Using Simulation and Graphics as an Aid in Planning Complicated Experiments

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What is The Most Commonly Asked Question in Reliability Consulting?

- How many samples do I need to estimate B10 life?

  (B10 is shorthand for the 10^{th} percentile of life)

- And for how long do I need to run the test?

Answer to the Questions

Literal: Assuming a two-parameter Weibull lifetime distribution, test two units; wait until just after one of them fails.

Practical: It depends on how much precision you need! How wide do you want your confidence interval to be?

Overview

- Confidence intervals and sample size for B-life – basic ideas (cf Stat 101).
- Test plan for a ***simple life test*** (no regression)
- Test plan for an ***accelerated life test*** (simple regression with censored data).
- Test plan for an accelerated ***destructive degradation test*** (nonlinear regression).
- Test plan for ***repeated measures degradation test*** (mixed effects regression model).

Computation of Confidence Intervals

Stat 101: $[\,\underset{\sim}{\mu},\quad \tilde{\mu}\,] = \overline{x} \pm z_{(1-\alpha/2)} s/\sqrt{n} \quad = \quad \overline{x} \pm d$

d is the ***half-width*** of the confidence interval.

Computation of Confidence Intervals

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Stat 533: $[\ \underset{\sim}{t}_p, \quad \tilde{t}_p\ ] \quad = \quad [\hat{t}_p/R, \quad \hat{t}_p \times R]$

R > 1 is a ***precision factor.***

If R=2 and , then the CI is [7/2, 7 × 2] = [3.5, 14].

Sample Size Formulas

Stat 101 (normal mean): $$n = \frac{z^2_{(1-\alpha/2)}\sigma^2}{d_T^2}$$

Sample Size Formulas

Stat 101 (normal mean):
$$n = \frac{z_{(1-\alpha/2)}^2 \sigma^2}{d_T^2}$$

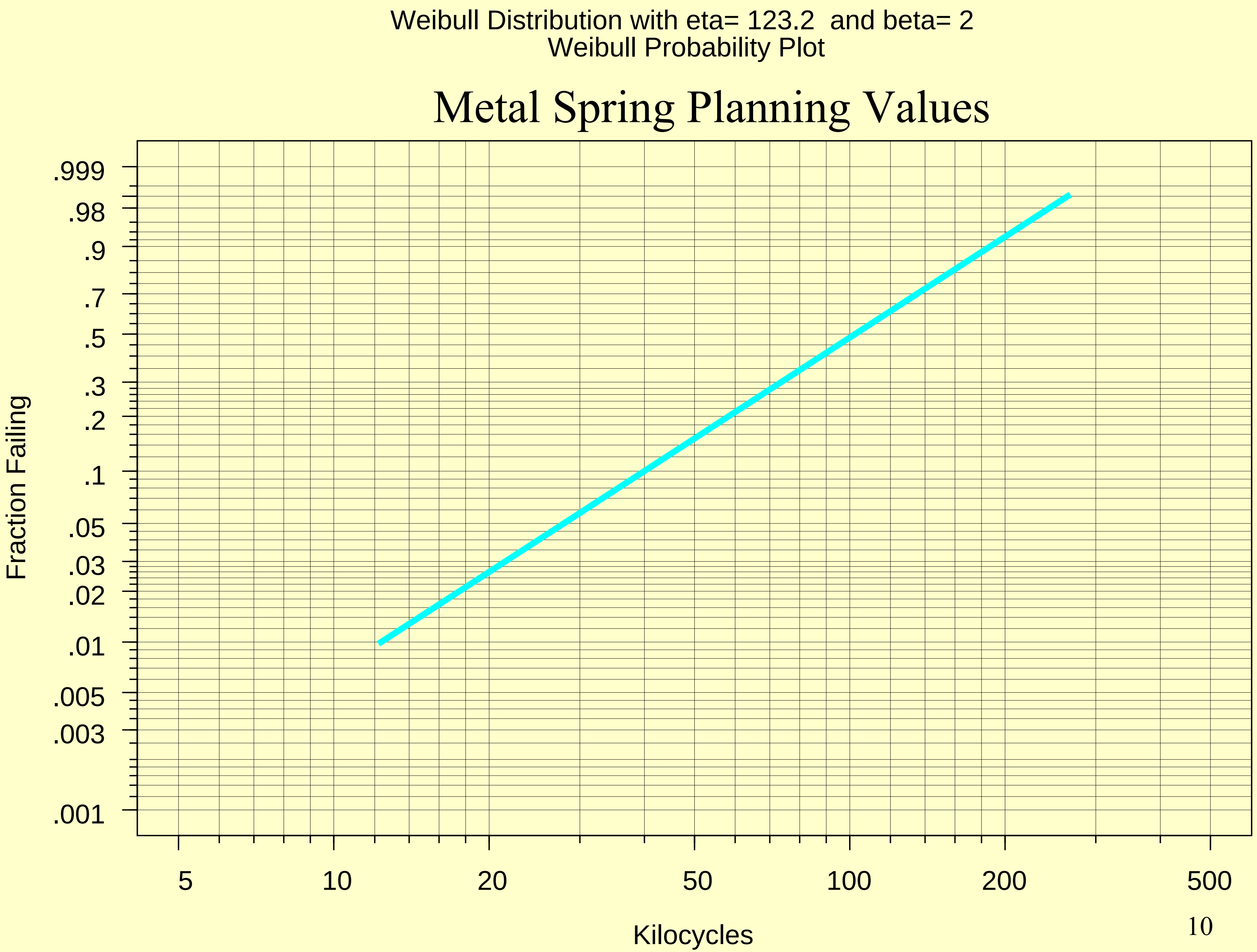
Stat 533 (general):
$$n = \frac{z_{(1-\alpha/2)}^2 V_{p_e, p_c}}{[\log(R_T)]^2}$$

Note: Need planning information.

Planning a Life Test for a Metal Spring

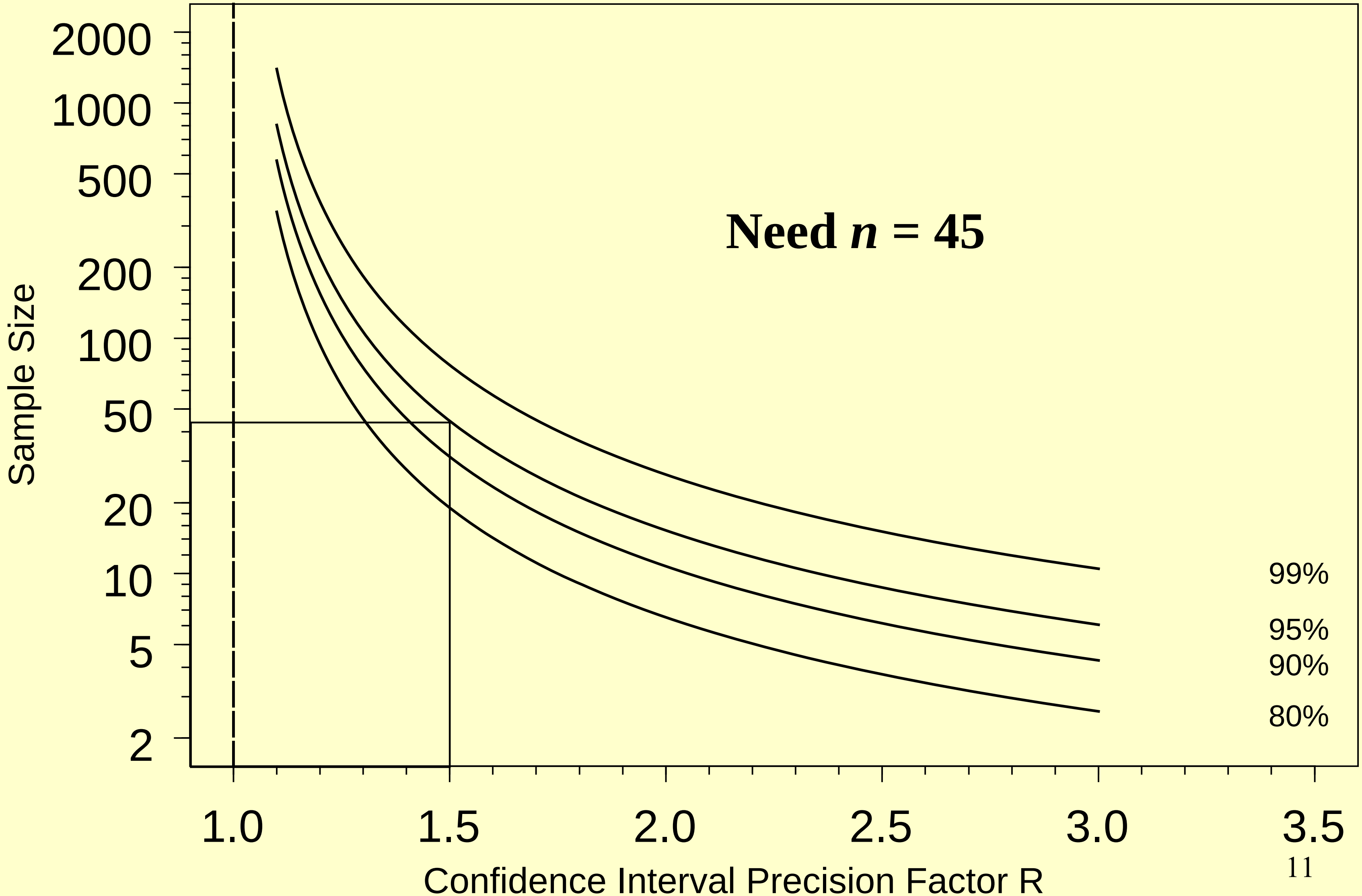
- Want to estimate B10 of spring life
- Planning information
  - Weibull distribution
  - Weibull shape $\beta = 2$
  - B10 = 40 thousand cycles
- Time censoring at 30 or 50 thousand cycles

Metal Spring Planning Values



Sample Size Tool --- Censoring at 50 kilocycles

Needed sample size giving approximately a 50% chance of having
a confidence interval factor for the 0.1 quantile that is less than R
weibull Distribution with eta= 123 and beta= 2
Test censored at 50 Kilocycles with 15.2 expected percent failing



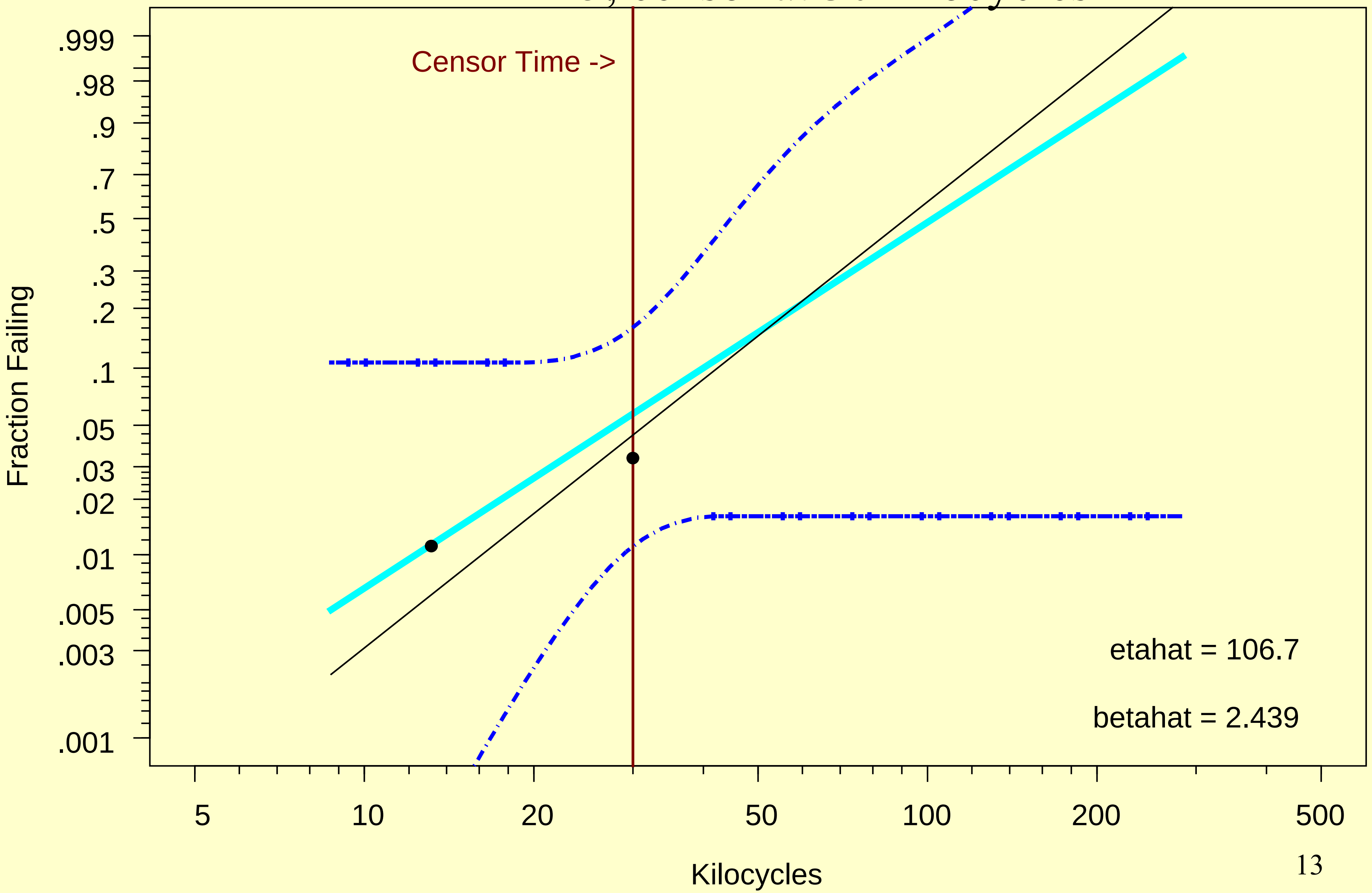
General Strategy for Test Planning: Anticipate Test Results

- Elicit planning information
- Use sample size tool (large sample approximation)
- Possibly optimize plan using large-sample approximation
- Simulate test test plan to provide
  - ❑ Visualization
  - ❑ Insight
  - ❑ Evaluation without approximation

Simulated Metal Spring Life Test

Simulated life test with sample size = 45
Weibull Distribution with eta= 123 and beta= 2
Weibull Probability Plot

n = 45, censor at 30 kilocycles



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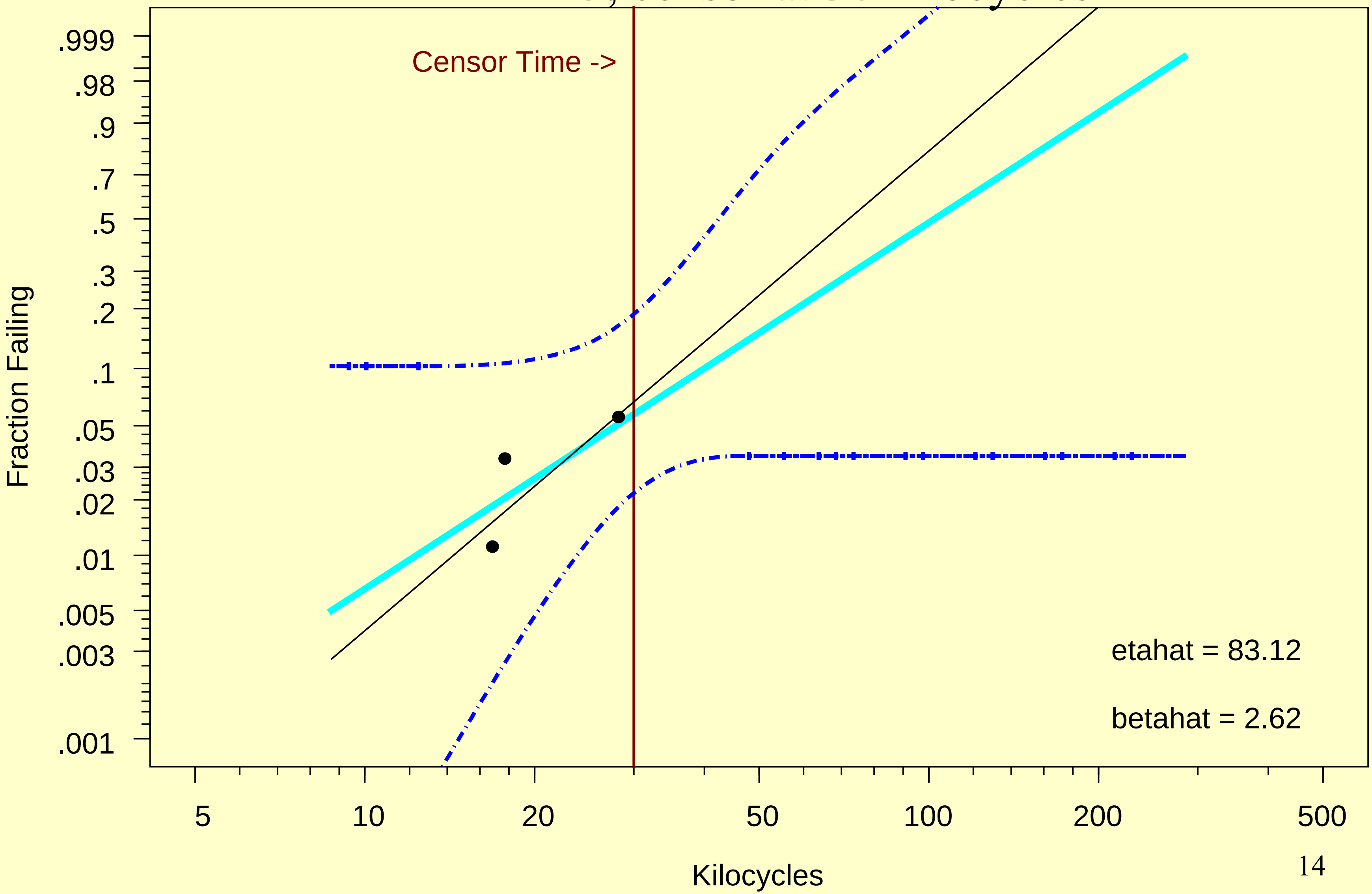

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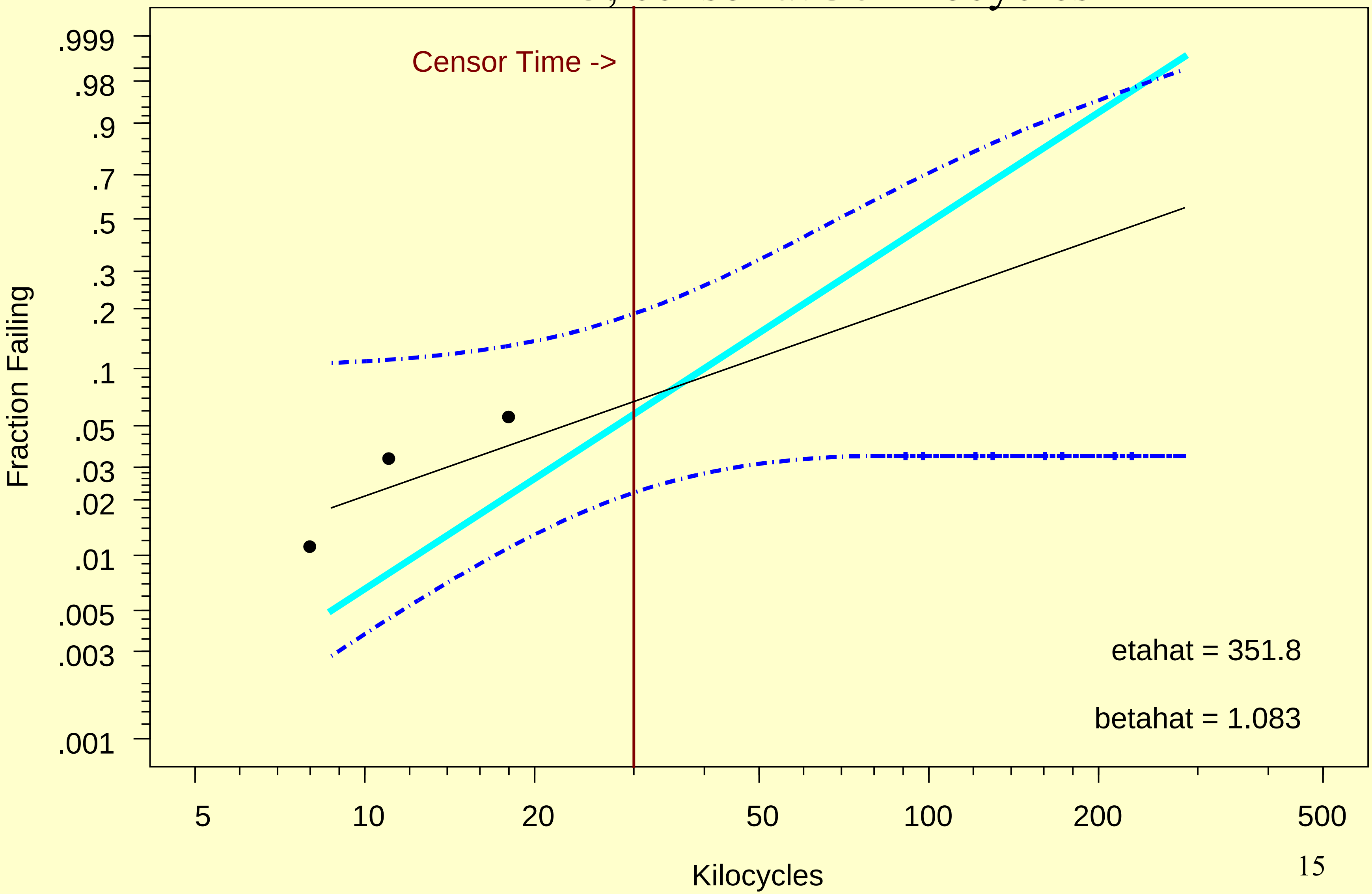
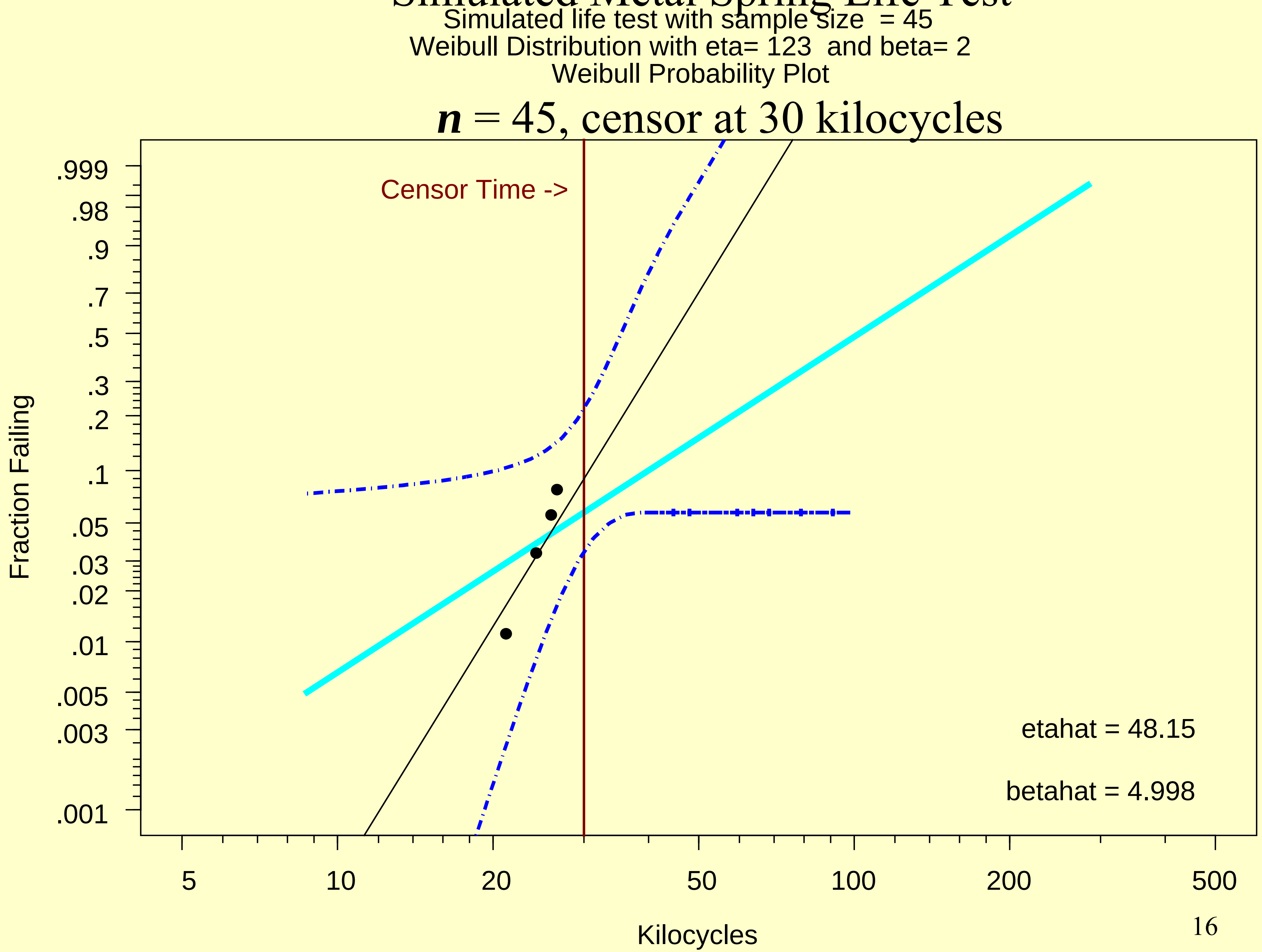


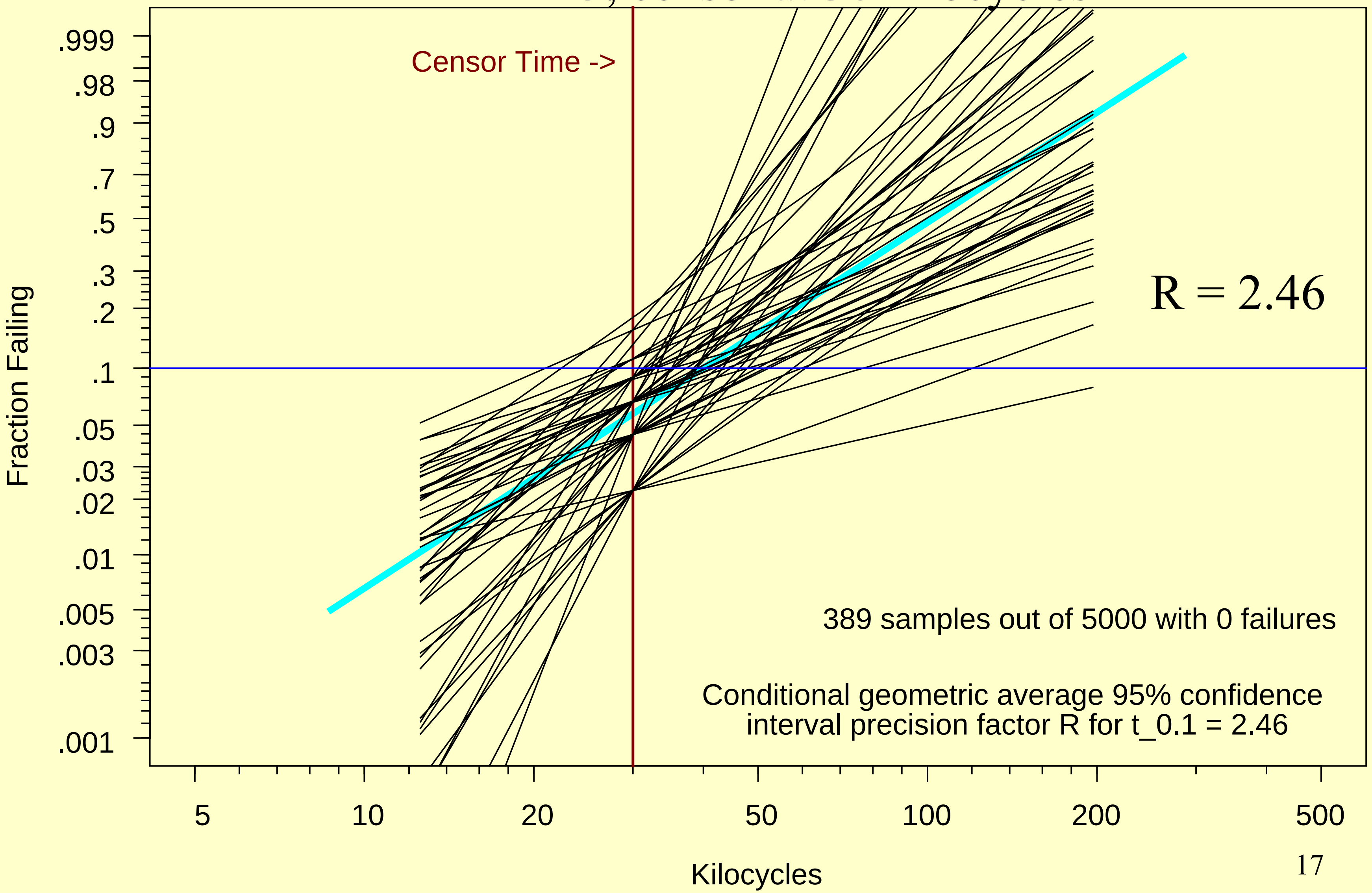
Simulated Metal Spring Life Test



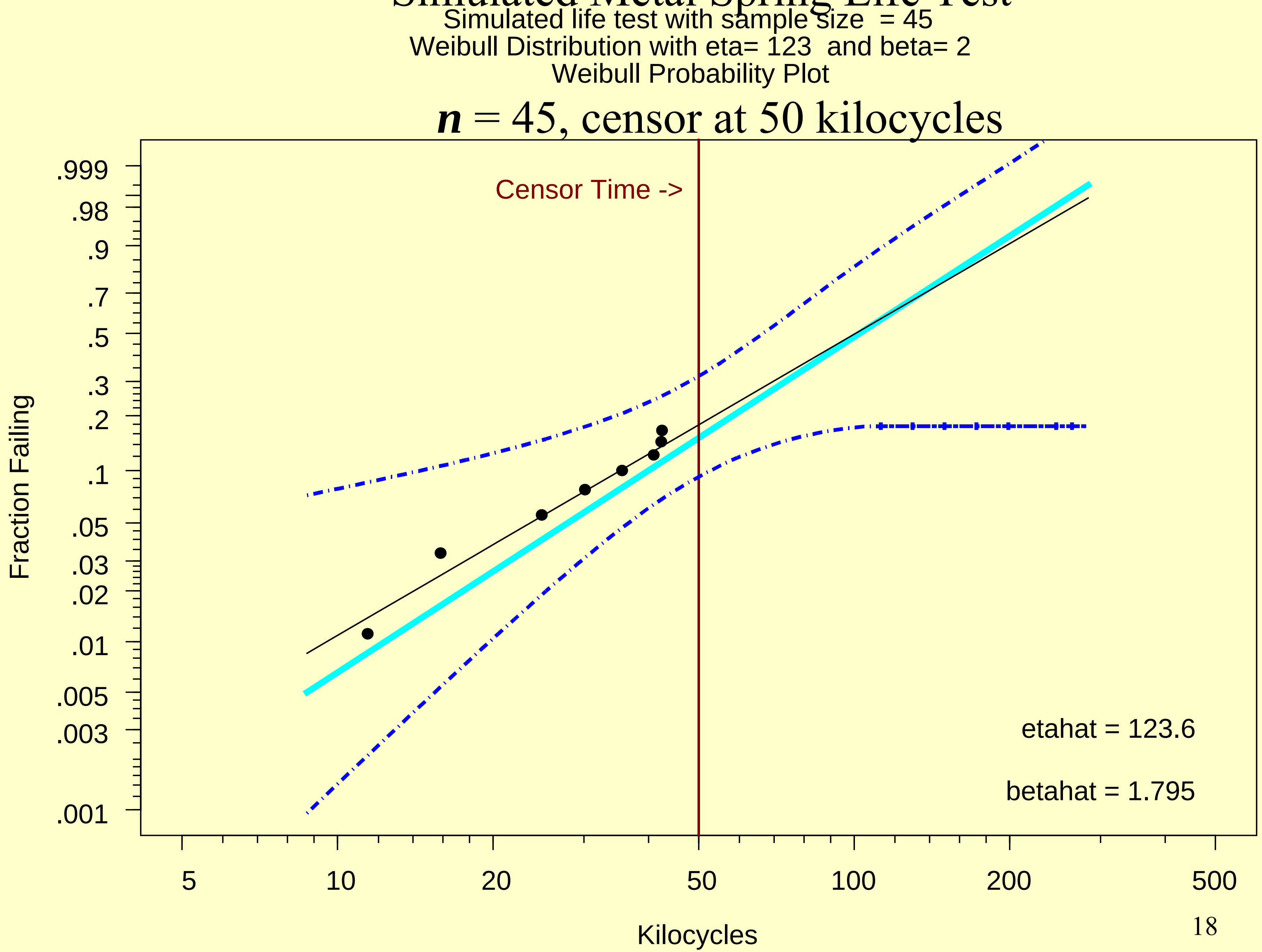
Summary of Simulated Metal Spring Life Test

5000 simulated life tests with sample size = 45 censored at 30 Kilocycles
Weibull Distribution with eta= 123 and beta= 2 with E(r)= 2.59
Weibull Probability Plot

n = 45, censor at 30 kilocycles

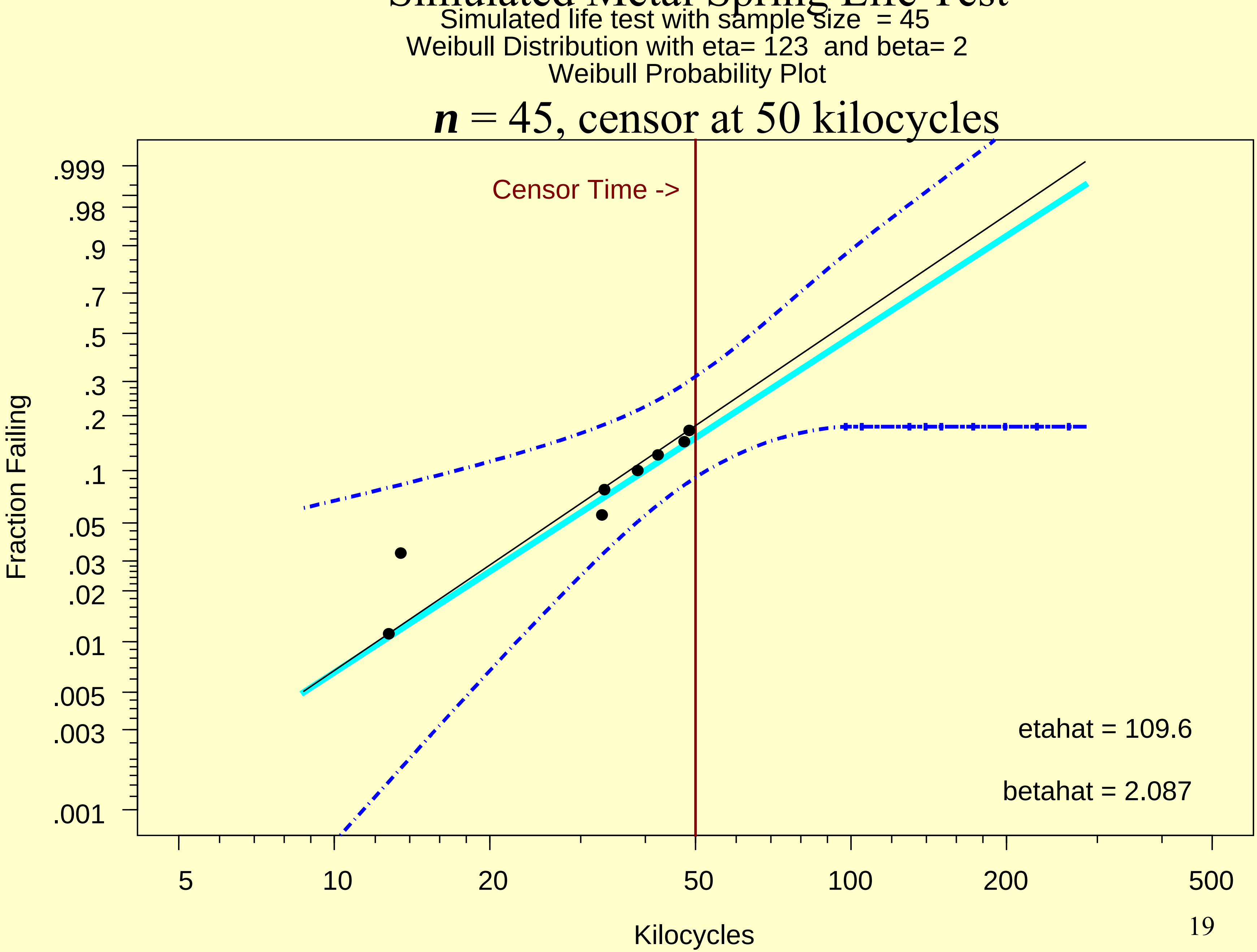


Simulated Metal Spring Life Test



n = 45, censor at 50 kilocycles

Simulated Metal Spring Life Test



Simulated Metal Spring Life Test

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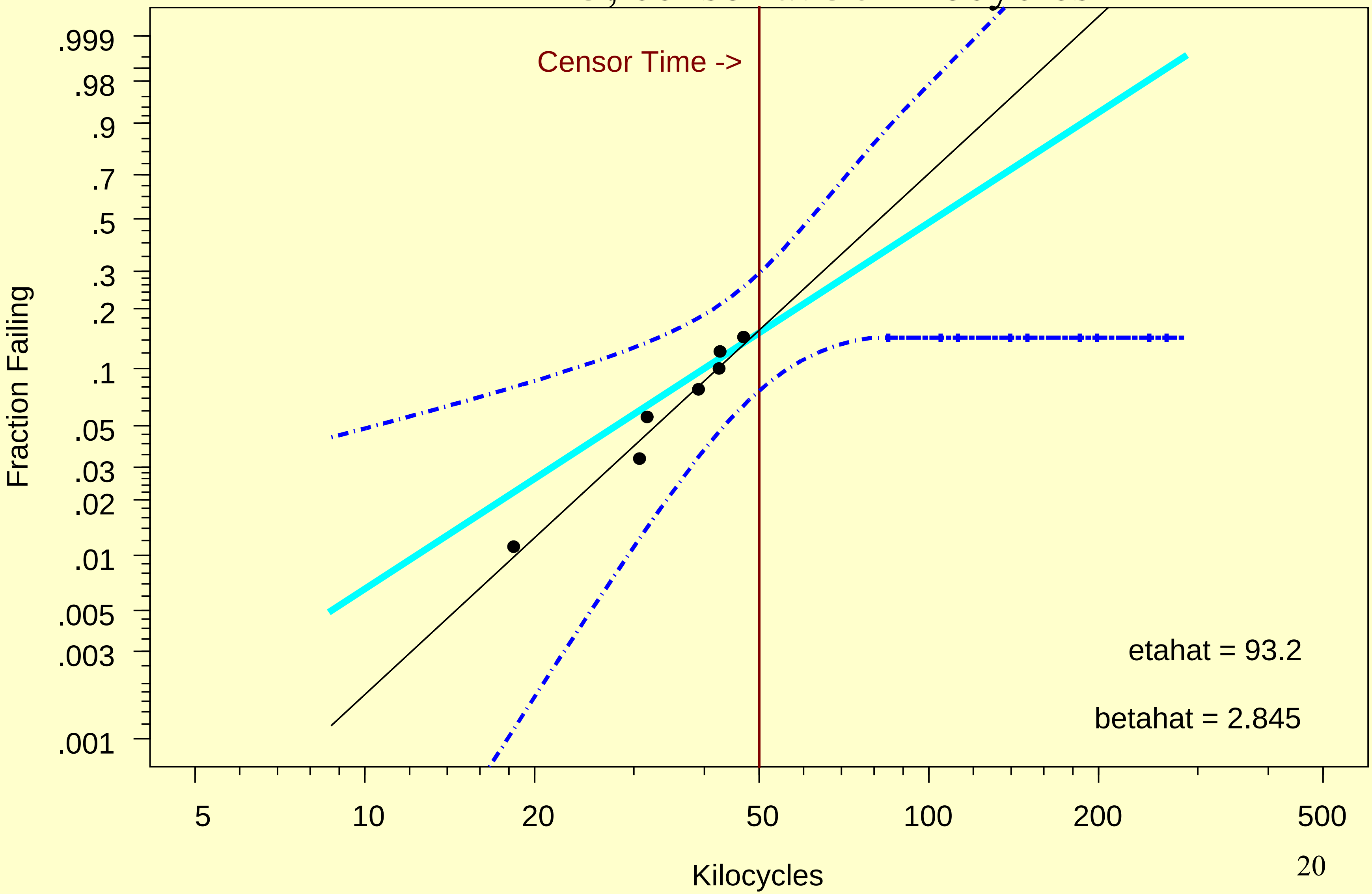
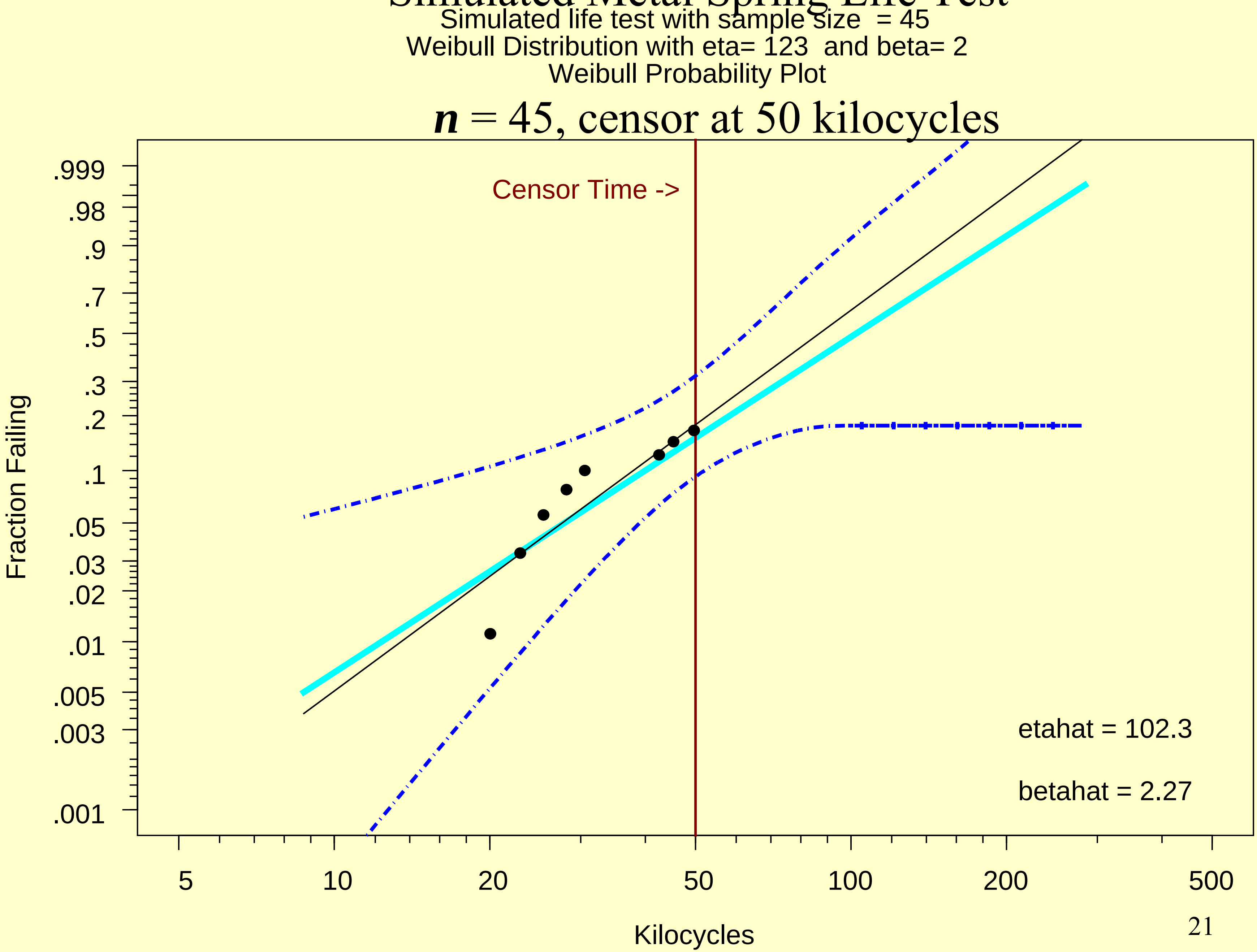
Weibull Probability Plot

n = 45, censor at 50 kilocycles



Simulated Metal Spring Life Test



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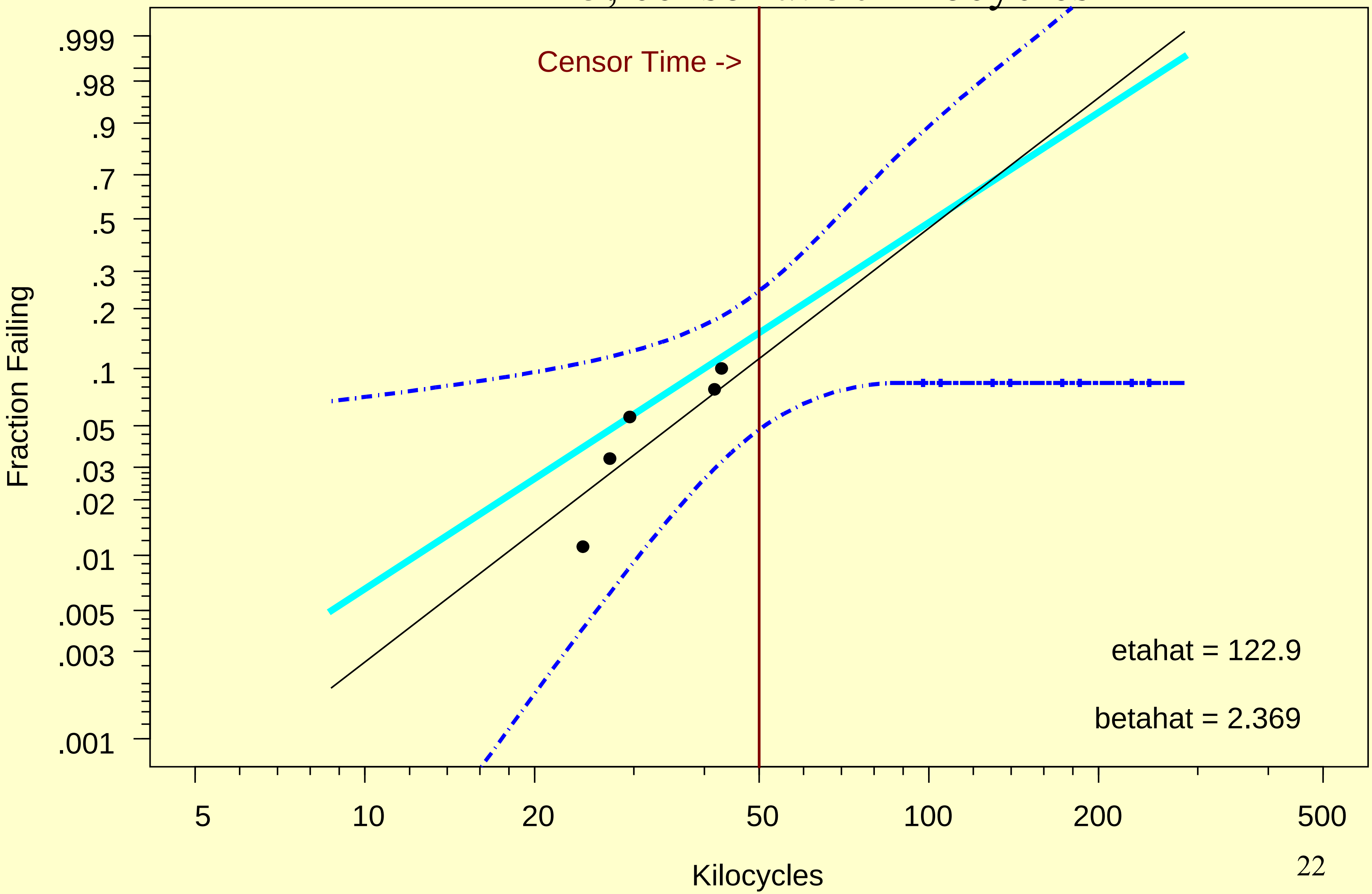
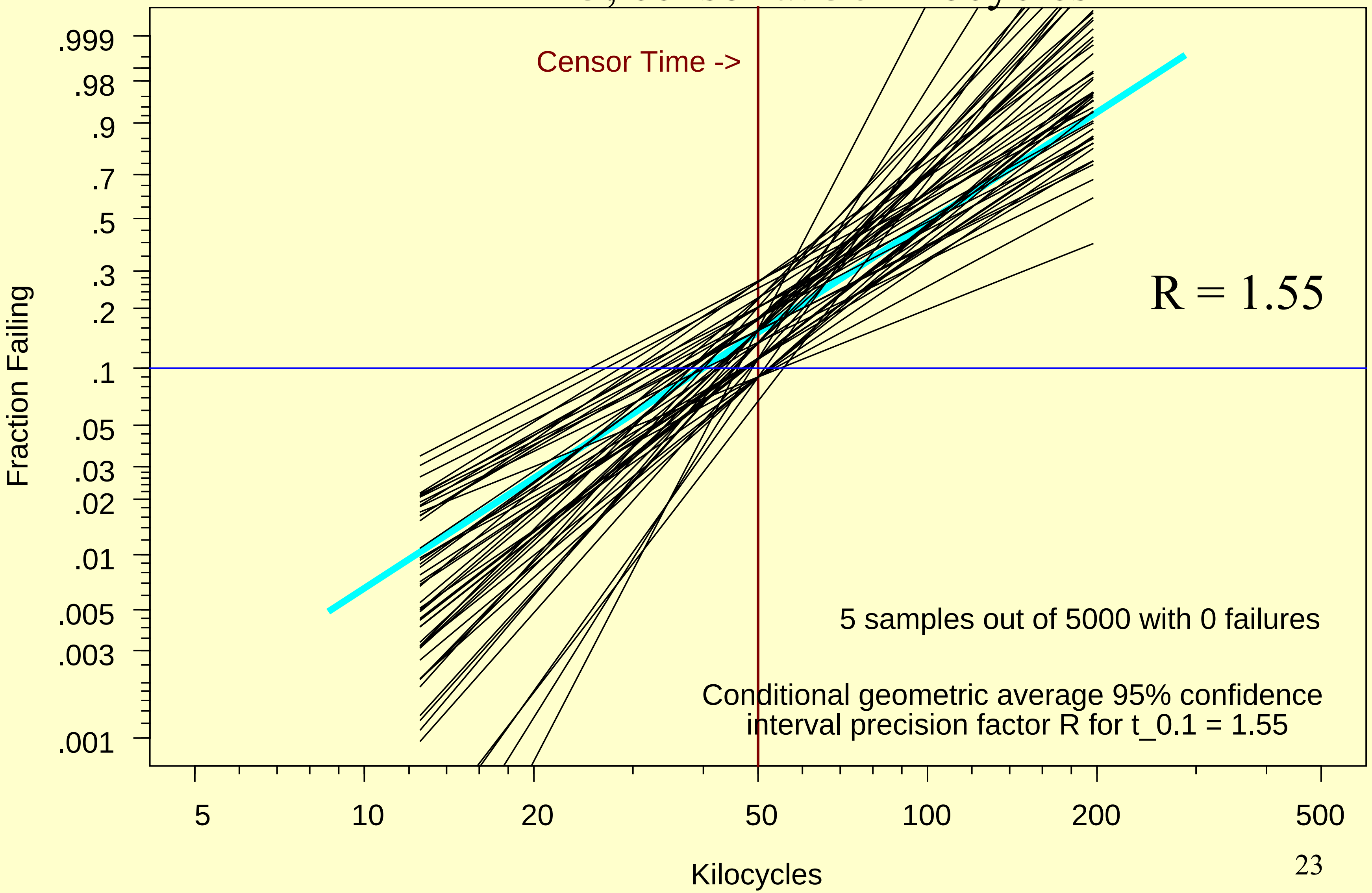


Summary of Simulated Metal Spring Life Test

5000 simulated life tests with sample size = 45 censored at 50 Kilocycles
Weibull Distribution with eta= 123 and beta= 2 with E(r)= 6.83
Weibull Probability Plot

n = 45, censor at 50 kilocycles



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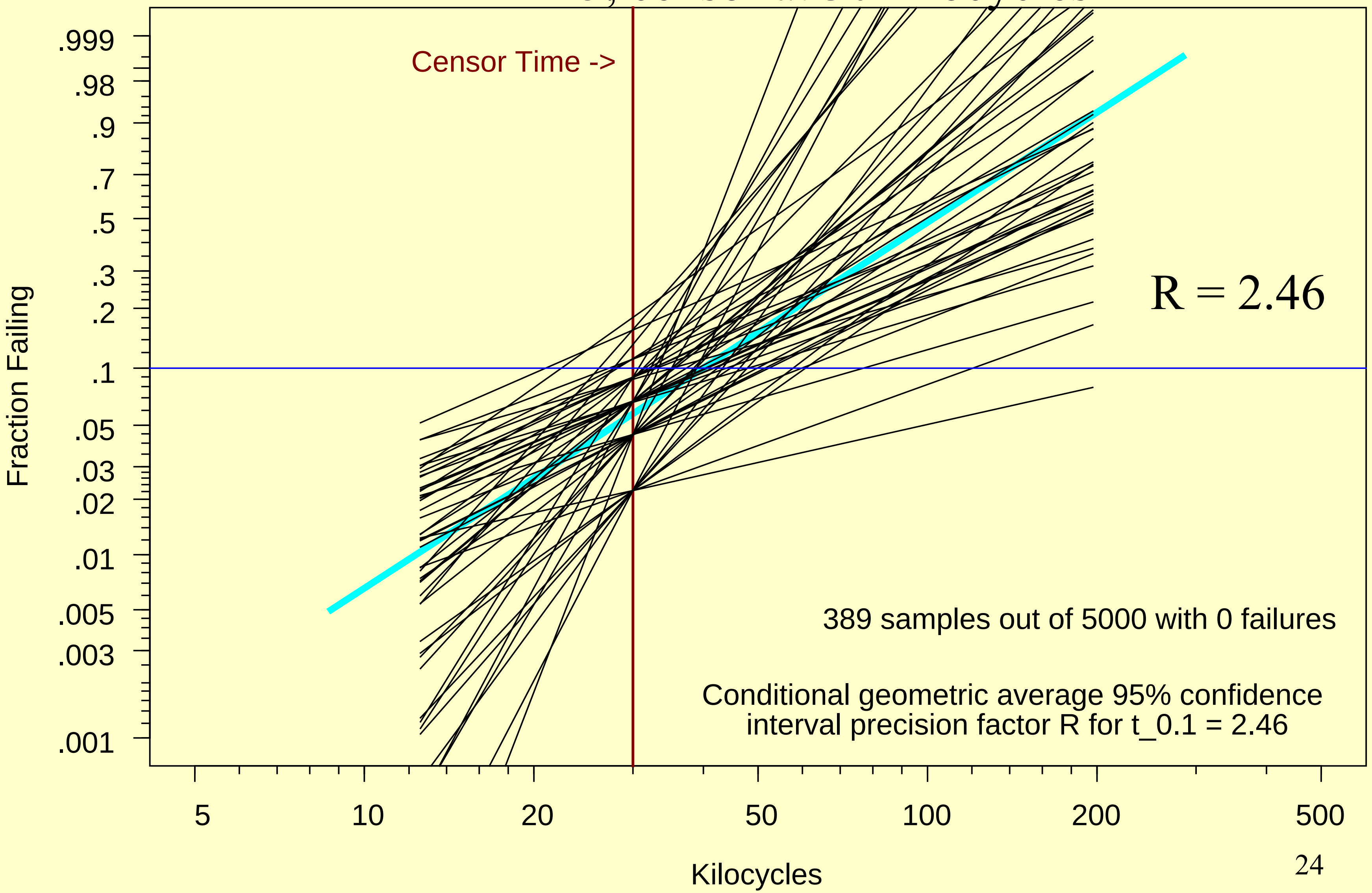
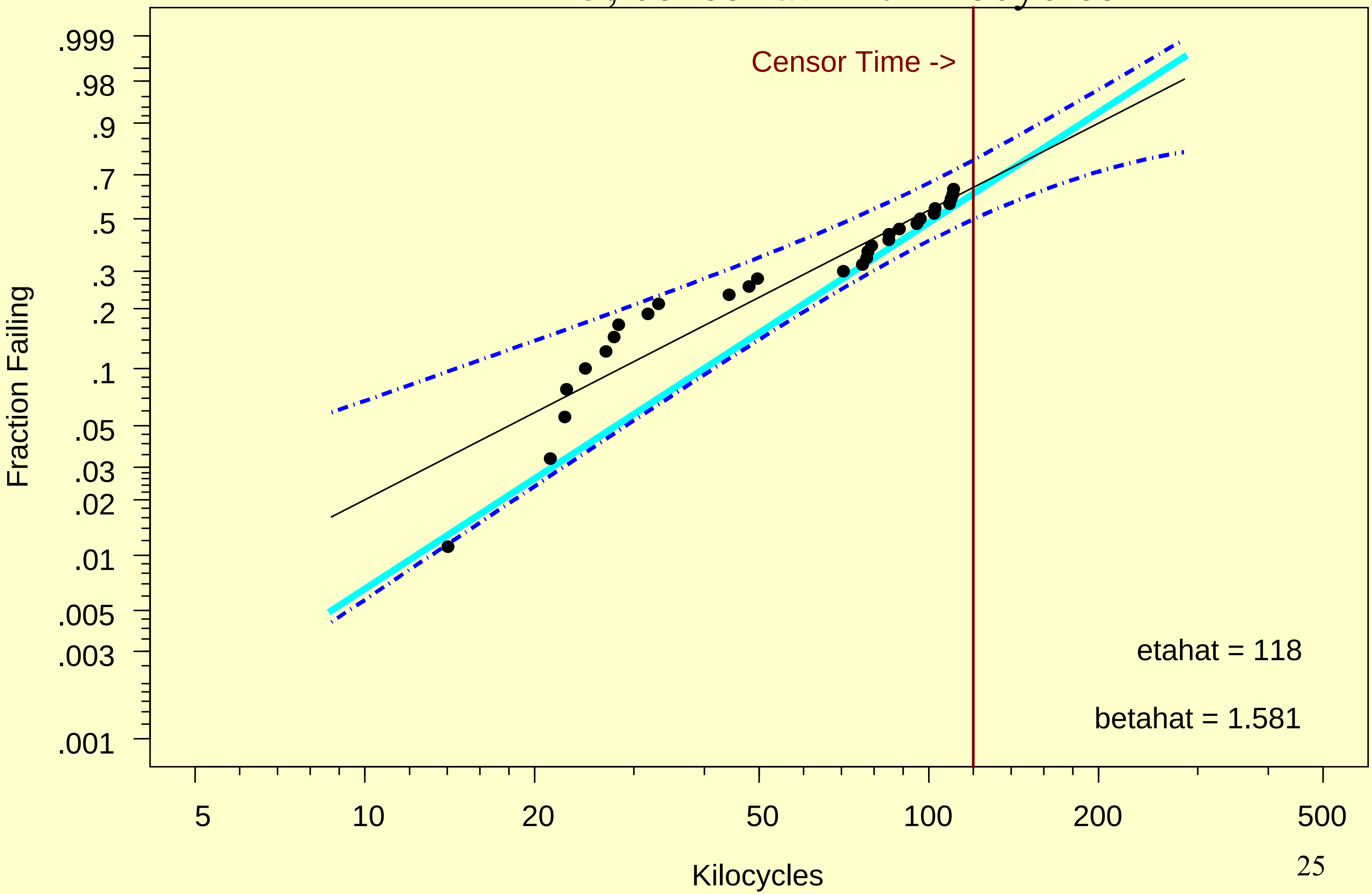


Simulated Metal Spring Life Test

Simulated life test with sample size = 45
Weibull Distribution with eta= 123 and beta= 2
Weibull Probability Plot

n = 45, censor at 120 kilocycles



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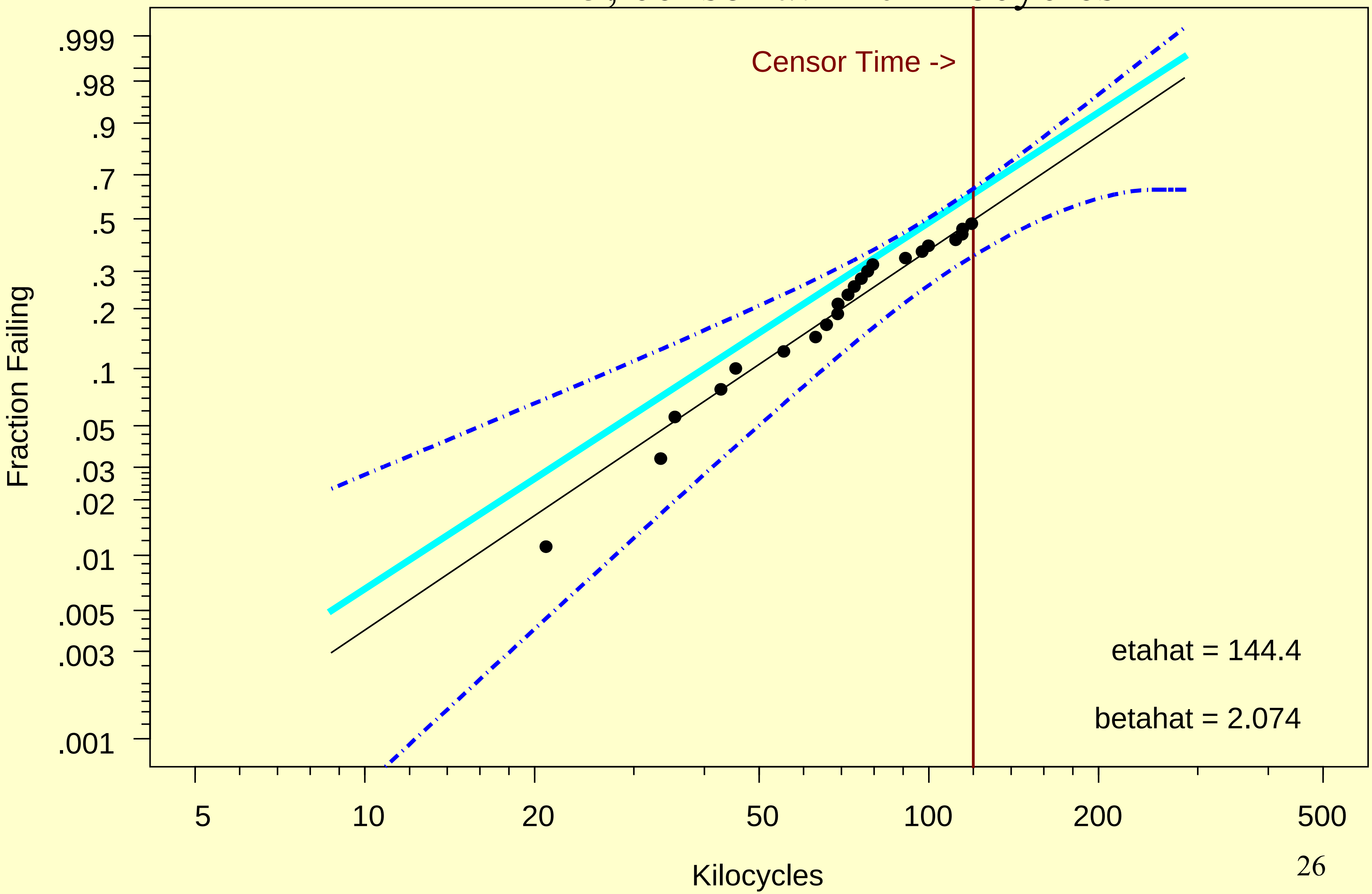
n = 45, censor at 120 kilocycles



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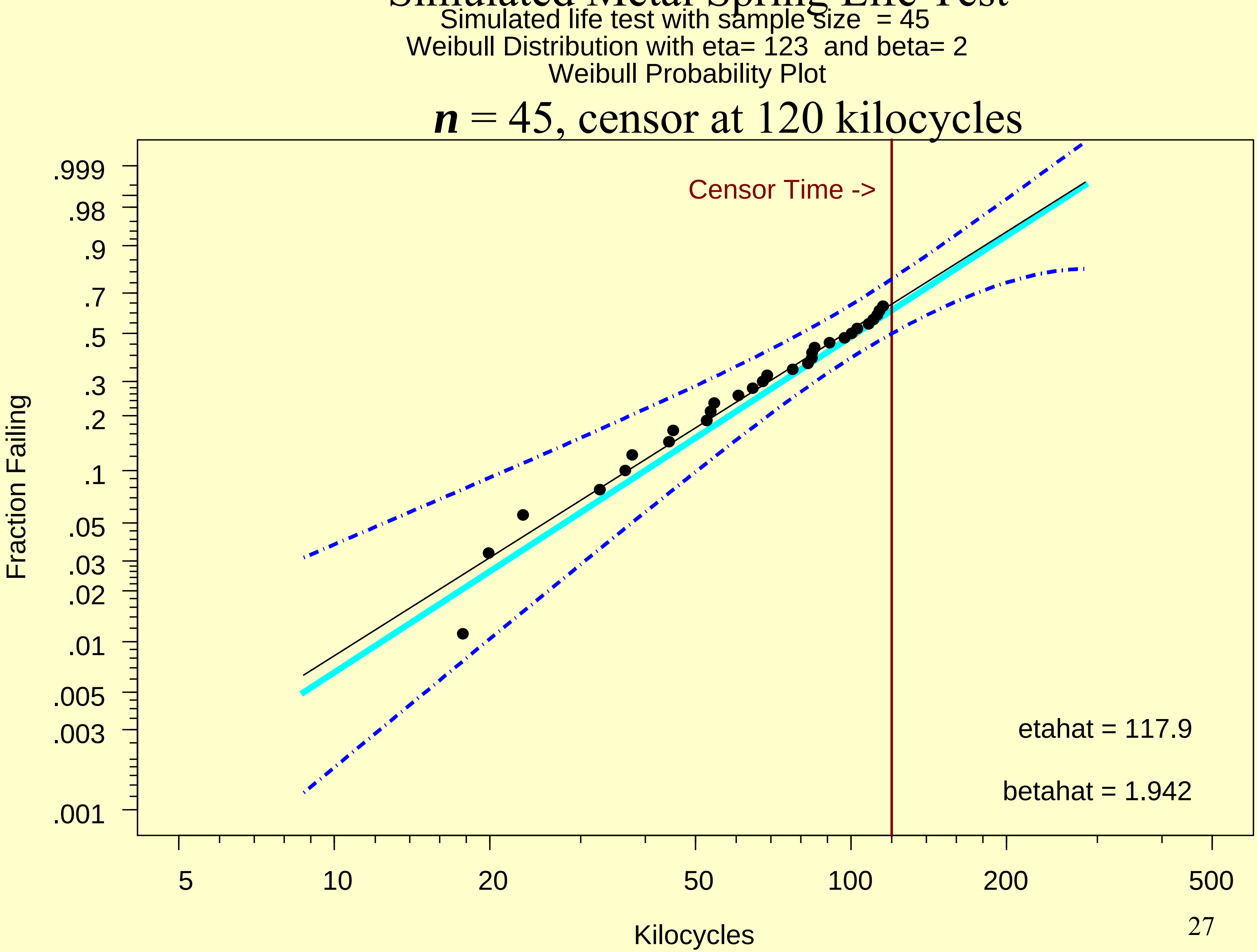

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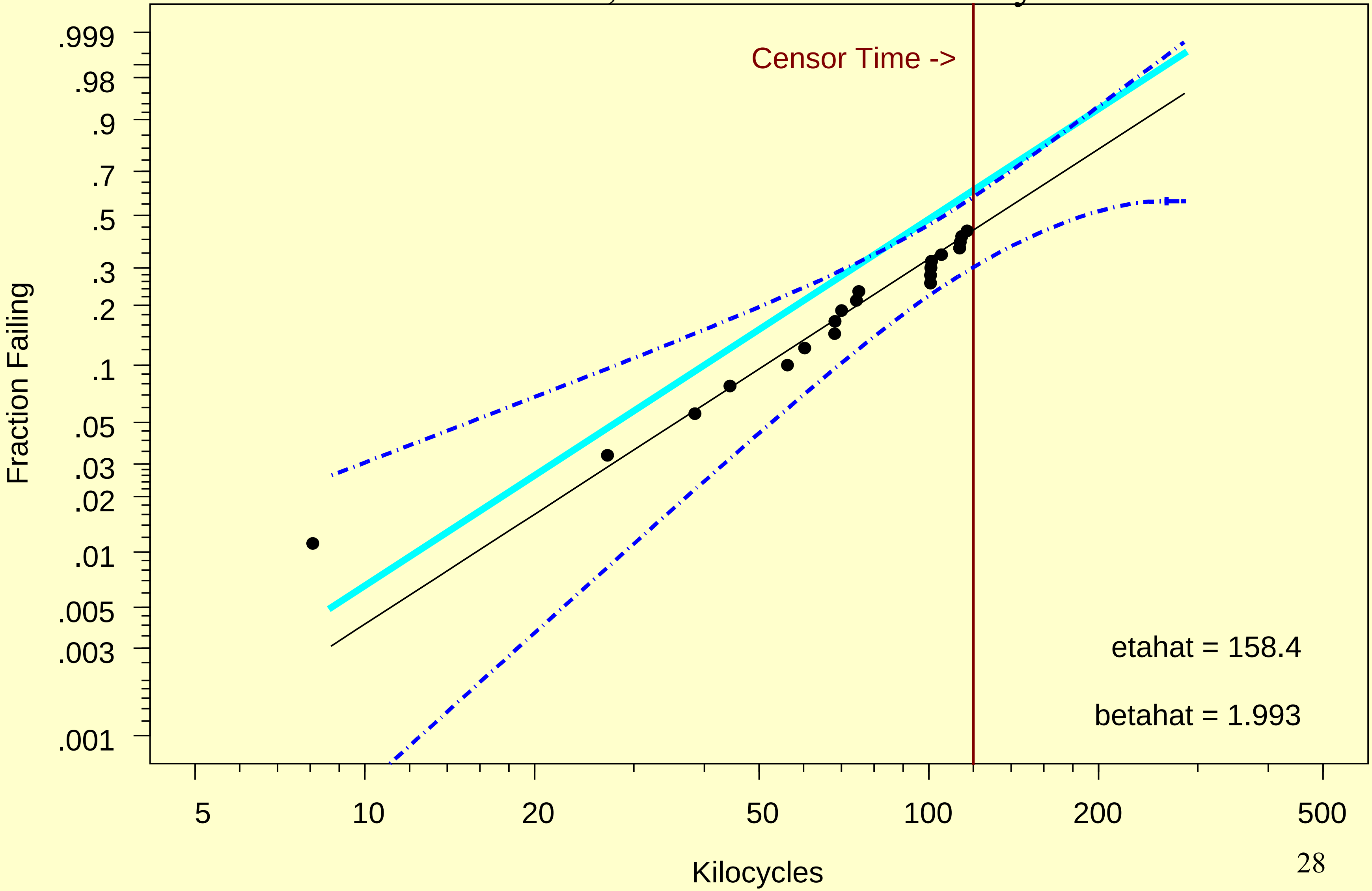

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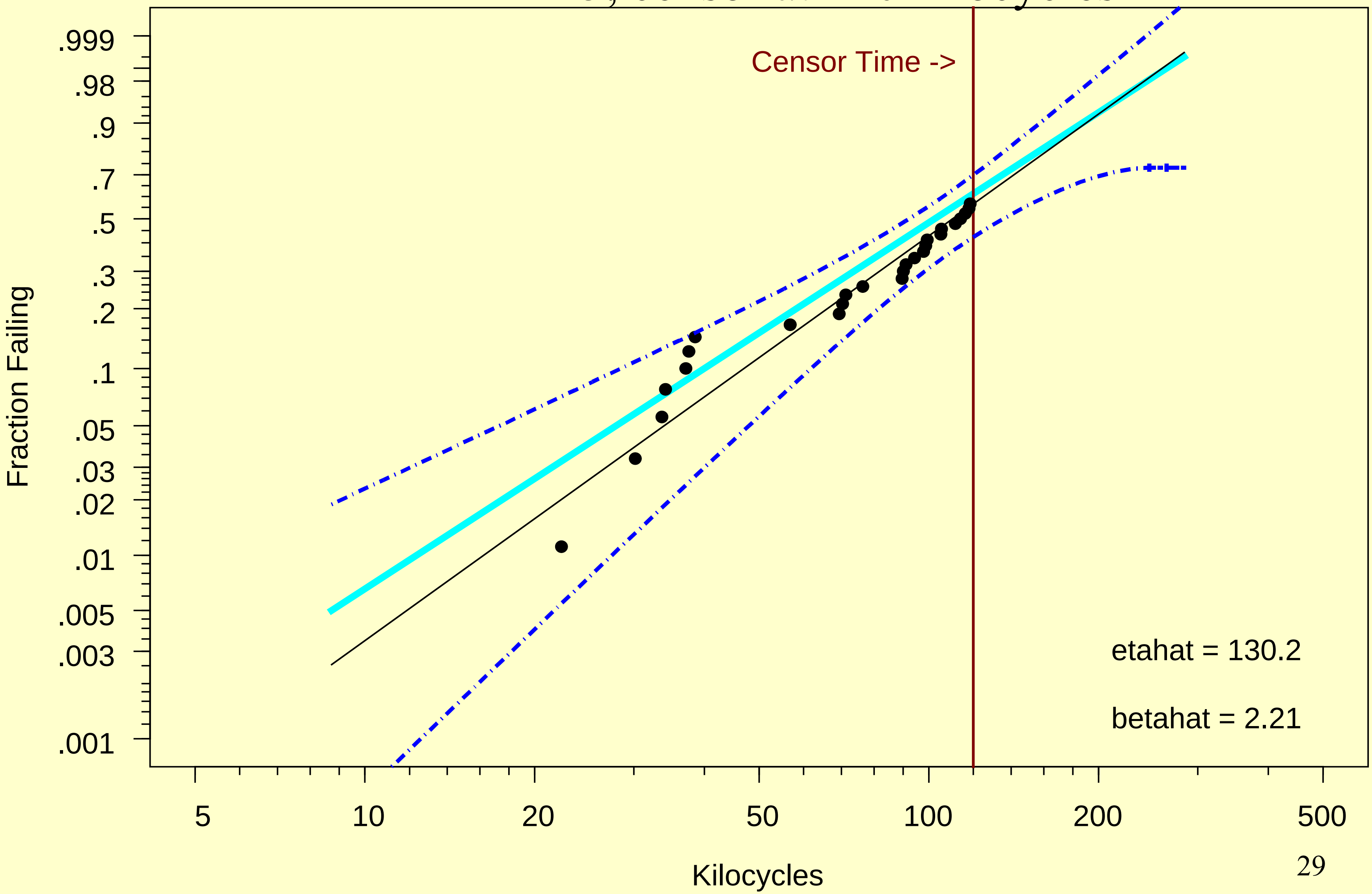
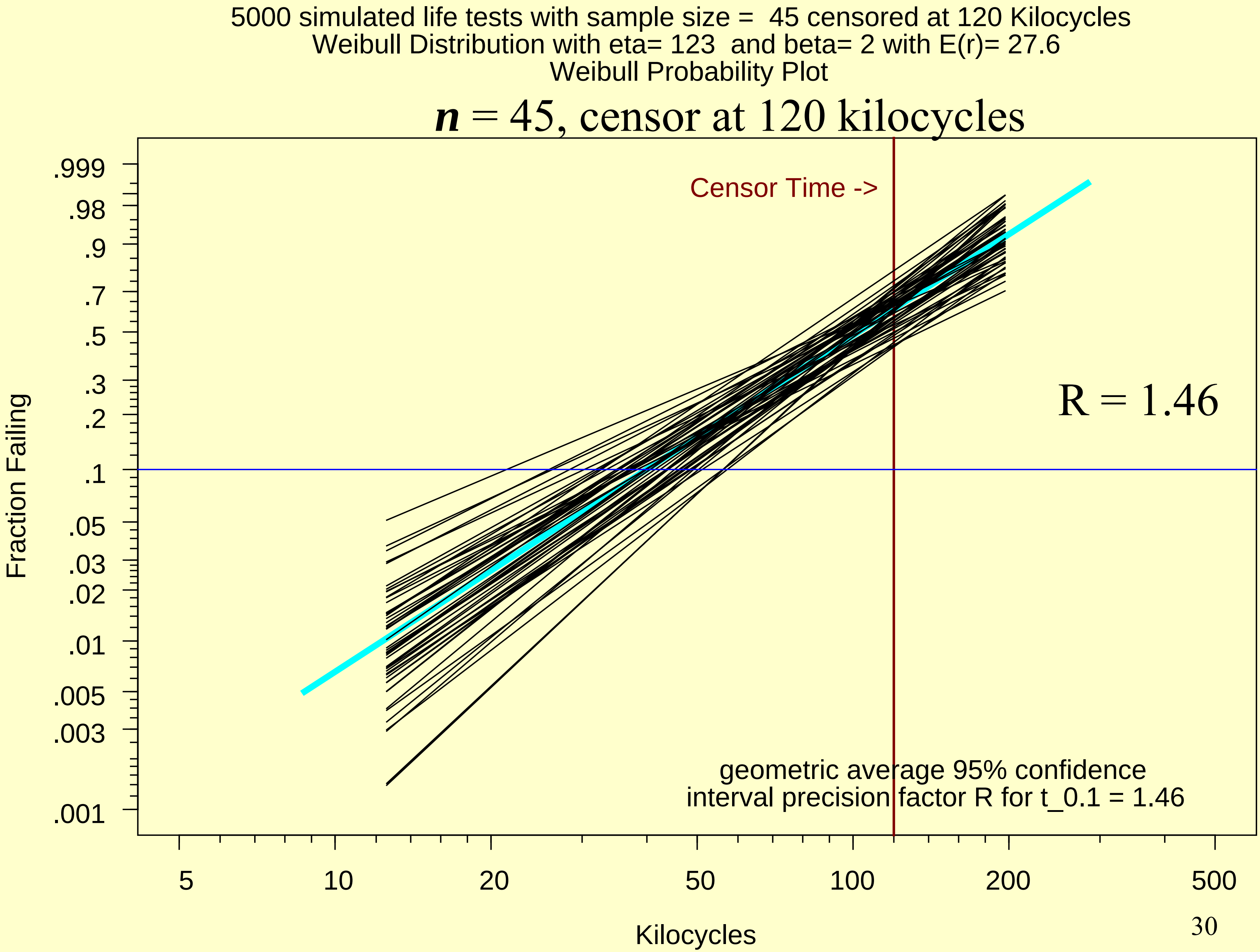


n = 45, censor at 120 kilocycles

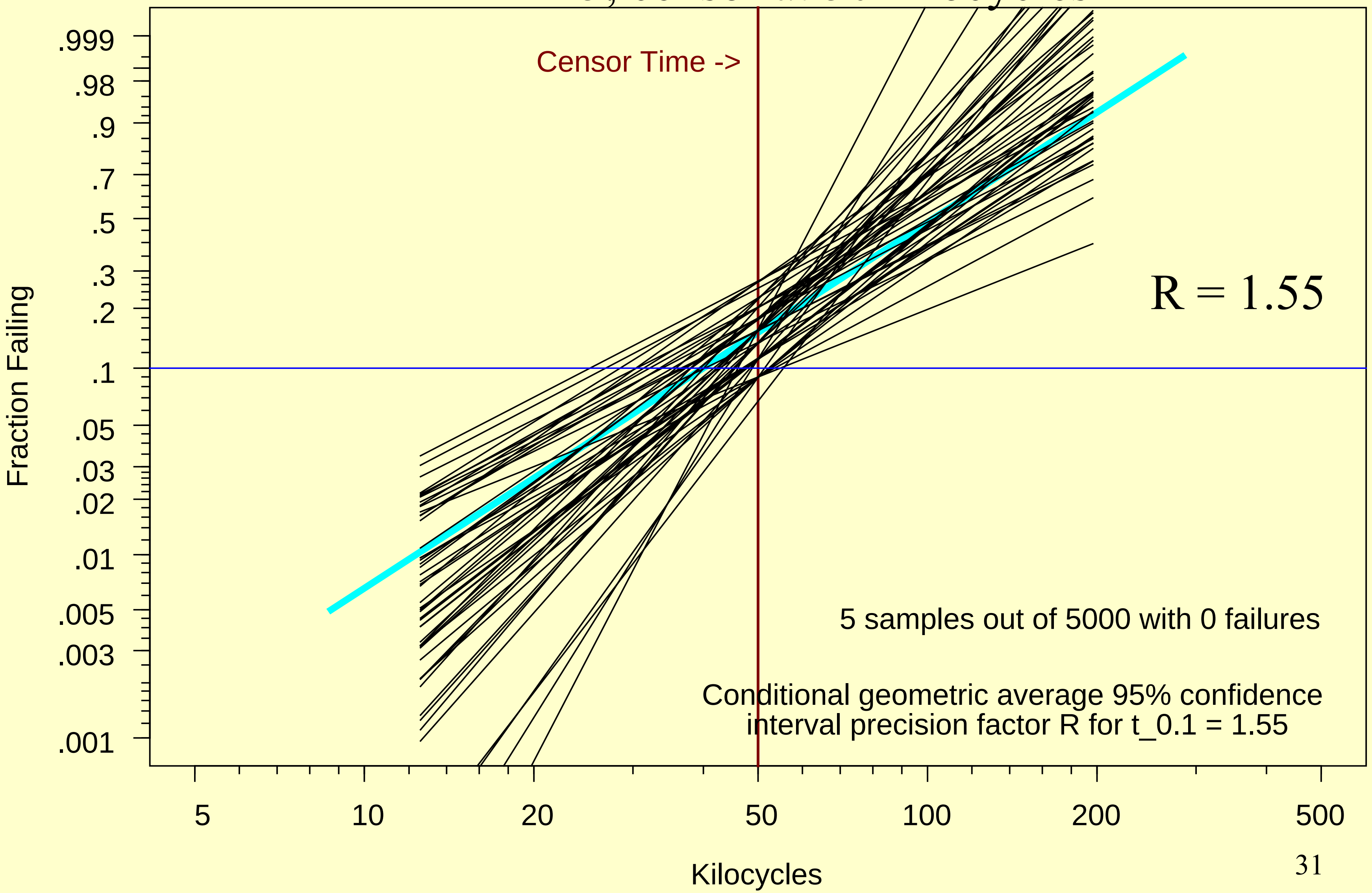


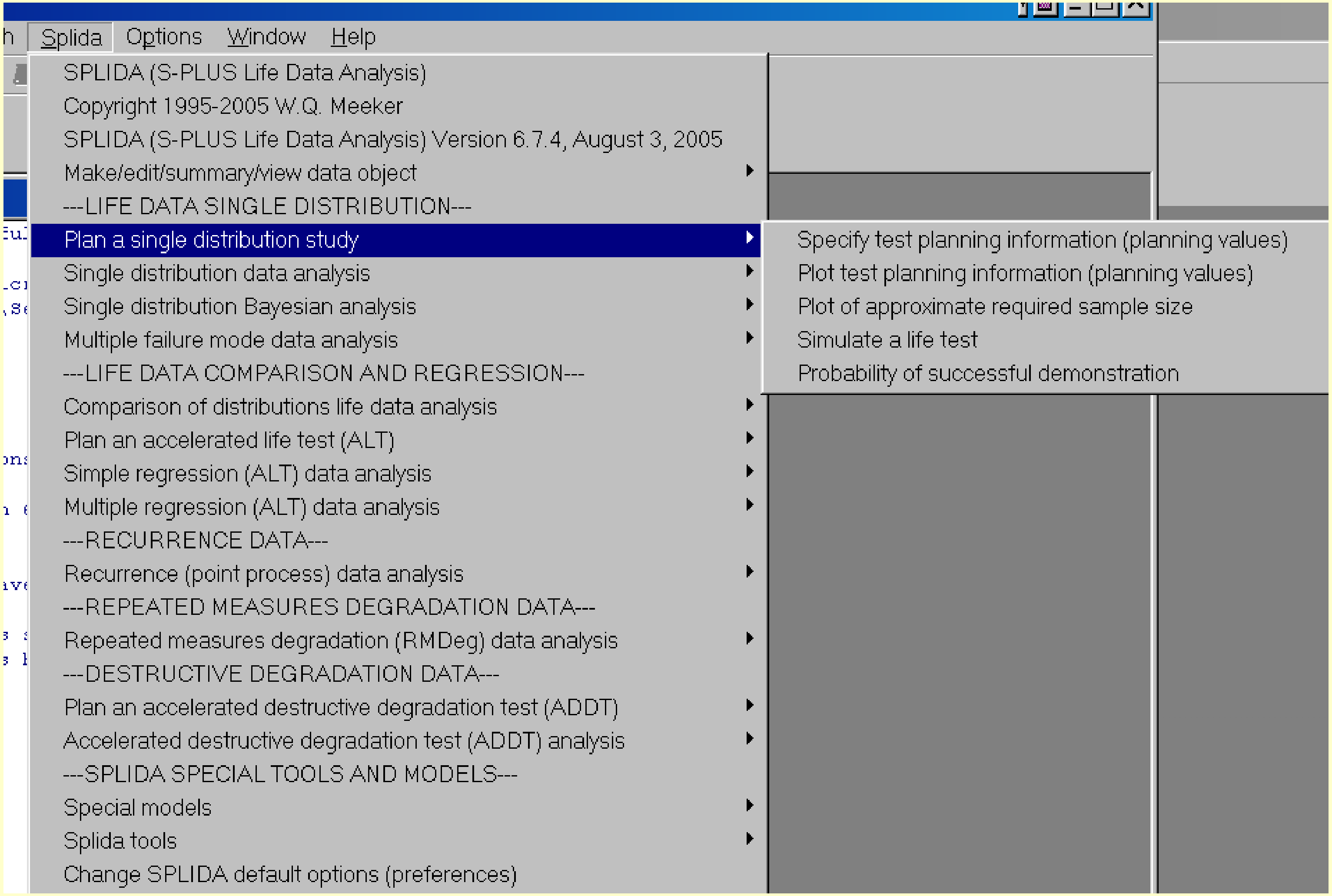
R = 1.46

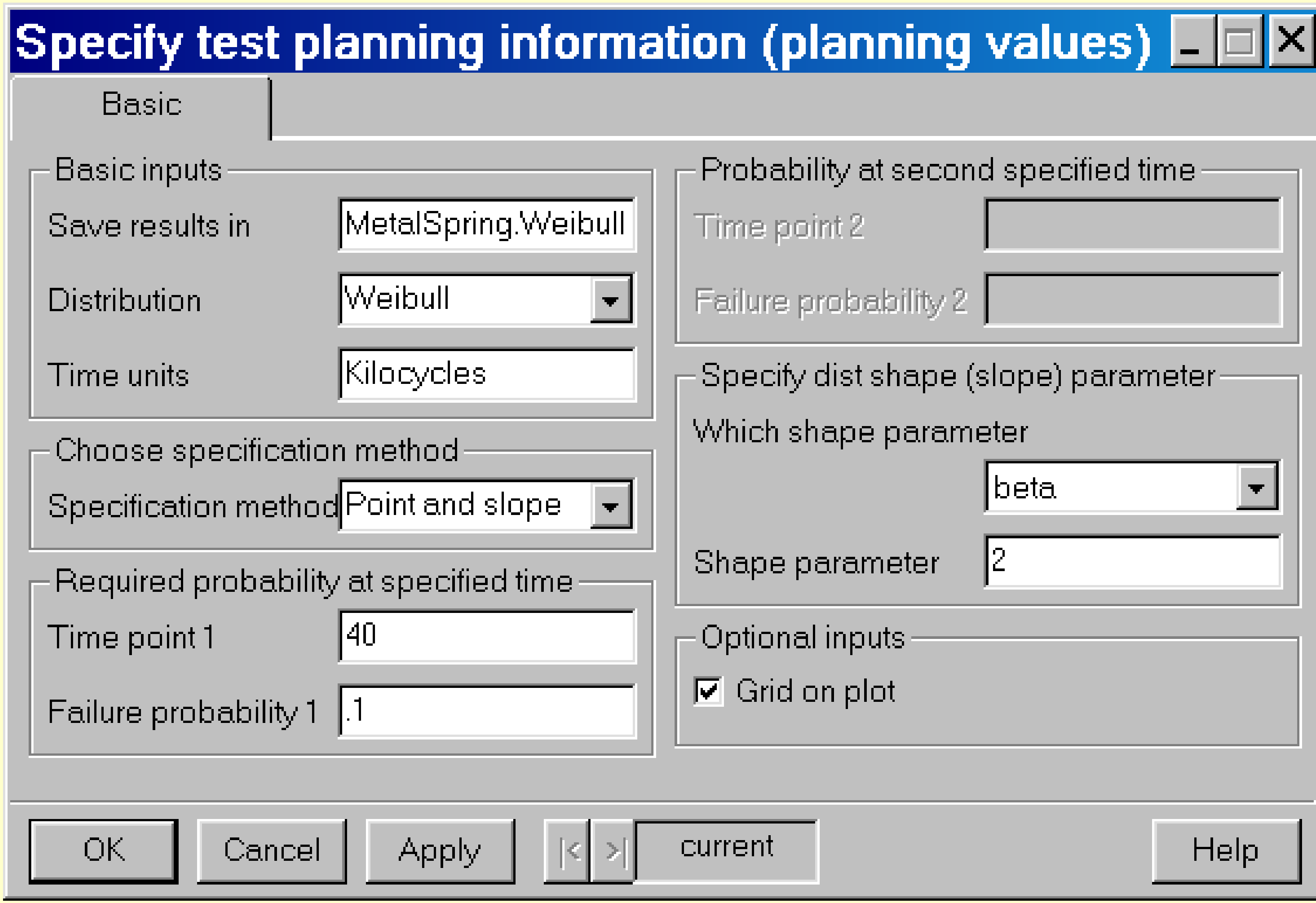
Summary of Simulated Metal Spring Life Test

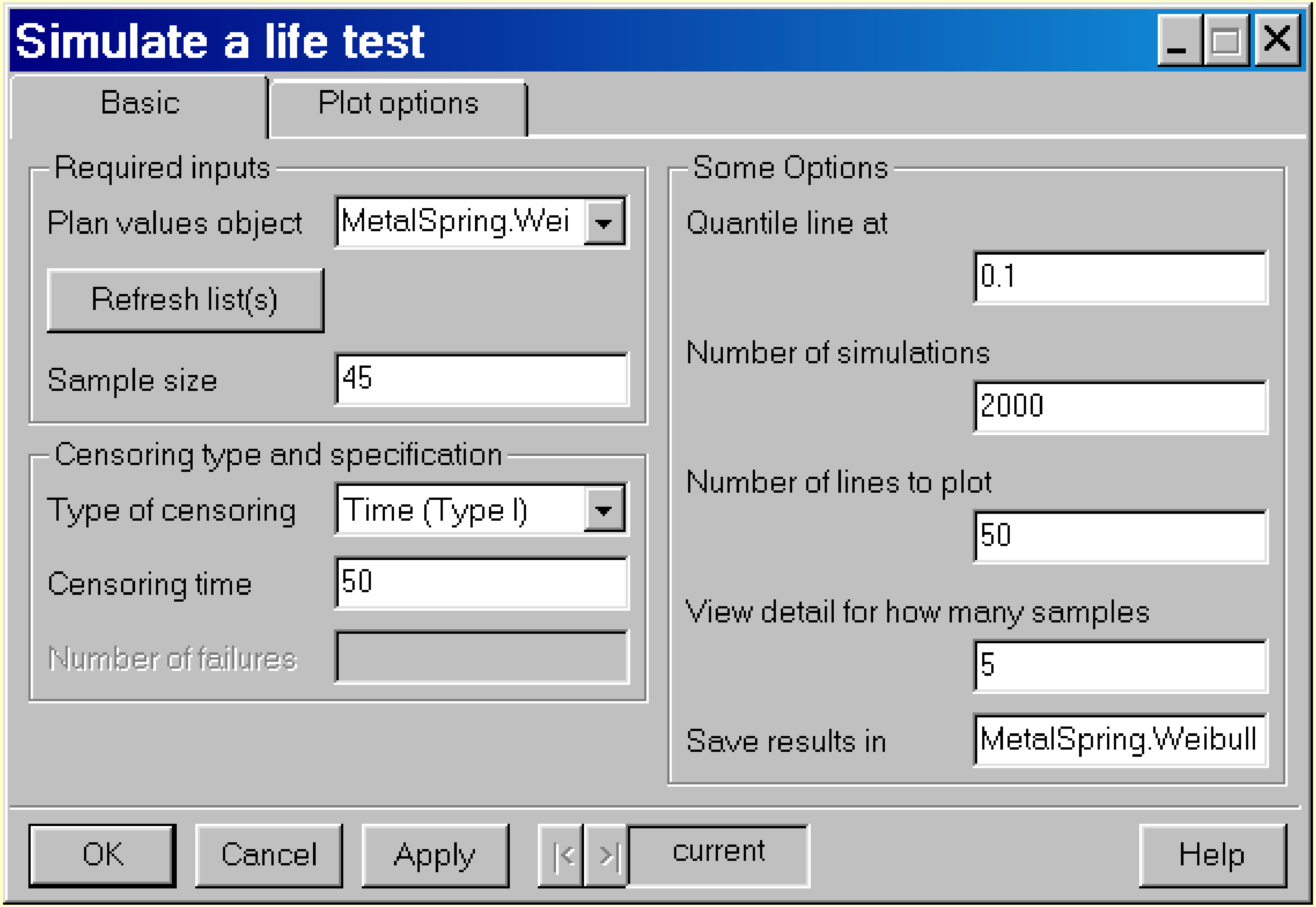
5000 simulated life tests with sample size = 45 censored at 50 Kilocycles
Weibull Distribution with eta= 123 and beta= 2 with E(r)= 6.83
Weibull Probability Plot

n = 45, censor at 50 kilocycles




Splida Options Window Help
SPLIDA (S-PLUS Life Data Analysis)
Copyright 1995-2005 W.Q. Meeker
SPLIDA (S-PLUS Life Data Analysis) Version 6.7.4, August 3, 2005
Make/edit/summary/view data object
---LIFE DATA SINGLE DISTRIBUTION---
Plan a single distribution study
Single distribution data analysis
Single distribution Bayesian analysis
Multiple failure mode data analysis
---LIFE DATA COMPARISON AND REGRESSION---
Comparison of distributions life data analysis
Plan an accelerated life test (ALT)
Simple regression (ALT) data analysis
Multiple regression (ALT) data analysis
---RECURRENCE DATA---
Recurrence (point process) data analysis
---REPEATED MEASURES DEGRADATION DATA---
Repeated measures degradation (RMDeg) data analysis
---DESTRUCTIVE DEGRADATION DATA---
Plan an accelerated destructive degradation test (ADDT)
Accelerated destructive degradation test (ADDT) analysis
---SPLIDA SPECIAL TOOLS AND MODELS---
Special models
Splida tools
Change SPLIDA default options (preferences)
Specify test planning information (planning values)
Plot test planning information (planning values)
Plot of approximate required sample size
Simulate a life test
Probability of successful demonstration

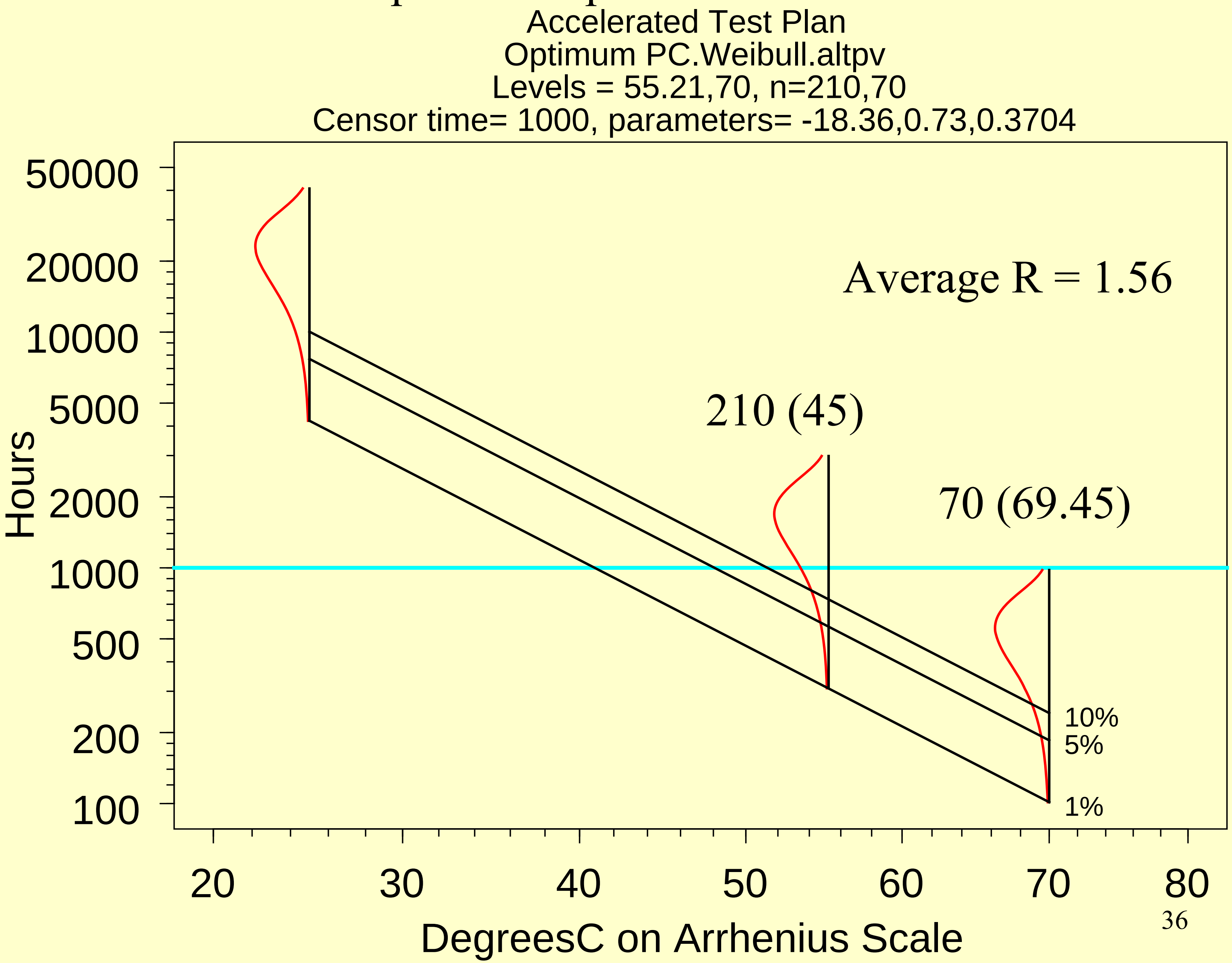

Specify test planning information (planning values)
Basic
Basic inputs
Save results in
MetalSpring.Weibull
Distribution
Weibull
Time units
Kilocycles
Choose specification method
Specification method
Point and slope
Required probability at specified time
Time point 1
40
Failure probability 1
.1
Probability at second specified time
Time point 2
Failure probability 2
Specify dist shape (slope) parameter
Which shape parameter
beta
Shape parameter
2
Optional inputs
Grid on plot
OK
Cancel
Apply
current
Help


Simulate a life test
Basic
Plot options
Required inputs
Plan values object
MetalSpring.Wei
Refresh list(s)
Sample size
45
Censoring type and specification
Type of censoring
Time (Type I)
Censoring time
50
Number of failures
Some Options
Quantile line at
0.1
Number of simulations
2000
Number of lines to plot
50
View detail for how many samples
5
Save results in
MetalSpring.Weibull
OK
Cancel
Apply
current
Help

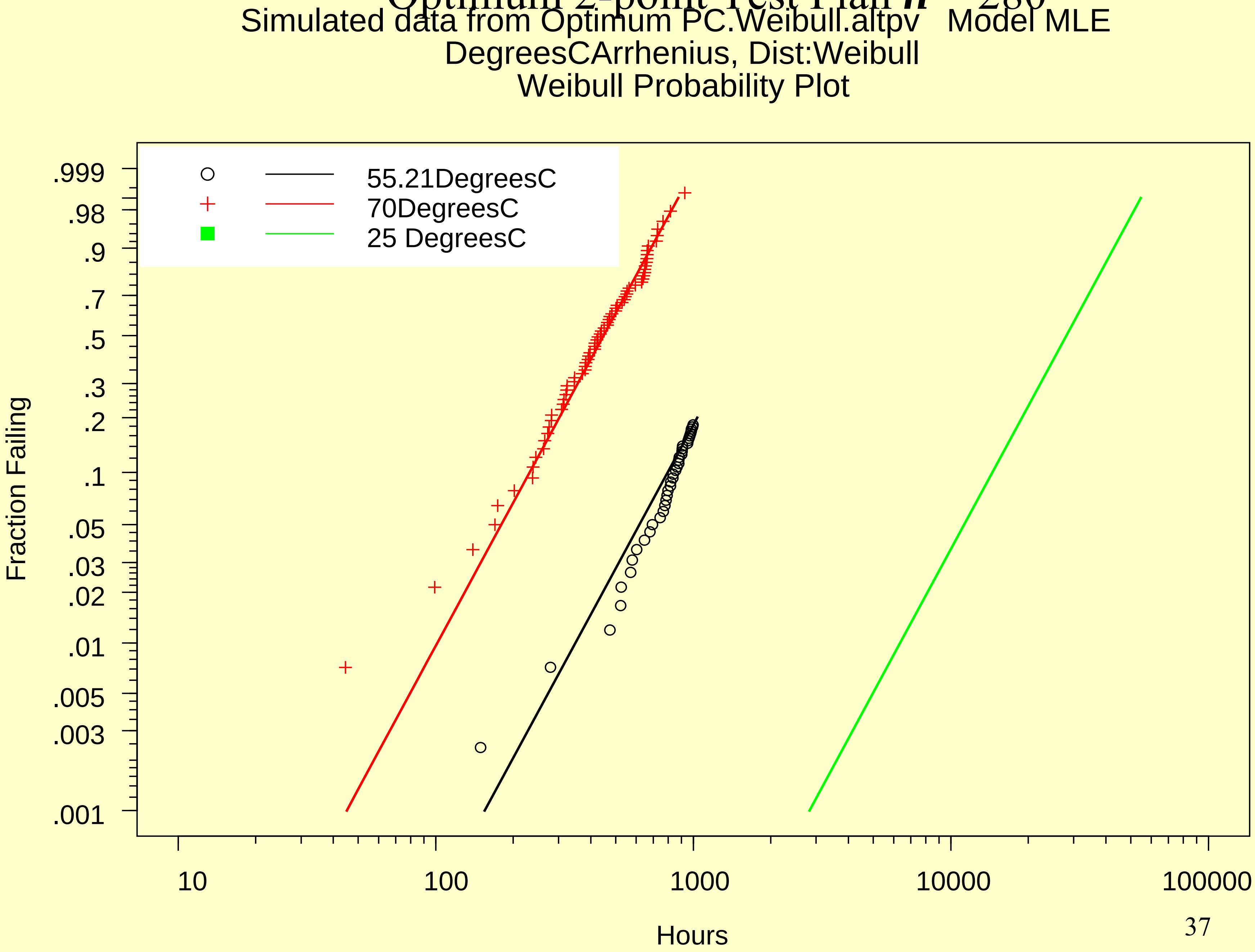
Planning an *Accelerated Life Test* for a Protective Coating for a Circuit Board

- Want to estimate B01 life at 25 Degrees C
- Can test at temperatures up to 70 Degrees C
- Planning values
  - Weibull distribution with shape $\beta = 2.7$
  - Activation energy = 0.73 eV
  - B01.2 = 500 hours at 50 Degrees C
- Time censoring required at 1000 hours
- Can test 280 units

Optimum 2-point Test Plan ***n*** = 280



Optimum 2-point Test Plan *n* = 280



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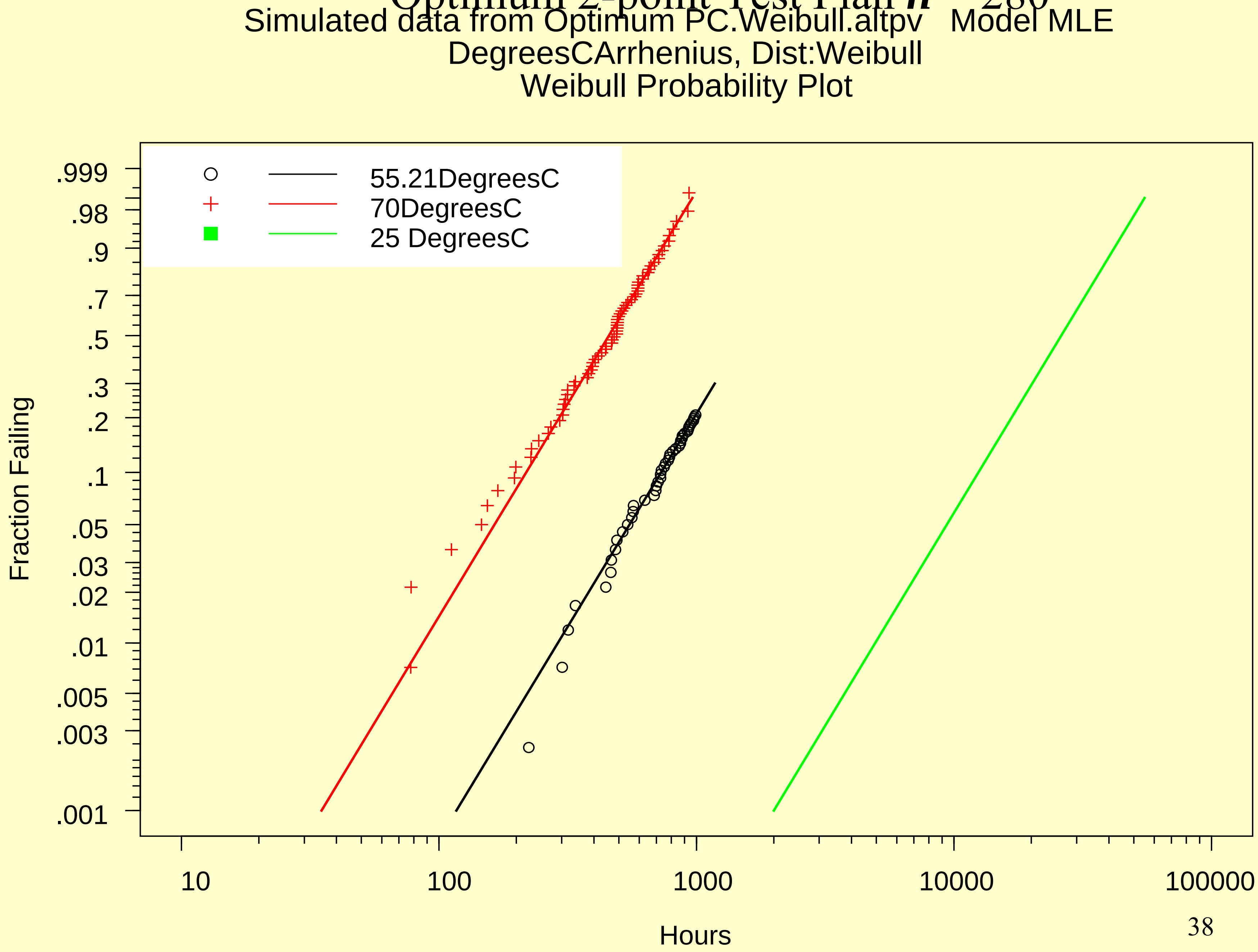

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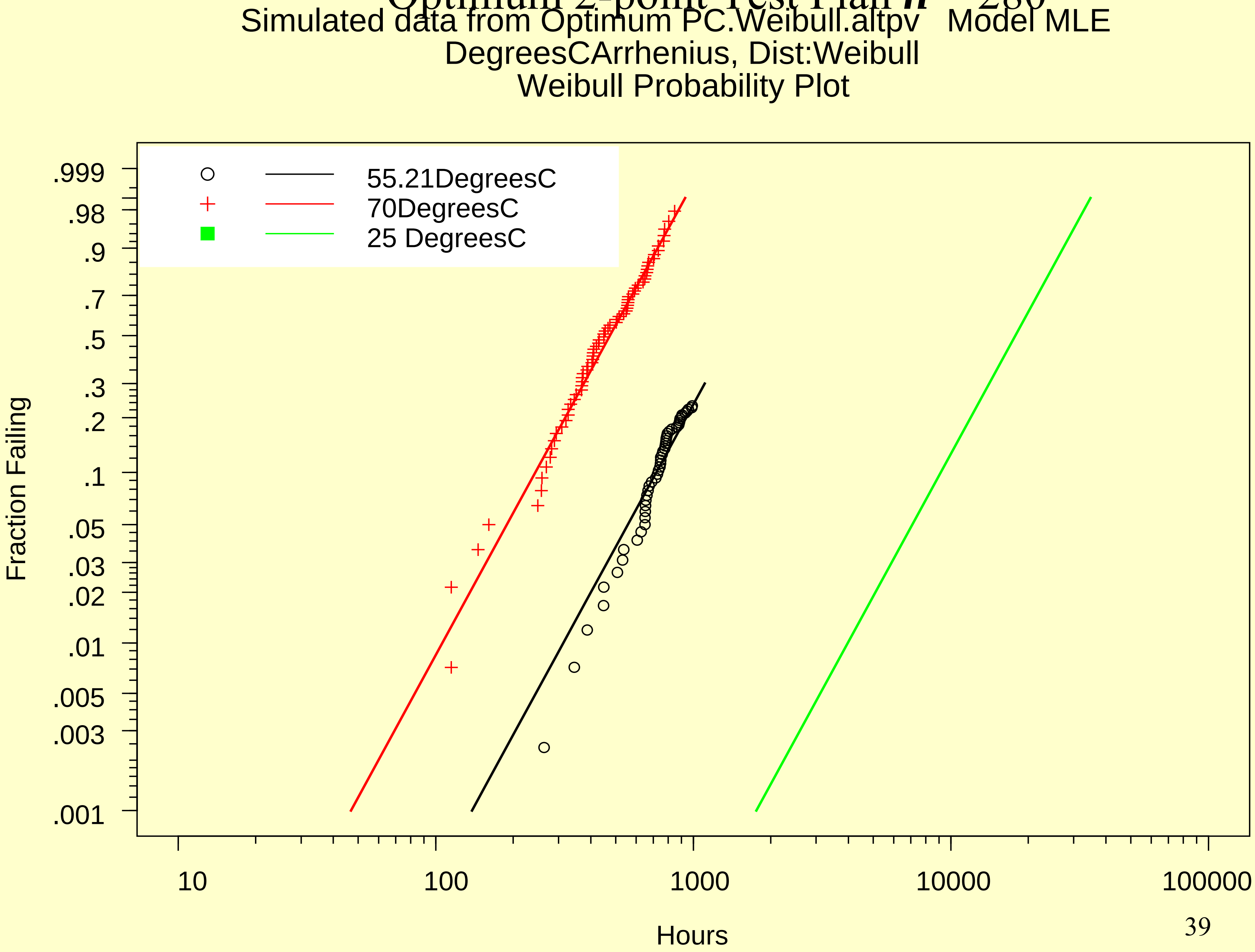

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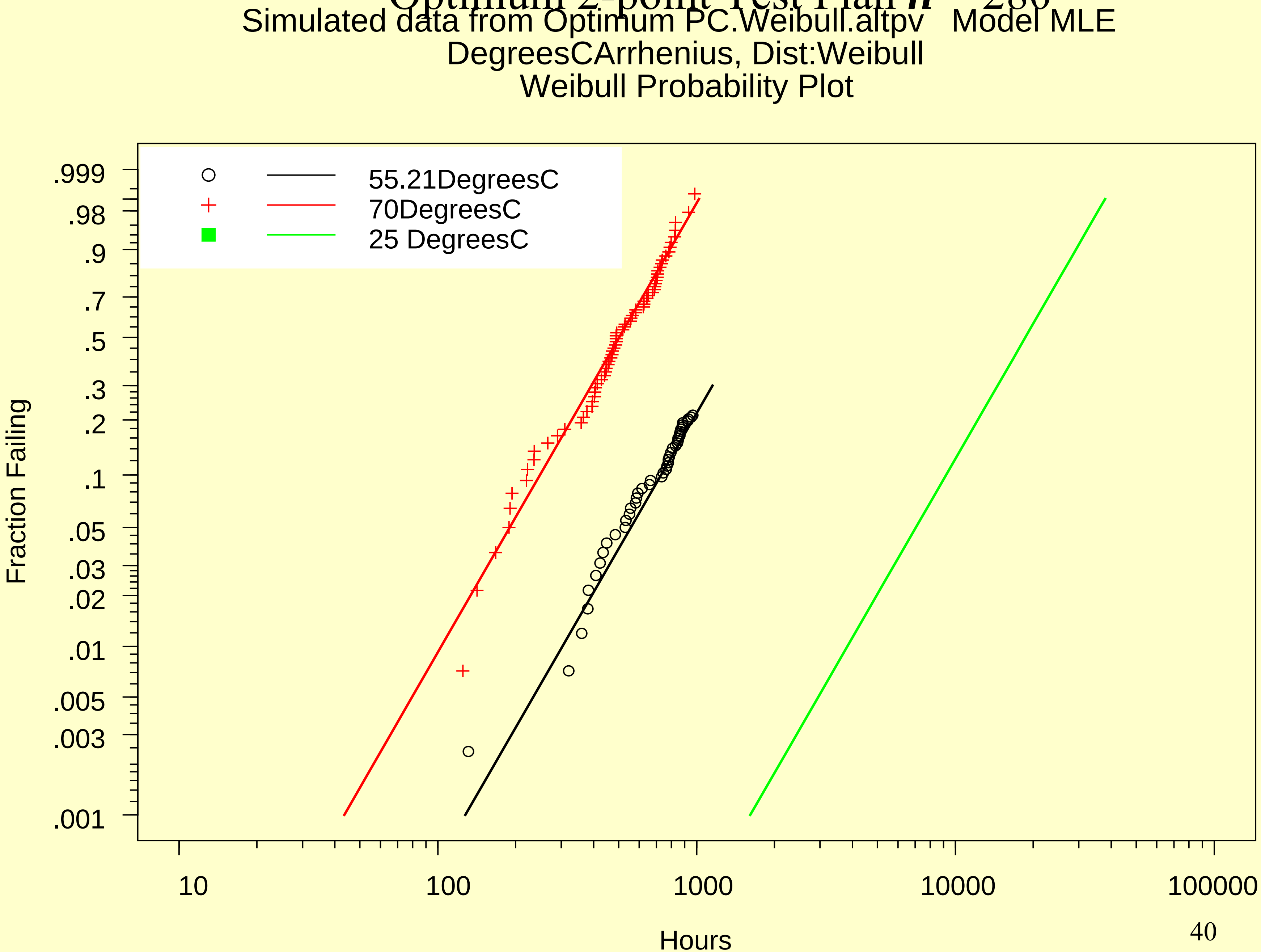

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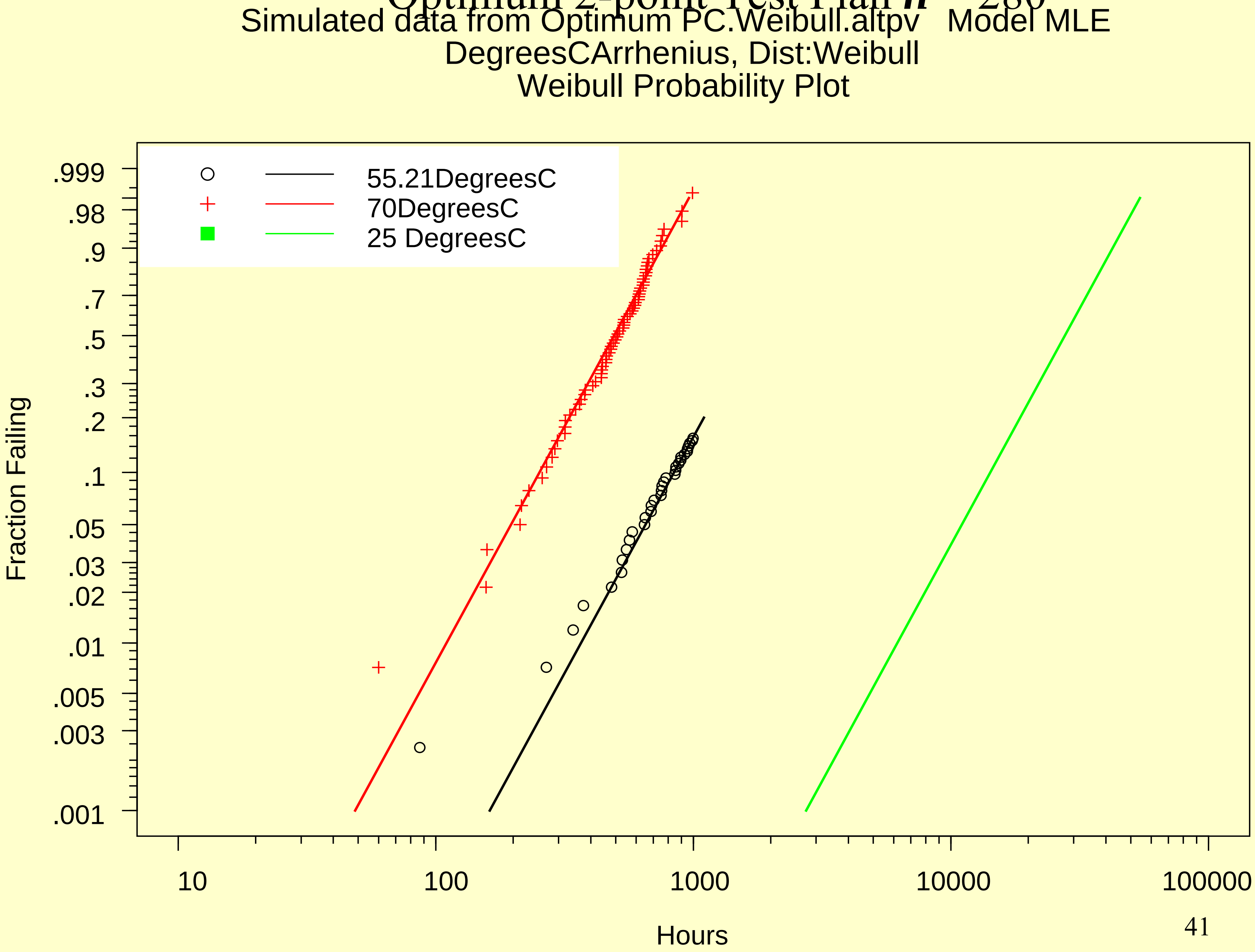
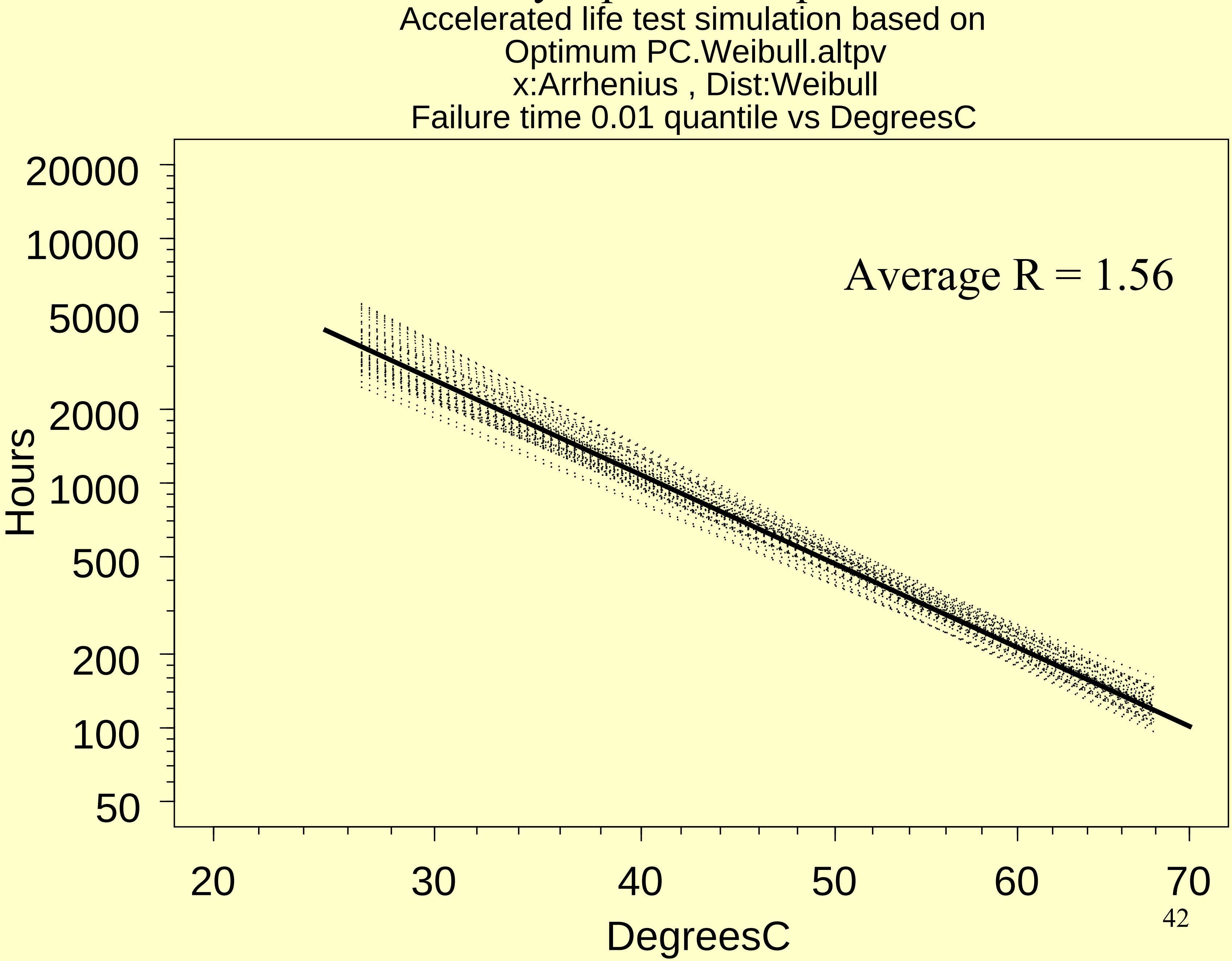
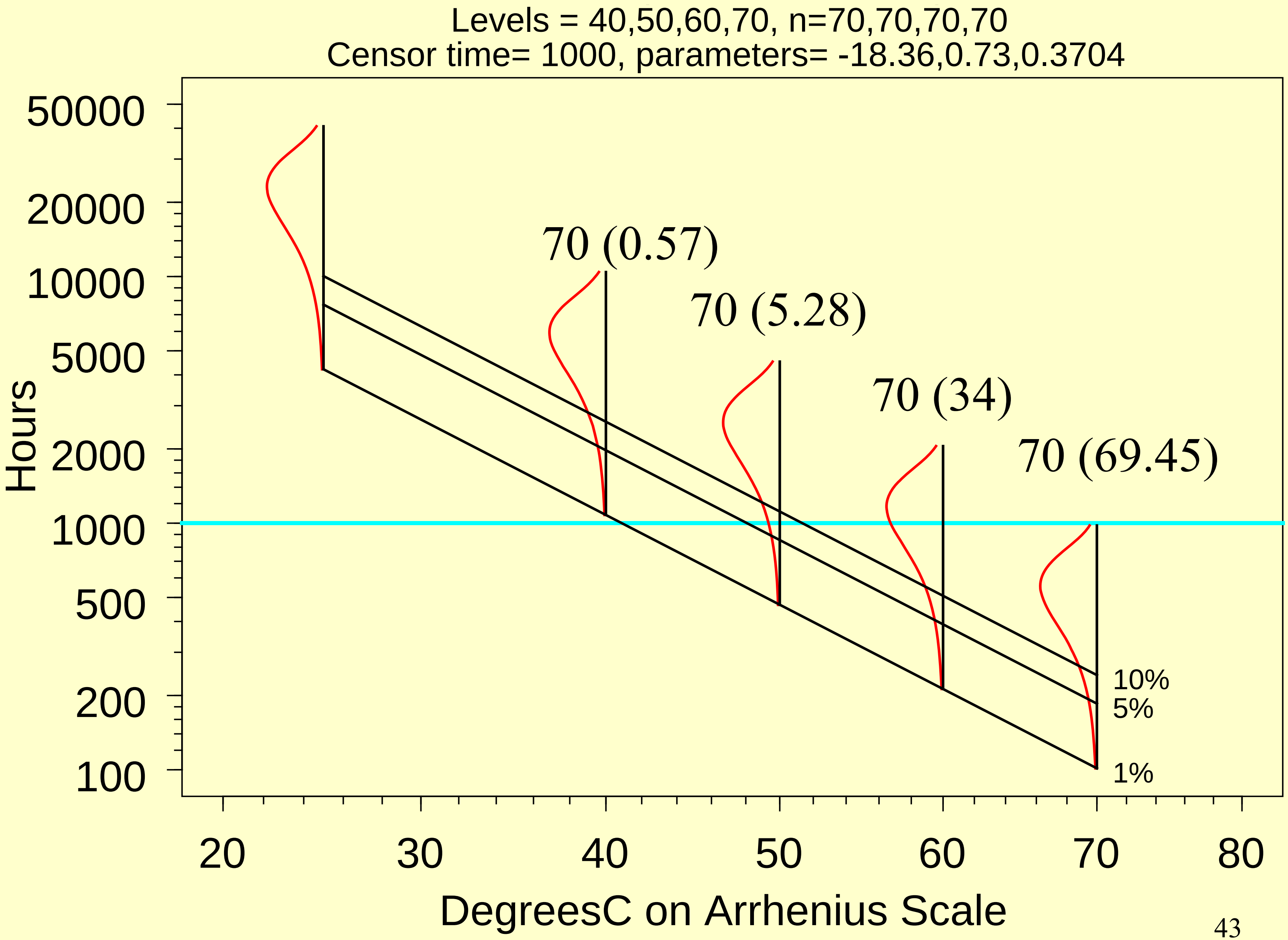


Simulation Summary Optimum 2-point Test Plan n = 280



Traditional 4-point Test Plan n = 280



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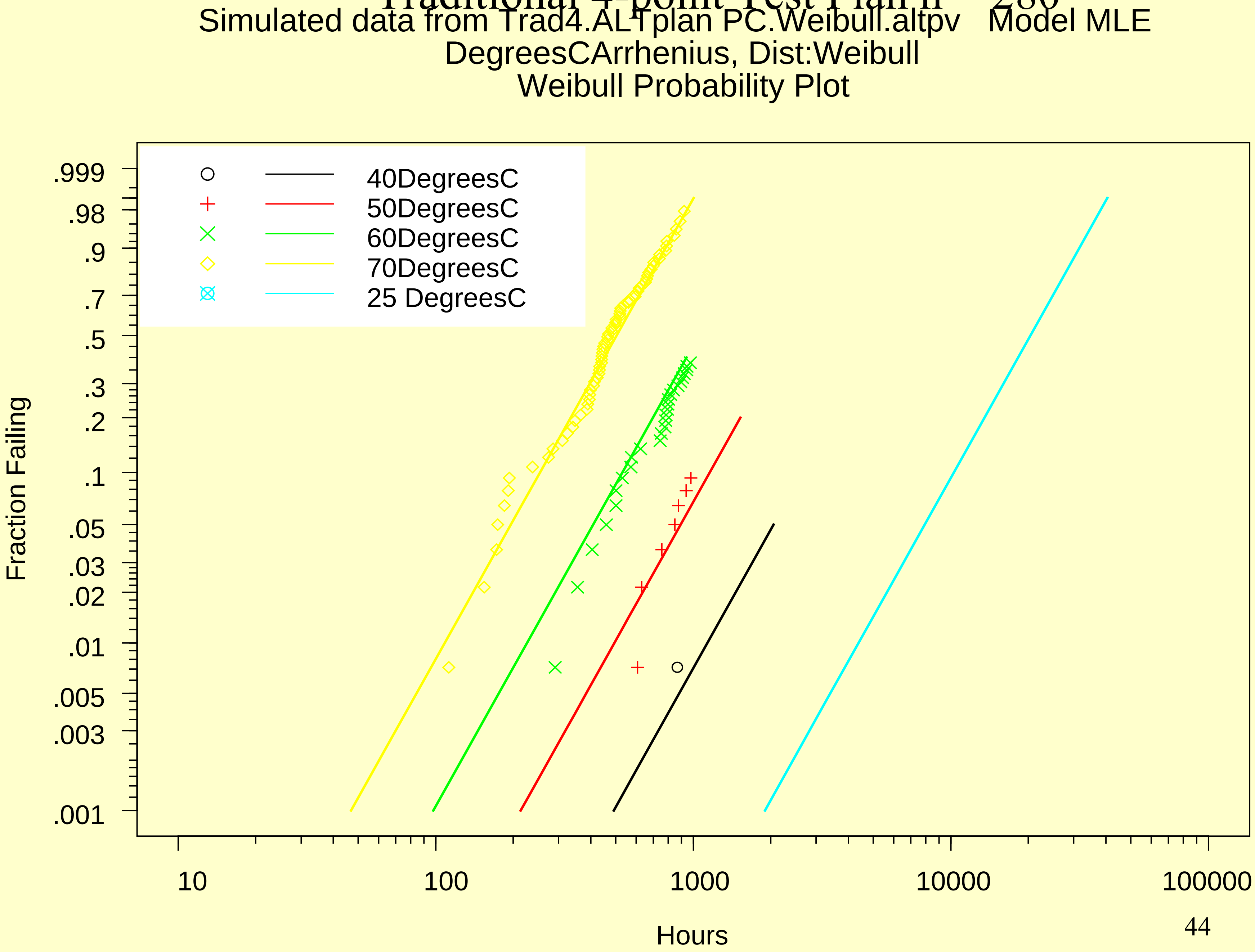

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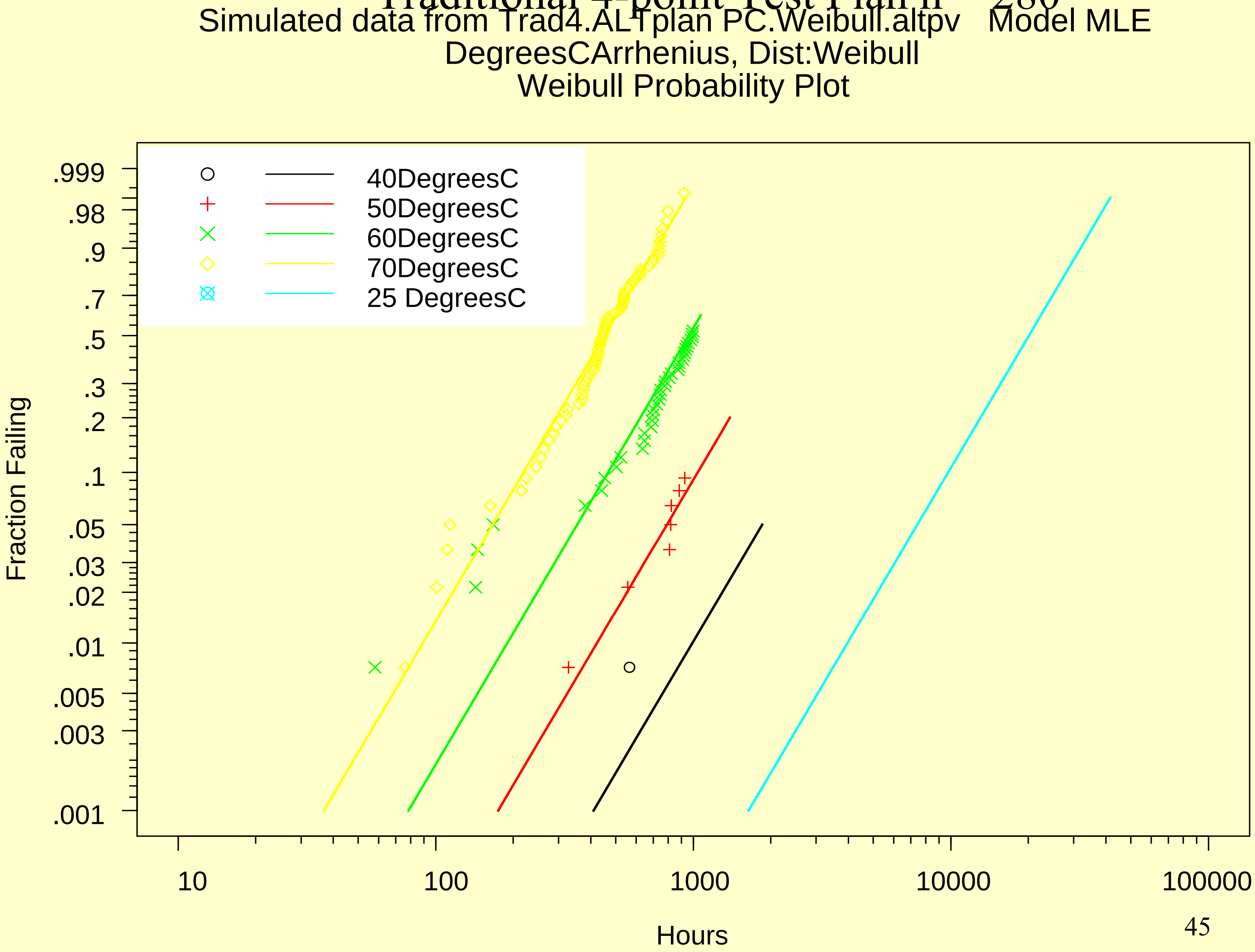

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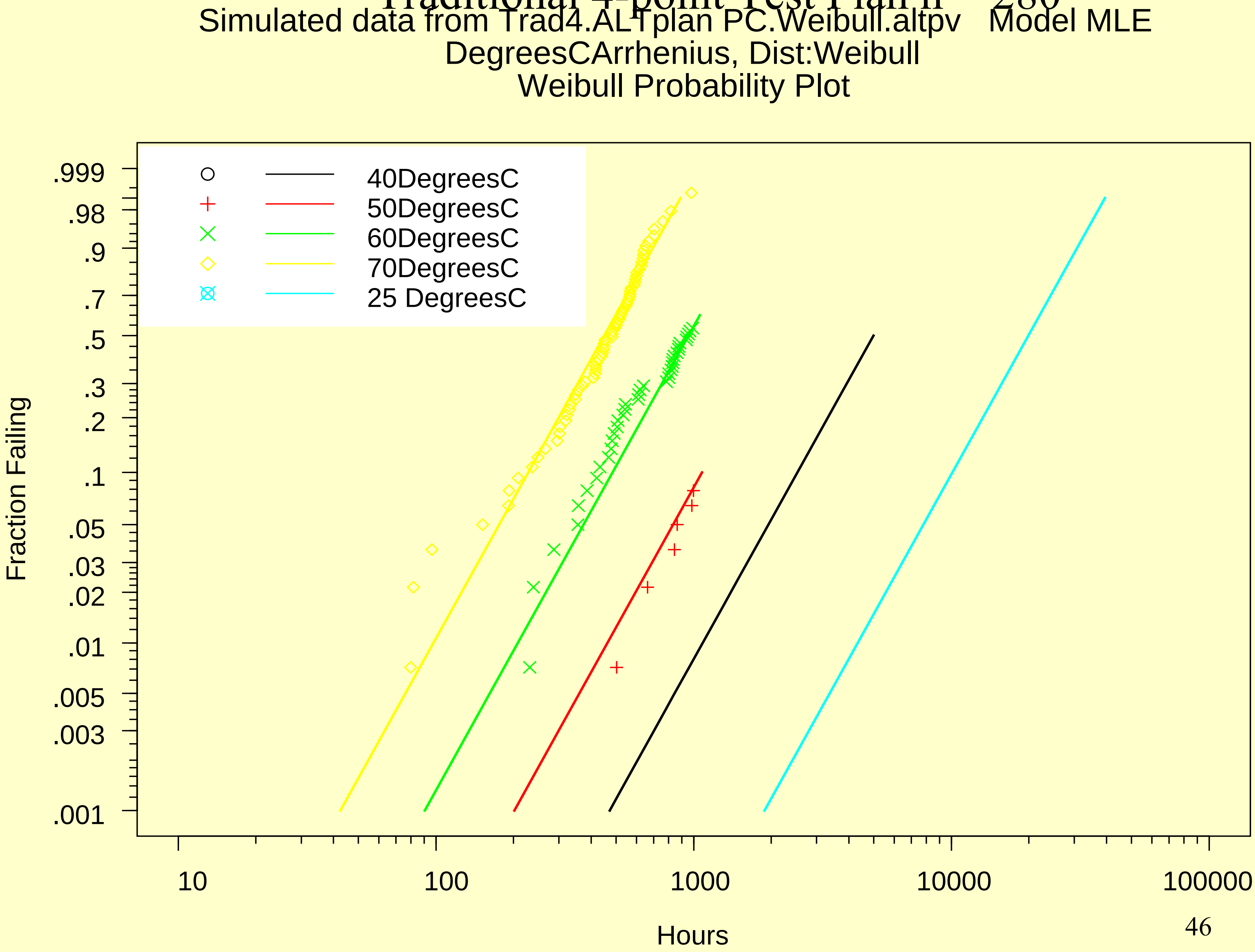

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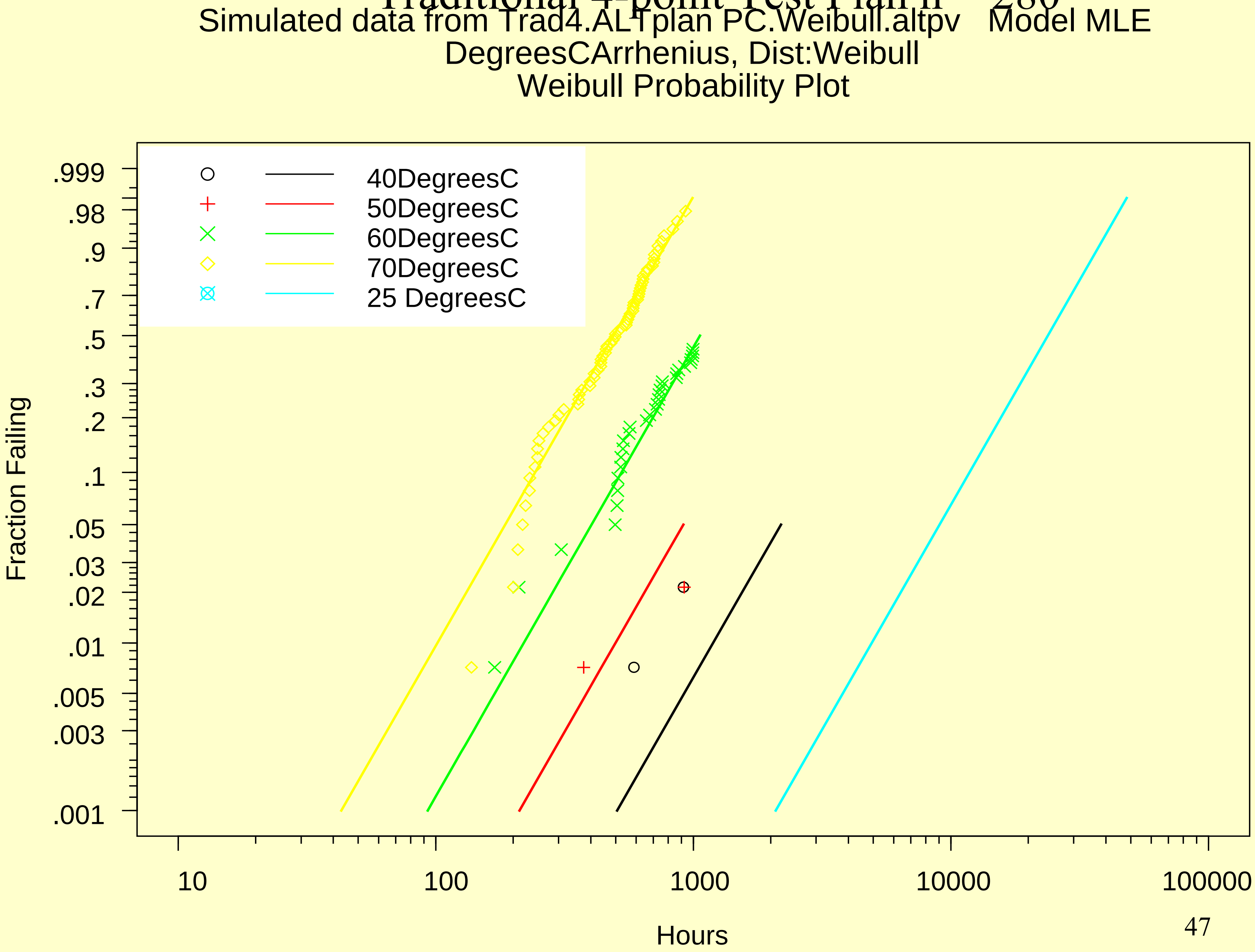

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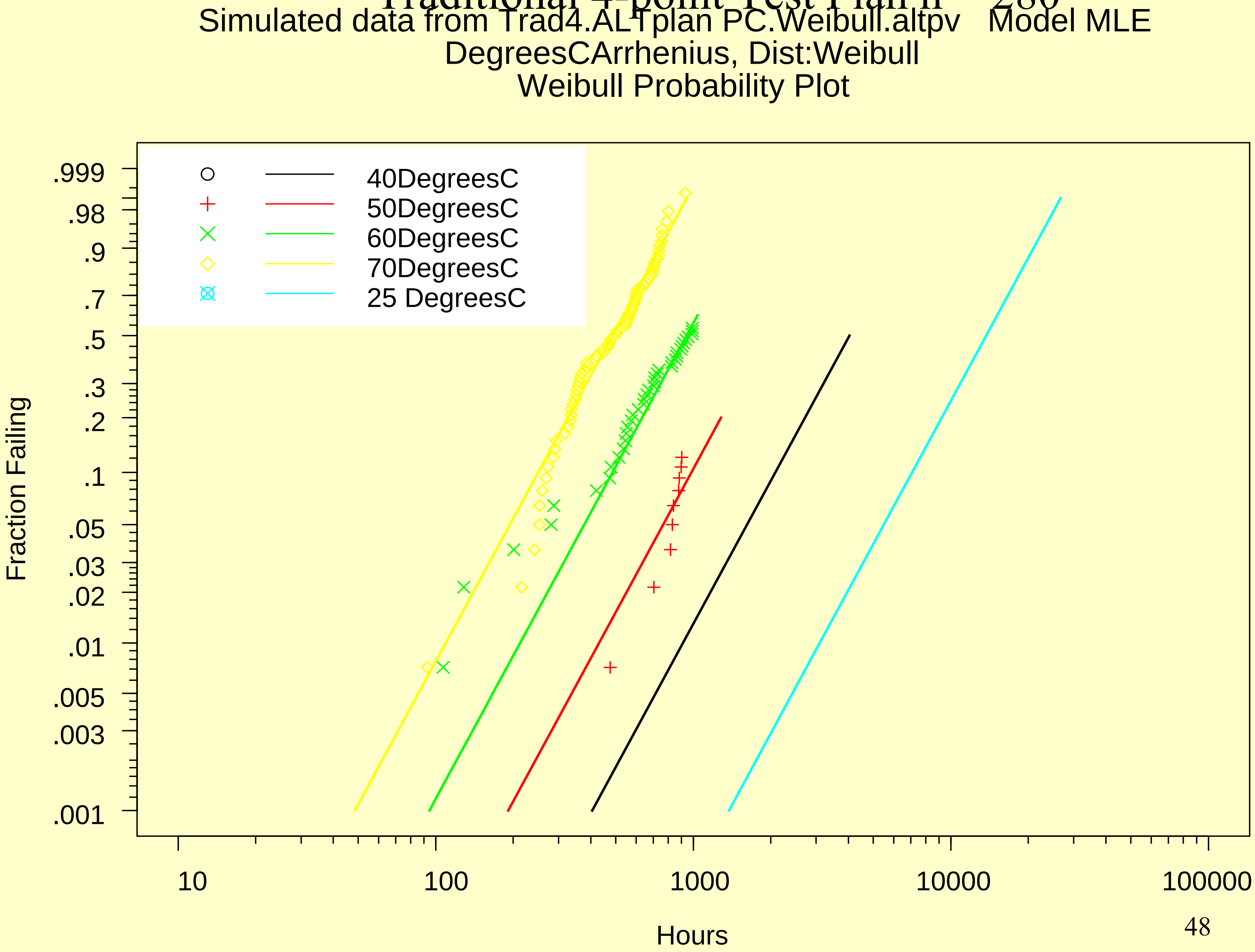
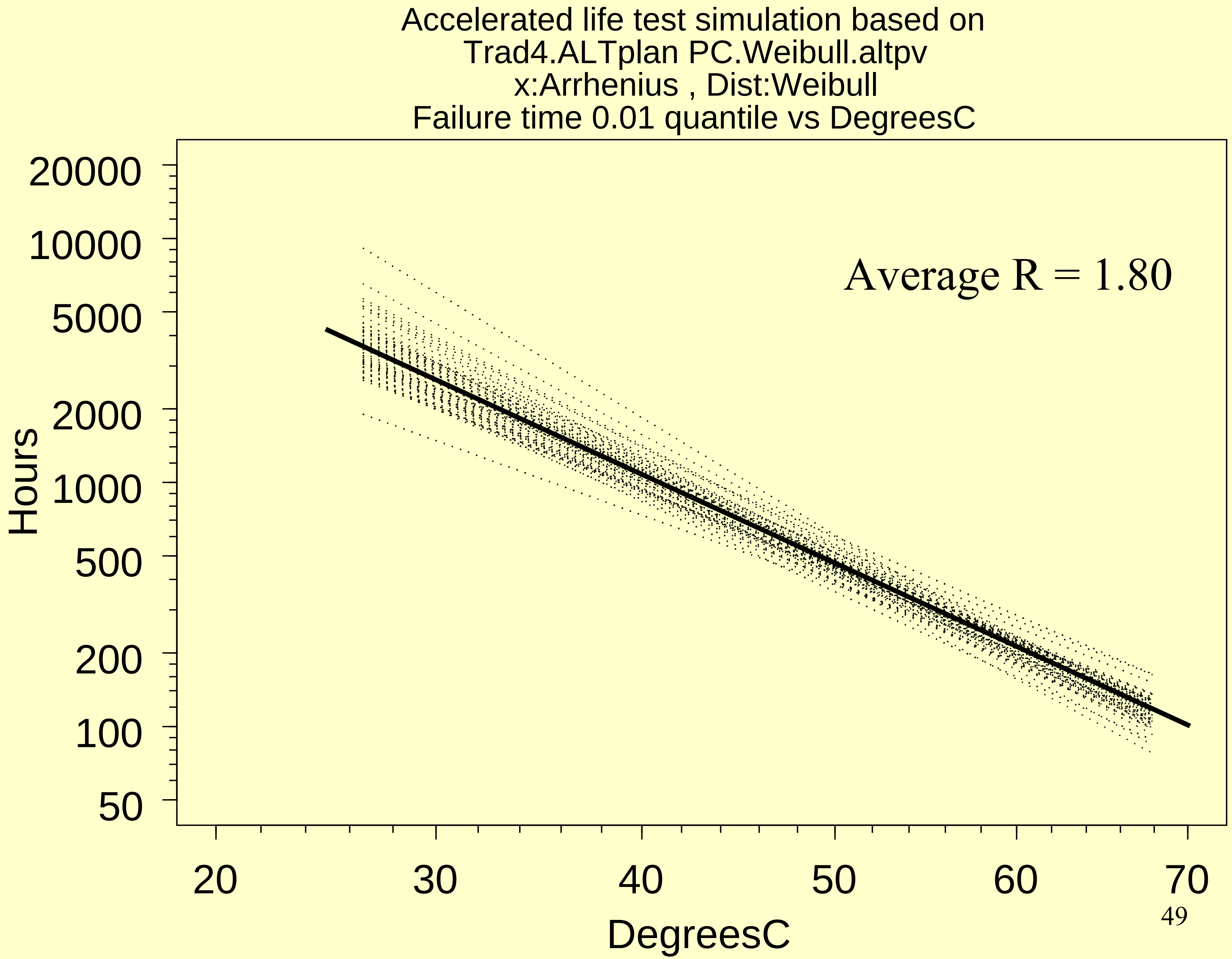
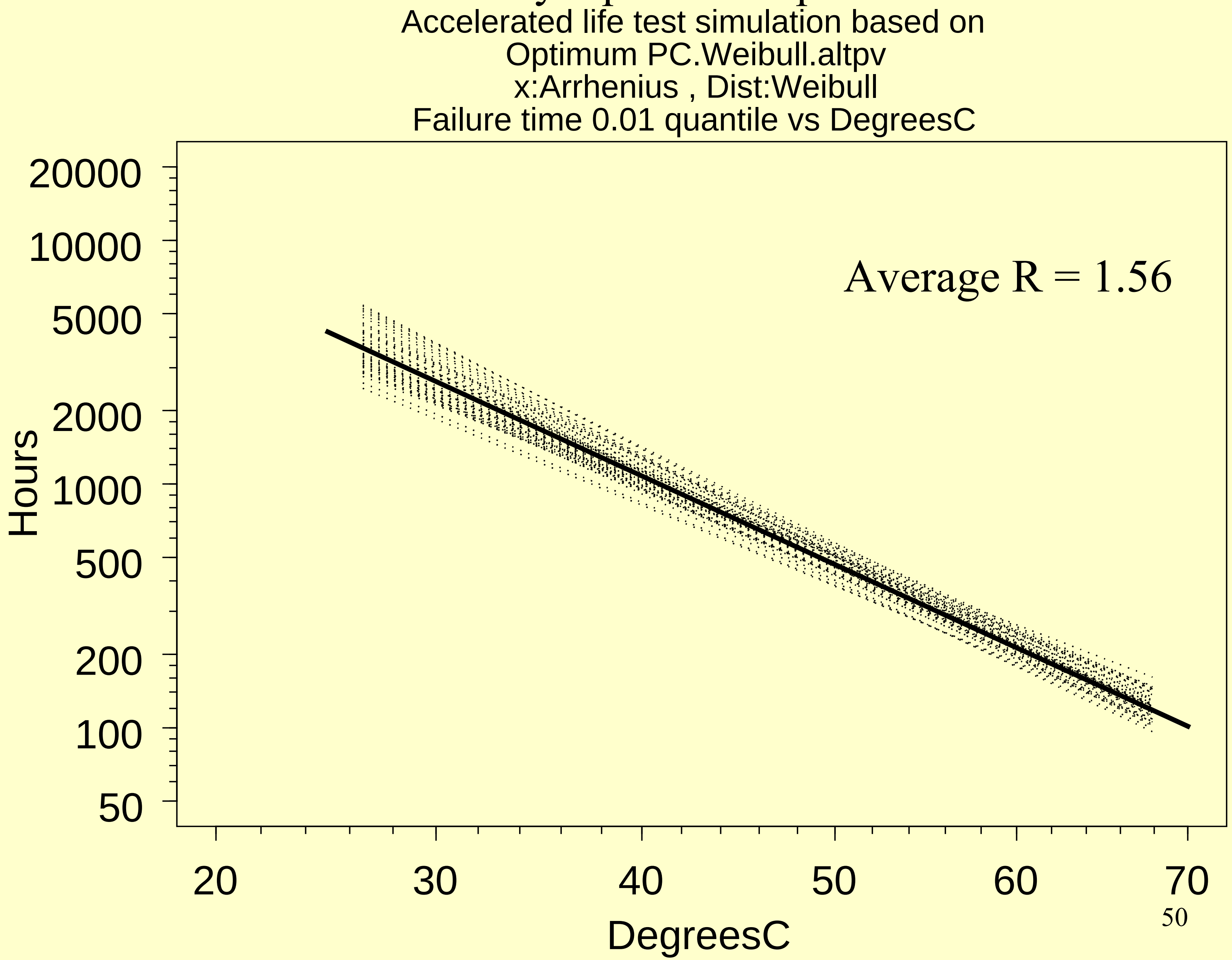


Simulation Summary Traditional 4-point Test Plan *n* = 280



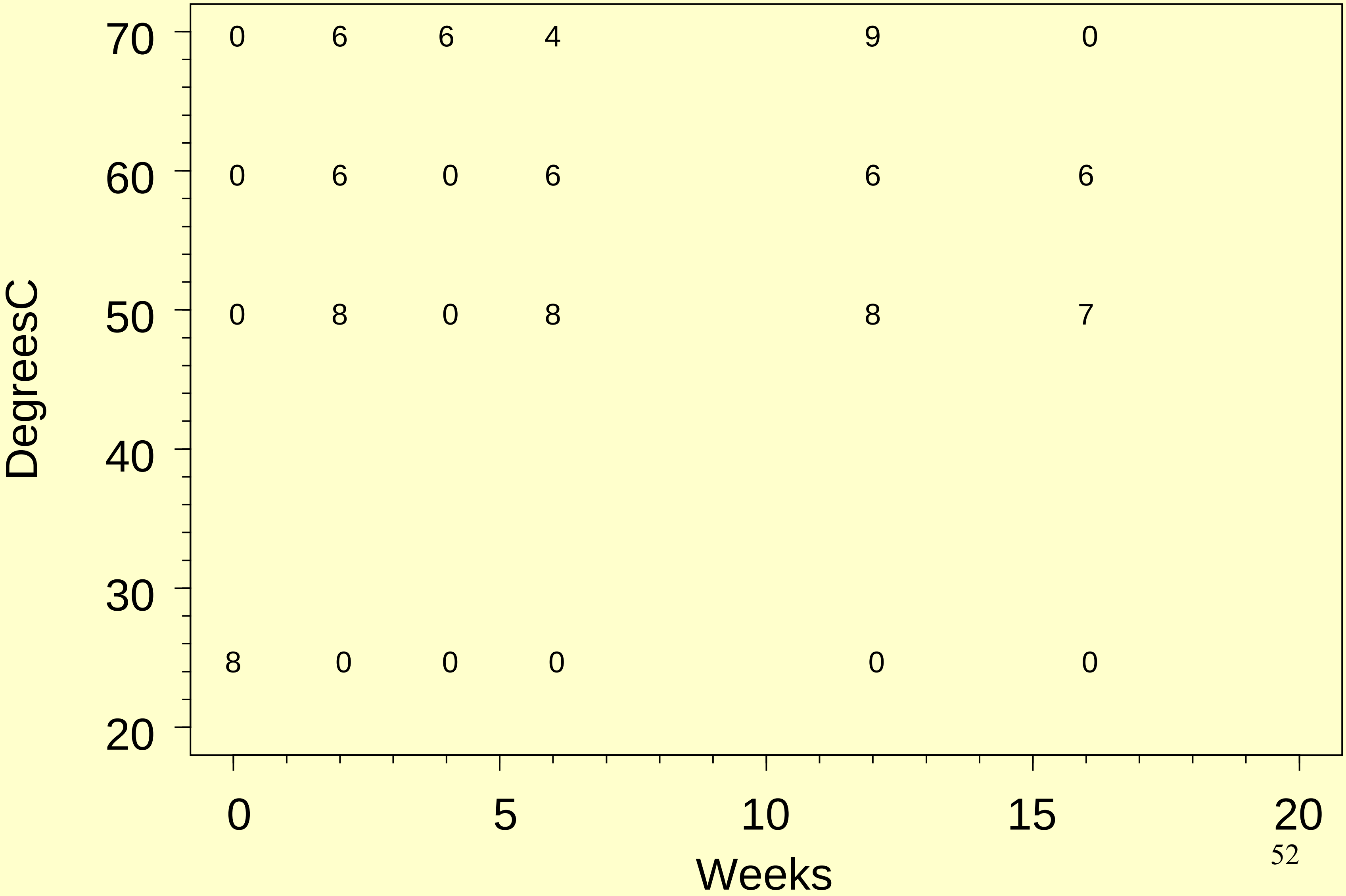
Simulation Summary Optimum 2-point Test Plan ***n*** = 280



Planning an *Accelerated Destructive Degradation Test* for an Adhesive Bond

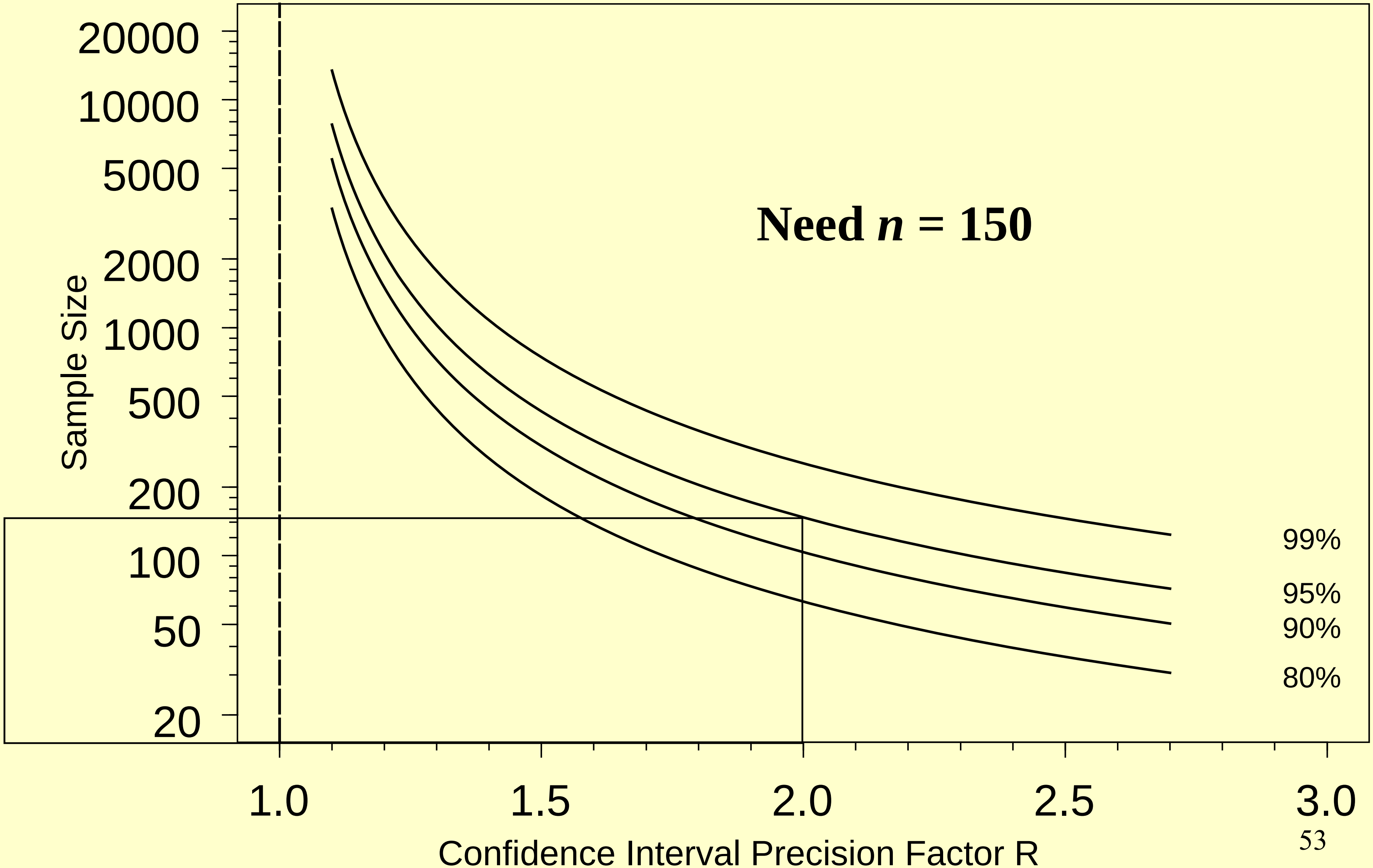
- Want to estimate B10 life at 25 Degrees C (260 week goal)
- Can test at temperatures up to 70 Degrees C
- Log strength is linear in square root of time
- Failure is defined as strength < 35 Newtons
- Planning values for strength (Weeks, Degrees C) distribution
  - Log strength distribution is normal with $\sigma = 2.7$
  - Median strength at time 0 is 78.3 Newtons
  - Log strength rate of change is –0.12 at 50 Degrees C
  - Effective activation energy = 0.58 eV
- Time censoring required at 16 weeks
- Can test 88 units

Initial Adhesive Bond Test Plan $n = 88$

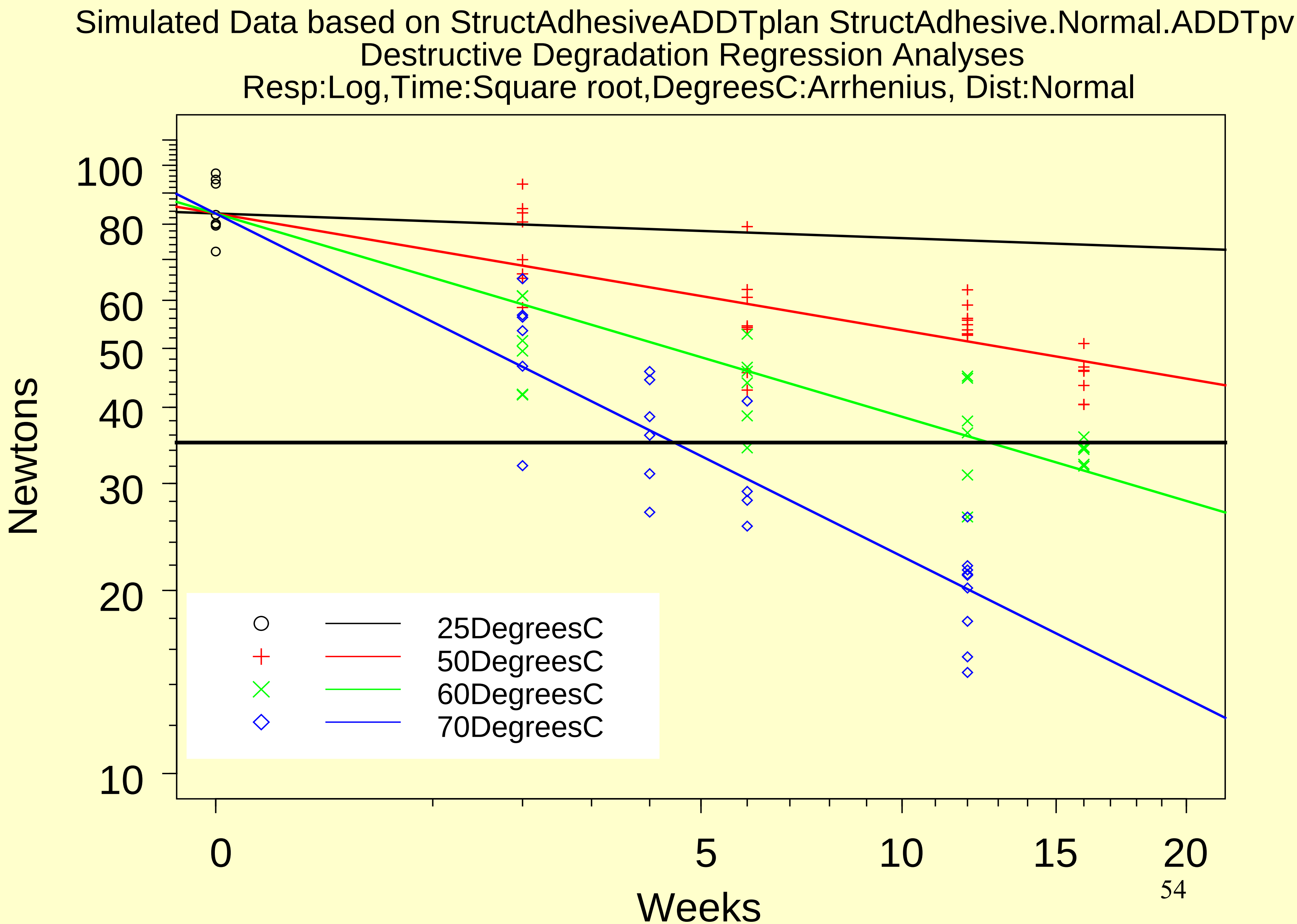


Sample Size Tool For the Initial Test Plan

Needed sample size giving approximately a 50% chance of having
a confidence interval factor for the 0.1 quantile that is less than R
use condition= 25. DegreesC and a failure definition= 35
StructAdhesiveADDTplan StructAdhesive.Normal.ADDTpv

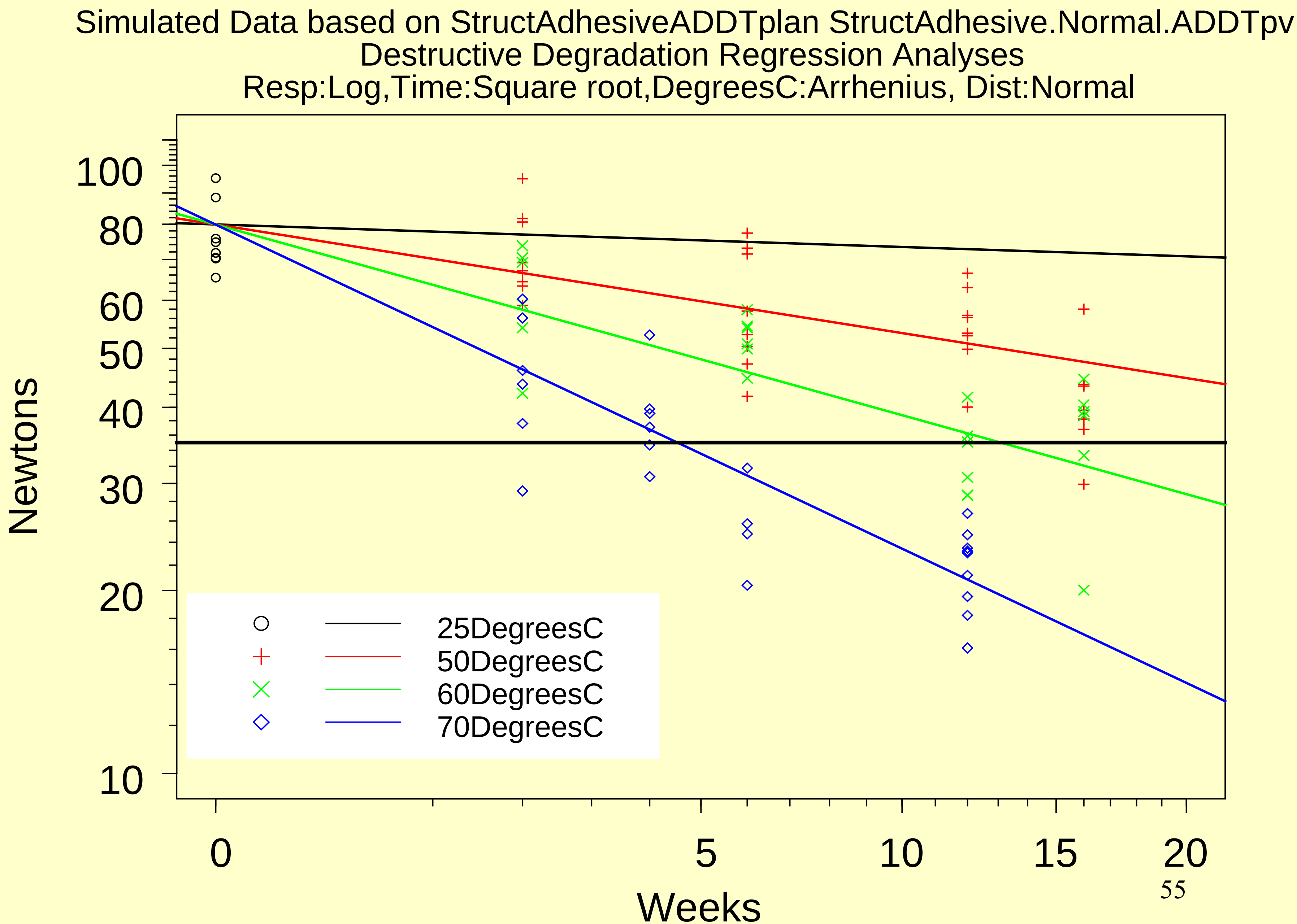


$n = 88$



Simulated Data based on StructAdhesiveADDTplan StructAdhesive.Normal.ADDTpv
Destructive Degradation Regression Analyses
Resp:Log,Time:Square root,DegreesC:Arrhenius, Dist:Normal
Newtons
100
80
60
50
40
30
20
10
0
5
10
15
20
Weeks
25DegreesC
50DegreesC
60DegreesC
70DegreesC

$n = 88$



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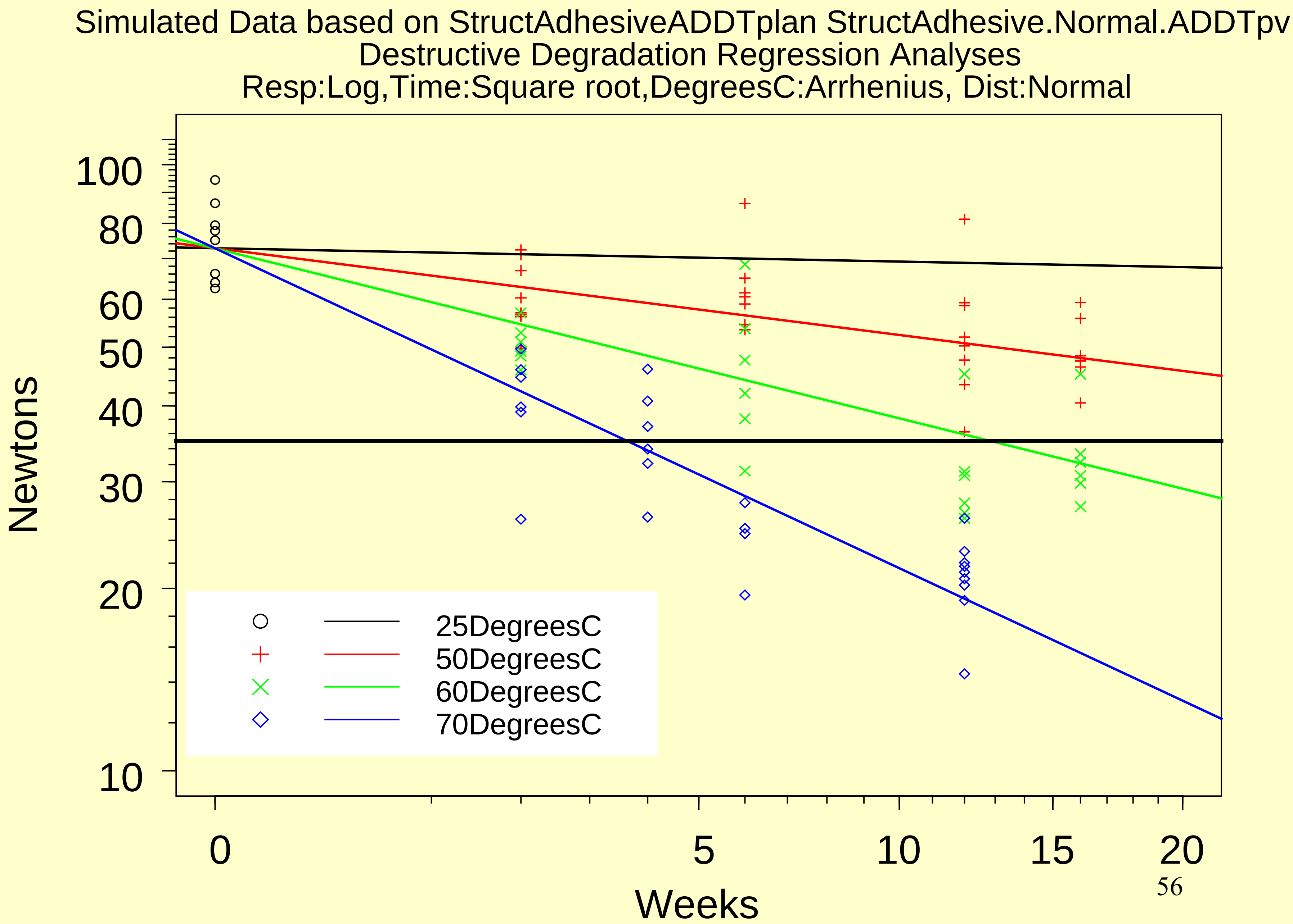
Destructive Degradation Regression Analyses
Resp:Log,Time:Square root,DegreesC:Arrhenius, Dist:Normal
Newtons
100
80
60
50
40
30
20
10
0
5
10
15
20
Weeks
25DegreesC
50DegreesC
60DegreesC
70DegreesC

$n = 88$



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Destructive Degradation Regression Analyses
Resp:Log,Time:Square root,DegreesC:Arrhenius, Dist:Normal
Newtons
100
80
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50
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30
20
10
0
5
10
15
20
Weeks
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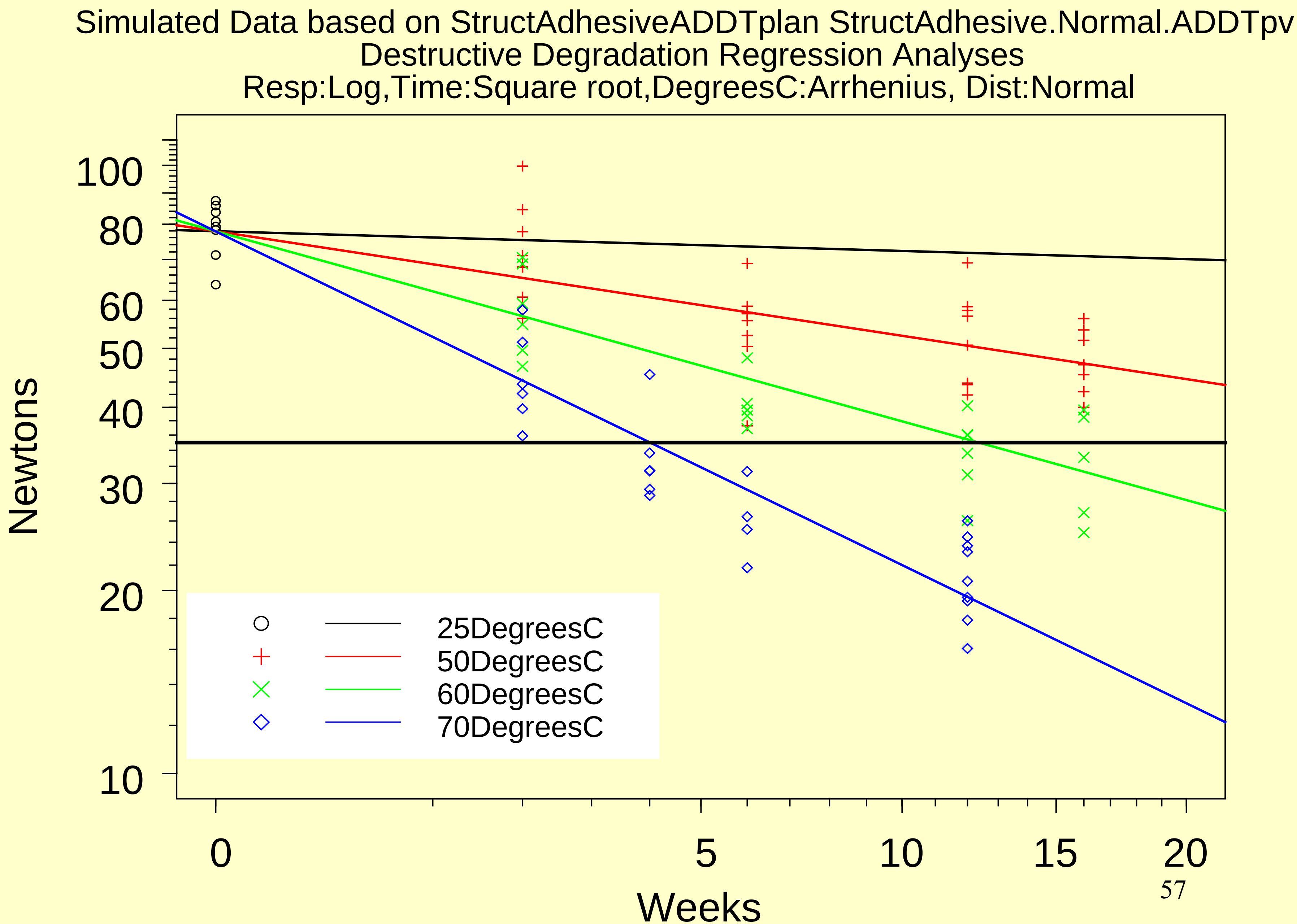

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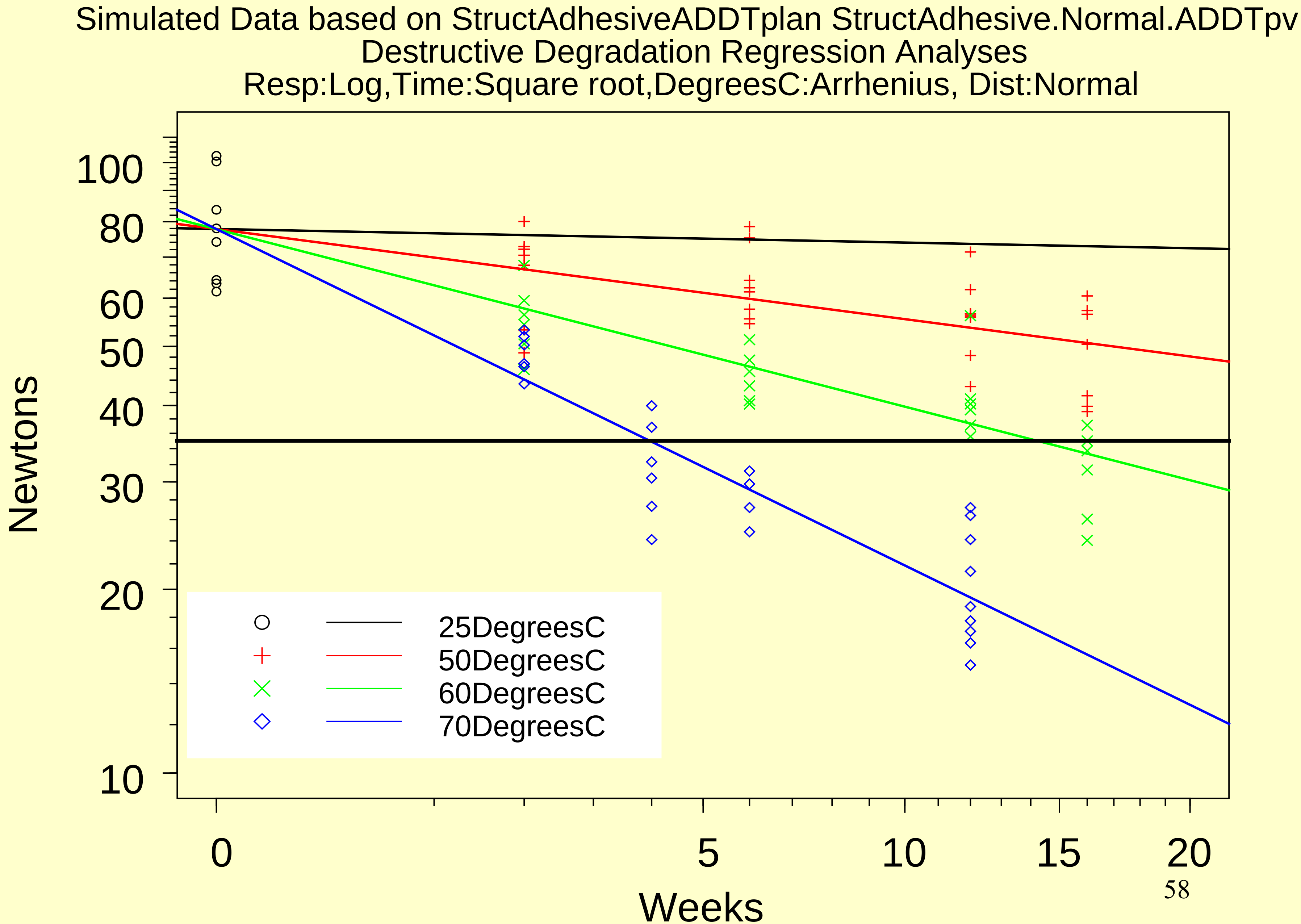
Destructive Degradation Regression Analyses
Resp:Log,Time:Square root,DegreesC:Arrhenius, Dist:Normal
Newtons
100
80
60
50
40
30
20
10
25DegreesC
50DegreesC
60DegreesC
70DegreesC
0
5
10
15
20
Weeks

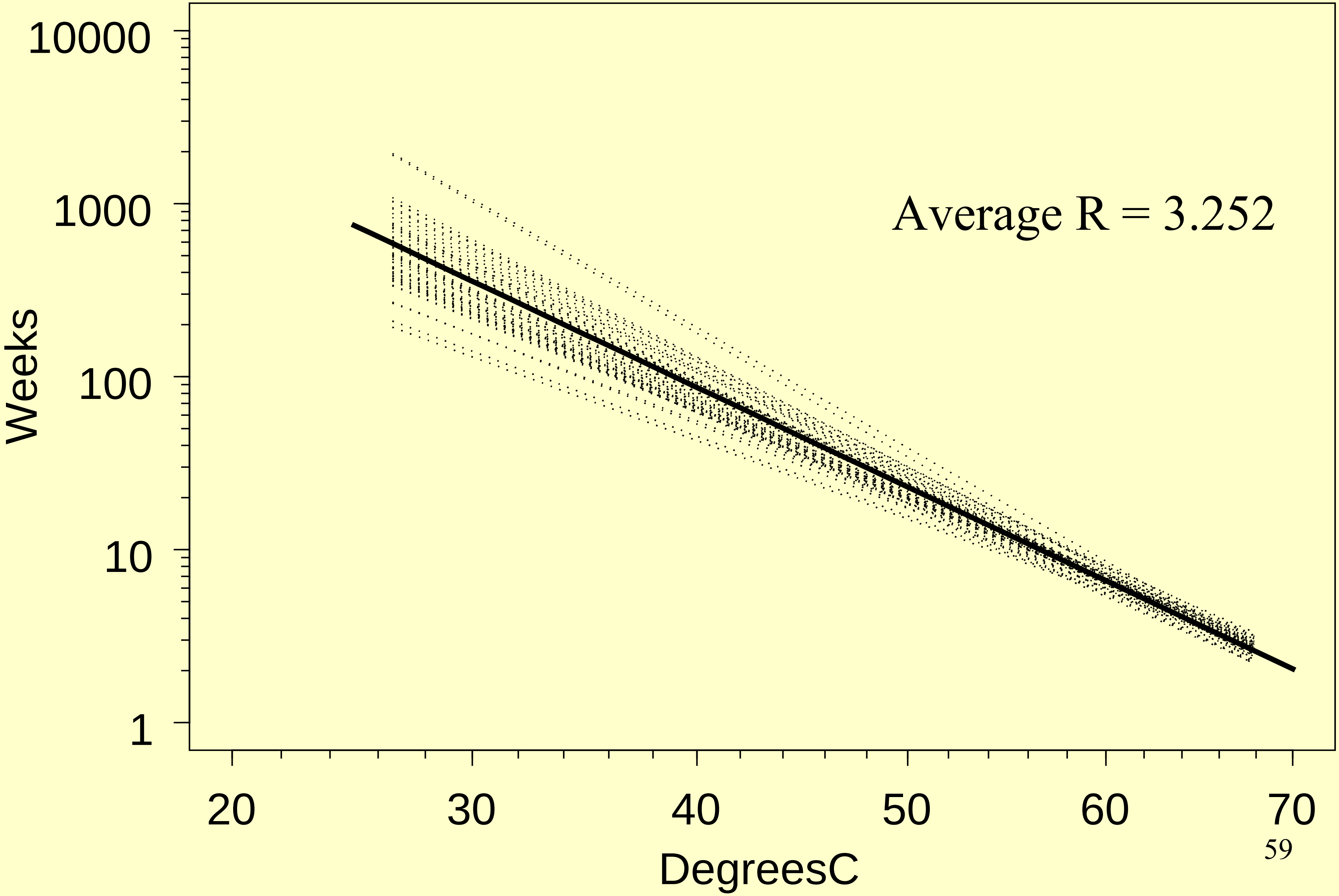
$n = 88$



Simulated Data based on StructAdhesiveADDTplan StructAdhesive.Normal.ADDTpv
Destructive Degradation Regression Analyses
Resp:Log,Time:Square root,DegreesC:Arrhenius, Dist:Normal
Newtons
100
80
60
50
40
30
20
10
0
5
10
15
20
Weeks
25DegreesC
50DegreesC
60DegreesC
70DegreesC

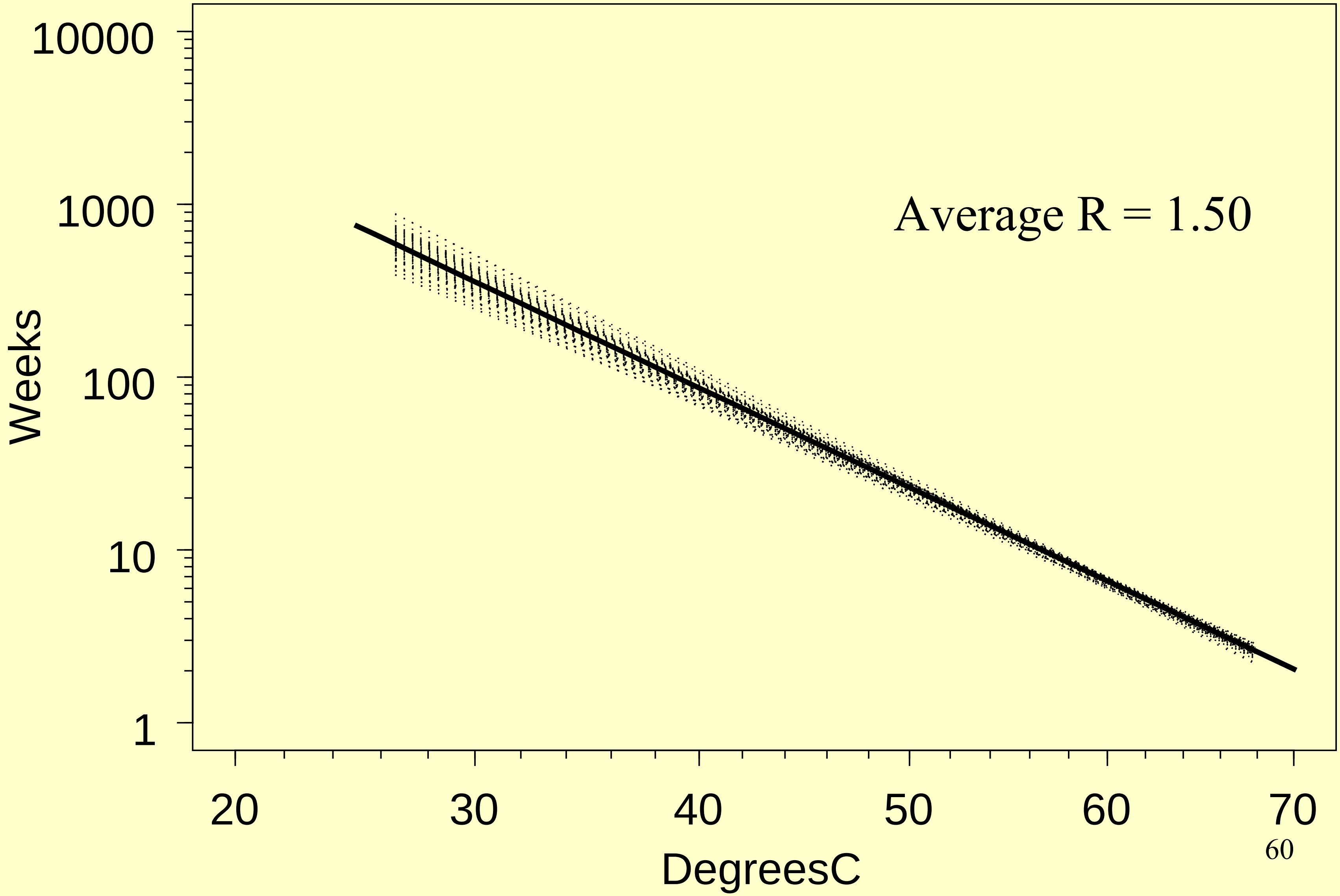
n = 88

Accelerated destructive degradation test simulation based on StructAdhesiveADDTplan StructAdhesive.Normal.ADDTpv
Failure time 0.1 quantile vs DegreesC for failure definition 35 Newtons
Resp:Log,Time:Square root,x:Arrhenius , Dist:Normal



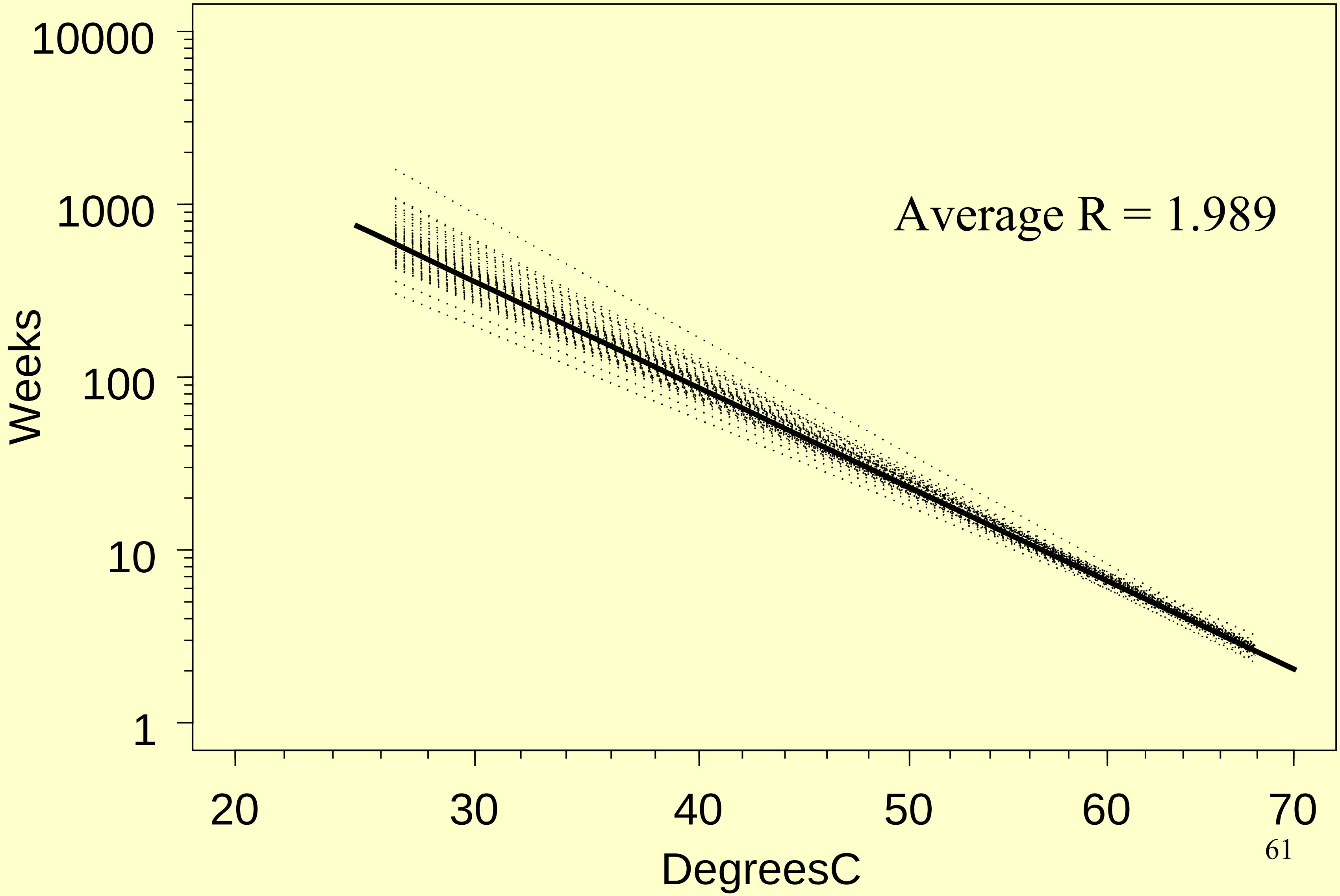
$n = 352$

Accelerated destructive degradation test simulation based on
StructAdhesive4ADDTplan StructAdhesive.Normal.ADDTpv
Failure time 0.1 quantile vs DegreesC for failure definition 35 Newtons
Resp:Log,Time:Square root,x:Arrhenius , Dist:Normal



n = 176

Accelerated destructive degradation test simulation based on
StructAdhesive2ADDTplan StructAdhesive.Normal.ADDTpv
Failure time 0.1 quantile vs DegreesC for failure definition 35 Newtons
Resp:Log,Time:Square root,x:Arrhenius , Dist:Normal



Concluding Remarks

- Analytical (large sample approximation) methods provide a useful tool for test plan *optimization* and approximate *sample size choice*.
- Simulation provides *visualization* and *insight* into reasons that a design has particular properties
- Modern computing and *graphical* tools make it possible to use these methods in concert.