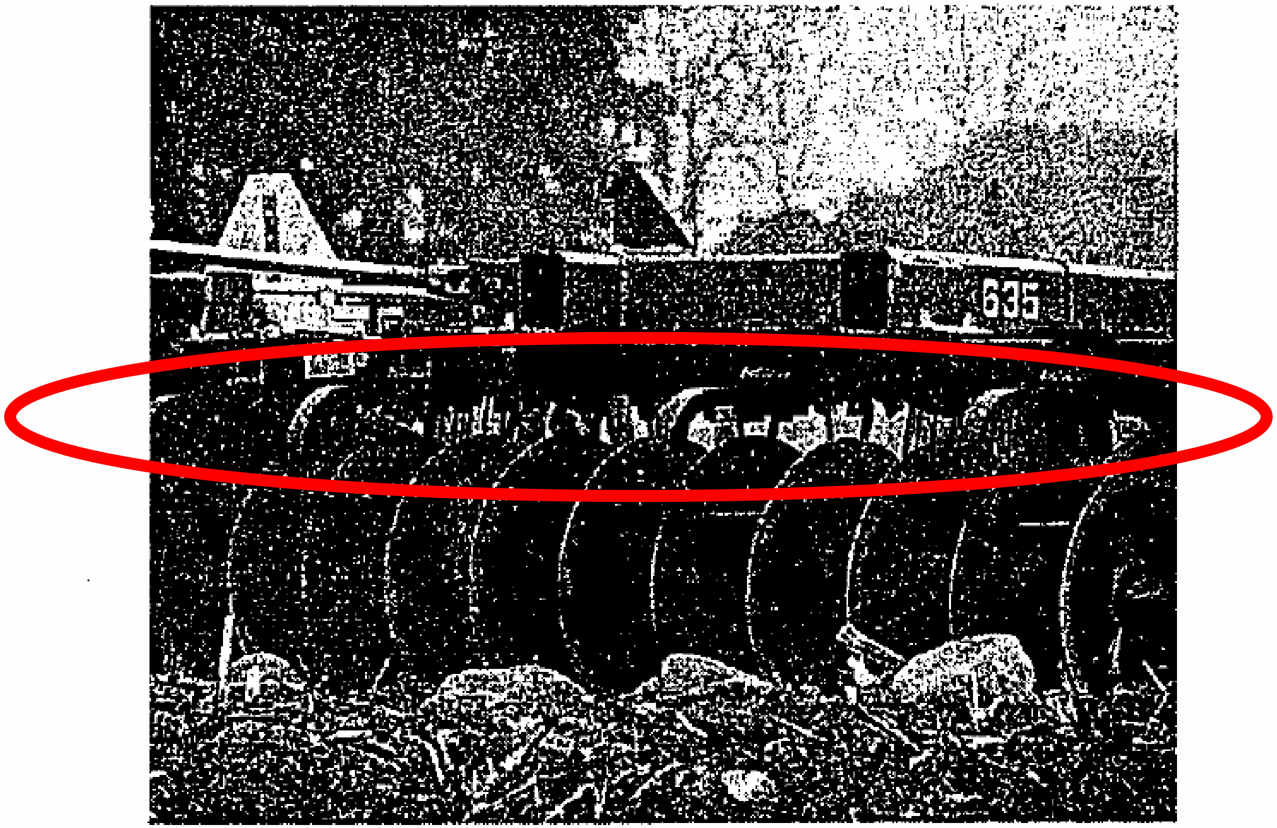


Fig. 1(*b*) Close-up view of disk with C-springs (Photograph courtesy of Deere & Company)

The Product: A C-Spring

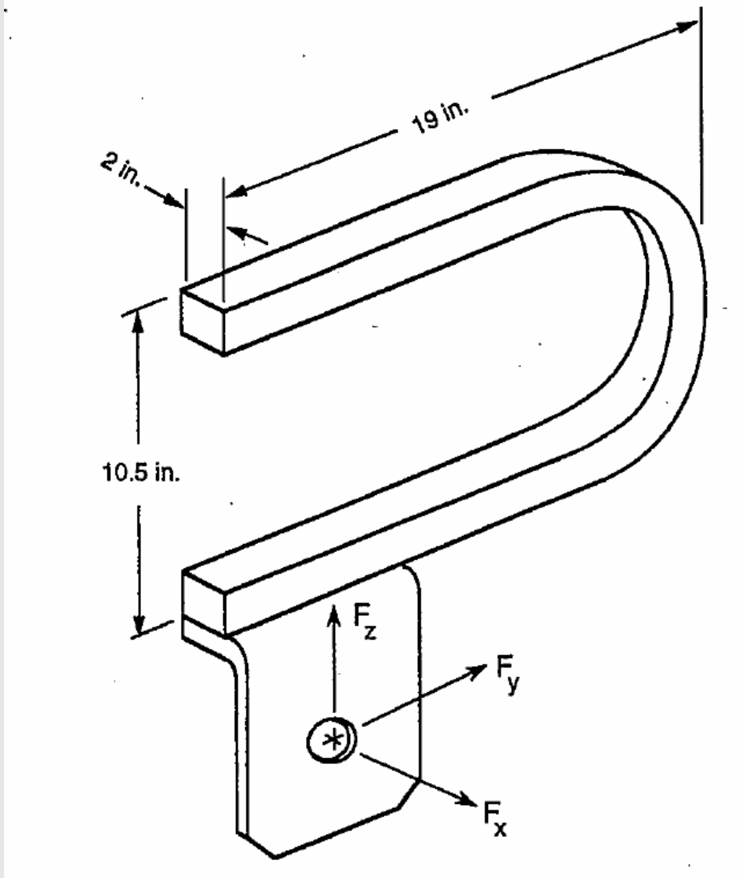


Fig. 2 C-Spring with three loads

FEA Model

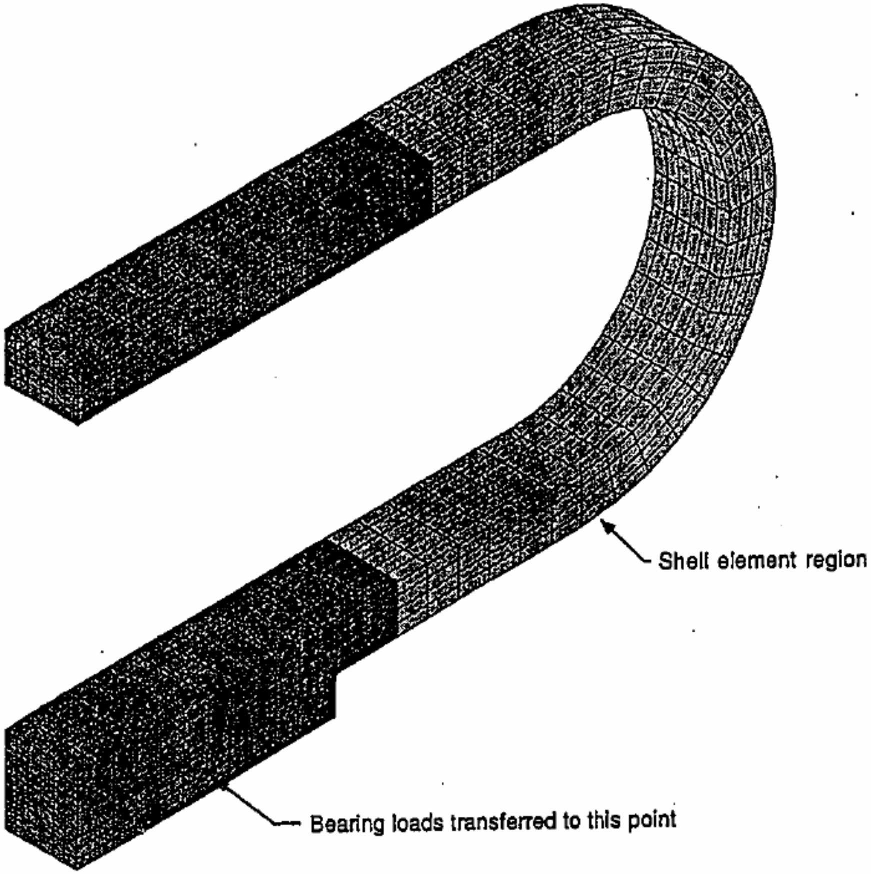


Fig. 3 Finite element analysis model

Design Problem

Where do we attach 8 strain gages in order to get the best estimates of loads placed on the C Spring?

Superposition principle: Loads linearly related to location strain sensitivities

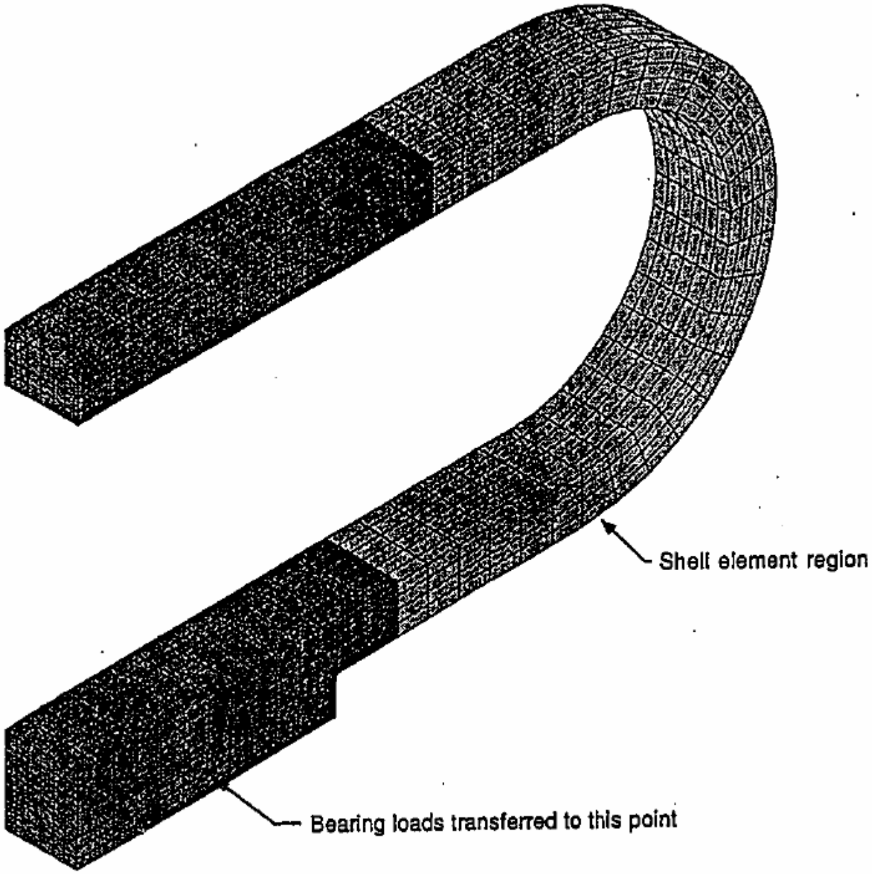


Fig. 3 Finite element analysis model

Design Space (3D, discrete)

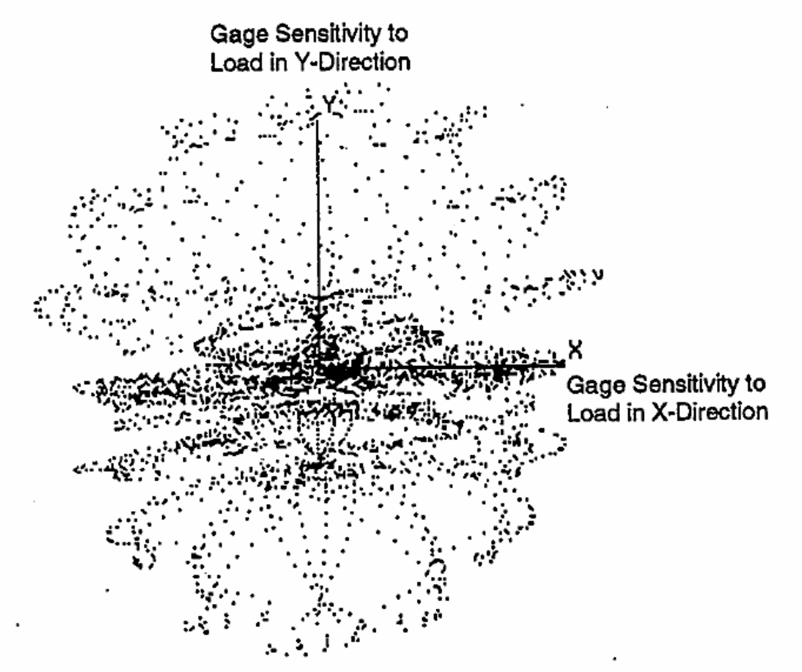
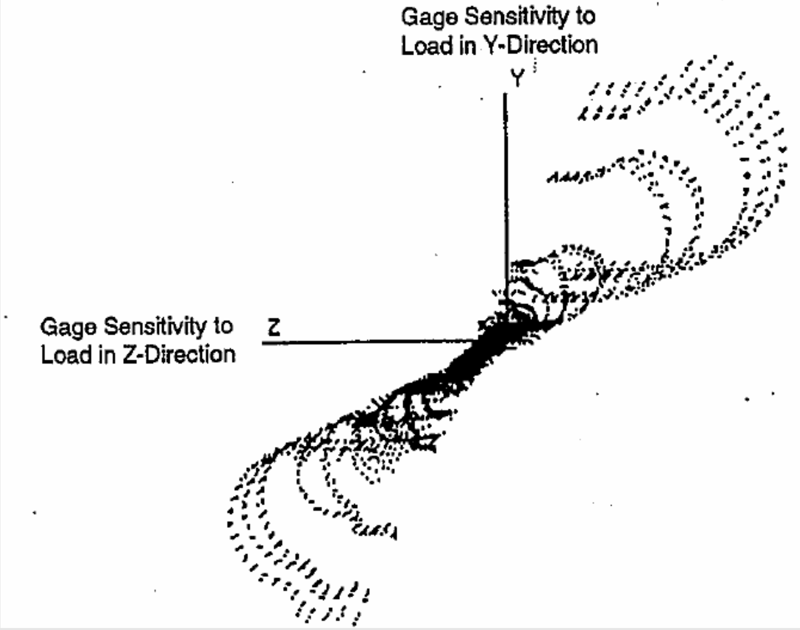


Fig. 4(*a*) Design space for C-spring application



Solution

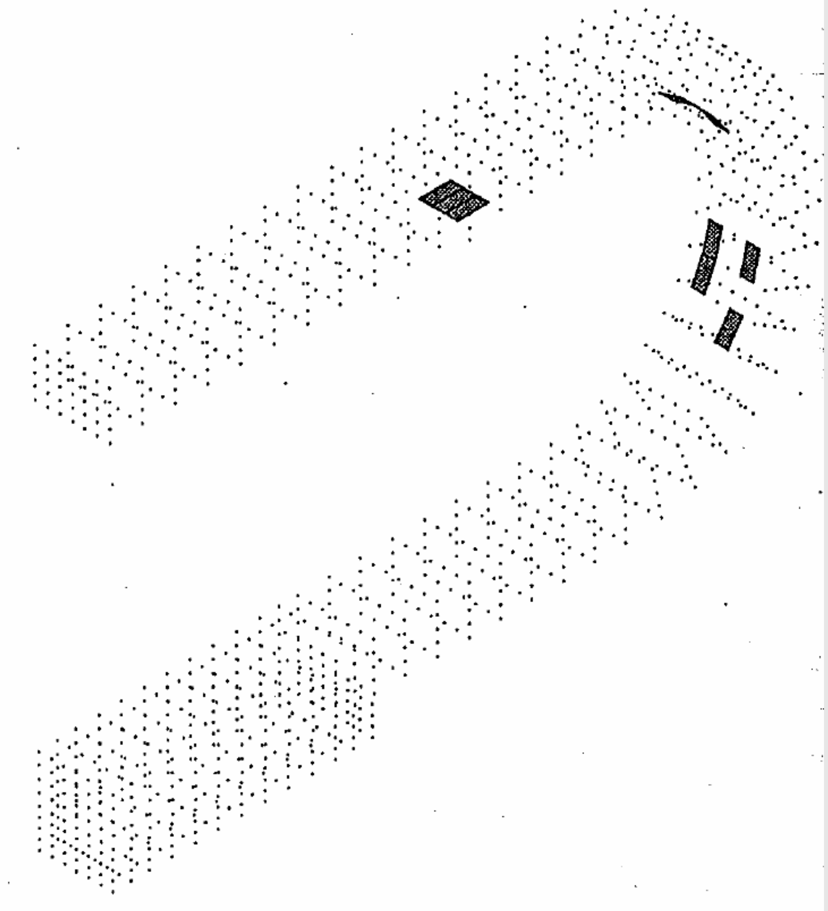


Fig. 6 Gage locations illustrated as highlighted elements

Synergy between classical/optimal design

Standard resistance (drivel):
Optimal designs depend on the a priori model

Three responses

1. **No more so than classical designs.**
2. **They should depend on the model (your objective)**
3. **See model uncertainty, model robustness, Bayesian design literature (e.g., Jones, Lin, Nachtsheim, JSPI, Bayesian Supersaturated Designs, in press)**

Second Interdisciplinary Example

Interdisciplinary team:
William Li, Chris Nachtsheim, Vincent Agboto

William---combinatorial design expert, Jeff Wu protege
Chris---OK, I'm an optimal design guy
Vincent---Grad student---tabula rasa, ignorant

Problem: Bayesian Model Robust Design

Second Interdisciplinary Example

Derived Criterion

$$\phi_{BR}(\xi) = \sum_{i=1}^{d} w_i \log | \sigma^{-2}(\mathbf{X}'_{i,\xi}\mathbf{X}_{i,\xi} + \mathbf{R}_i) | \; .$$

Used it to:

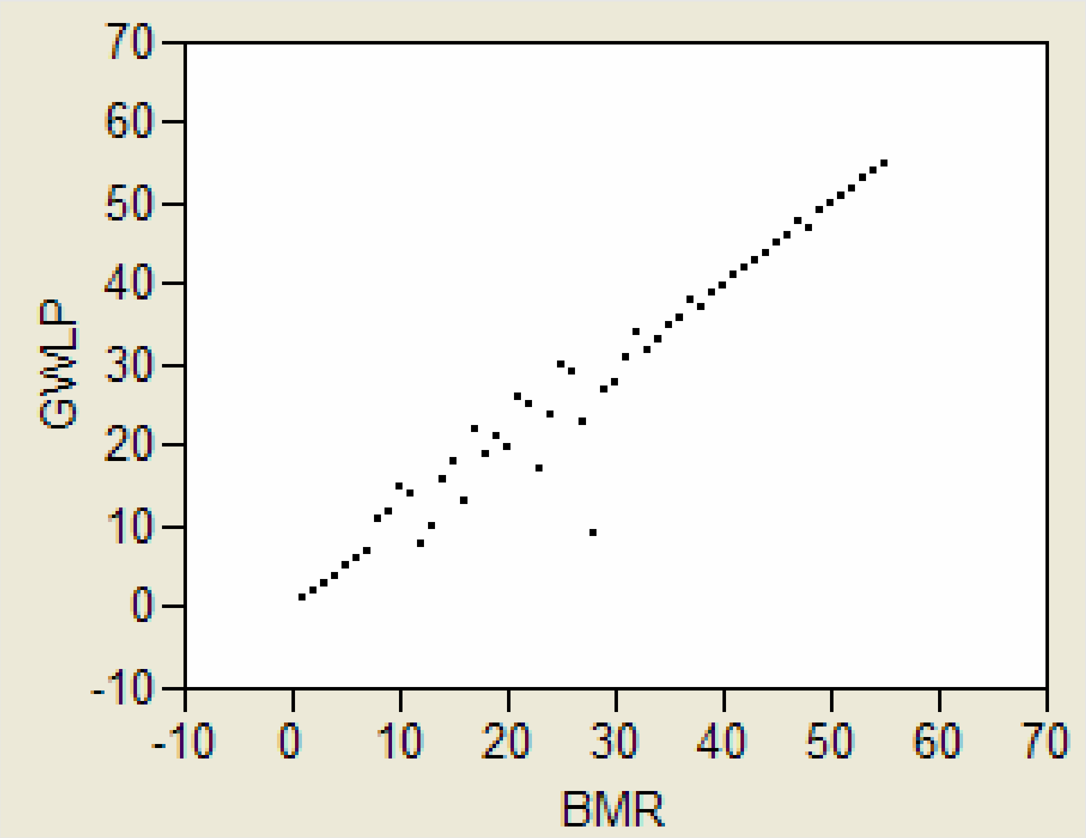
1. **Construct model robust designs**
2. **Evaluate standard orthogonal designs**

Second Interdisciplinary Example

Results for all 55 ODs for n = 12

1. The BMR design was none other than the minimum generalized abberation design!
2. The rankings of the 55 designs by the BMR and generalized abberation criterion were nearly identical.
3. Preliminary conclusion. BMR criterion values an alternative to GA, can be used to evaluate any design, not just orthogonal designs

Second Interdisciplinary Example



Final Notes

- With good software, optimal design is very easy to teach. One, unified approach to (nearly) all design problems:

  1. Describe your responses
  2. Describe your factors
  3. Describe your objectives (model terms)
  4. Tell me your budget (n)
  5. Press button to create the design
  6. Manually alter as with classical
  7. Check diagnostics, do sensitivity analysis

- Can eliminate a barrier to entry for users, and that’s the objective!

Final Wish

I hope I've set the standard for constructive, supportive discussion for my colleagues.

Seminar to conclude at 1:30 with:

1. **Speakers will sing one verse of "Cum-bi-ya" together**
2. **Group hug**