Inferring the Interactions in Complex Manufacturing Processes Using Graphical Models

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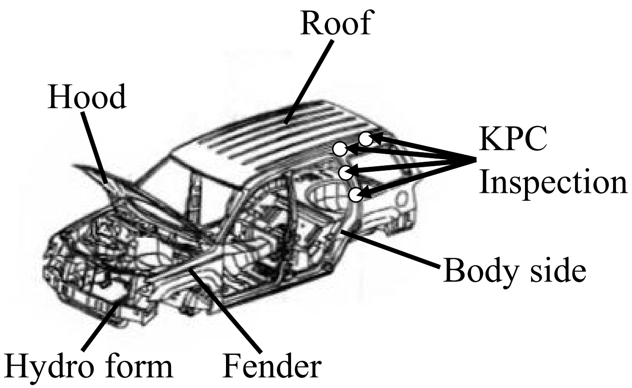
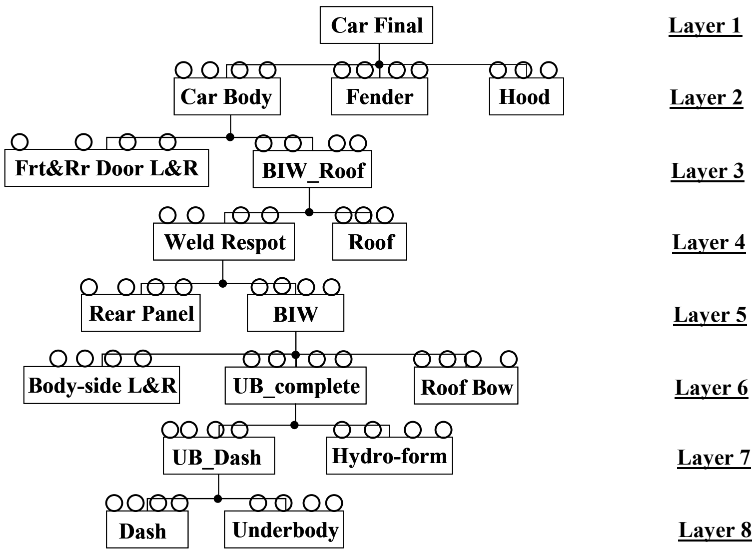
Outline

- Motivation and Current Techniques
- Problem Formulation
- Conventional Method to Build CG
- Proposed Methodology and Procedure
- Overview of Proof
- Case Study
- Summary and Future Work

Motivation

Multistage Car-body Assembly Process





Challenge: **How to deal with complex interactions among the KPCs?**

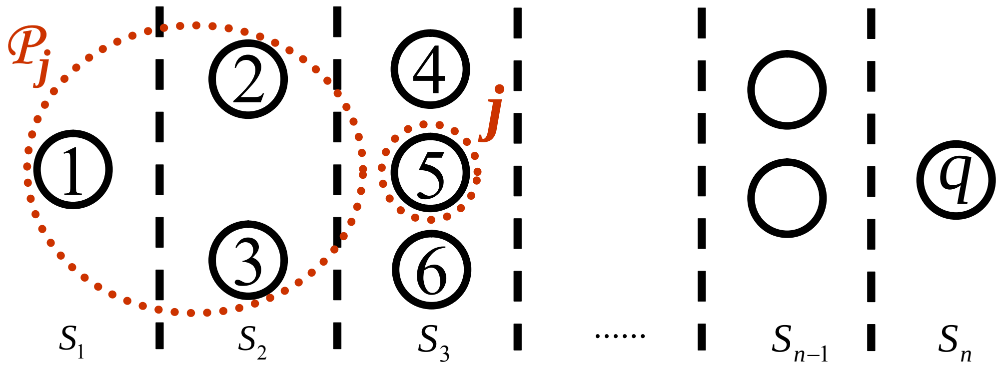
Current Techniques

- **Data-driven techniques** for simple discrete processes
  - Cause-selecting control chart (Zhang, Wade and Woodall)
  - Variation analysis in multistage processes (Lawless and Mackay)
  - Zantek's method
- **Analytical methods** known physical mechanism
  - Stream of Variation (SOV) methodologies

⇩

Generic methodology to identify interactions in complex multistage processes are needed!

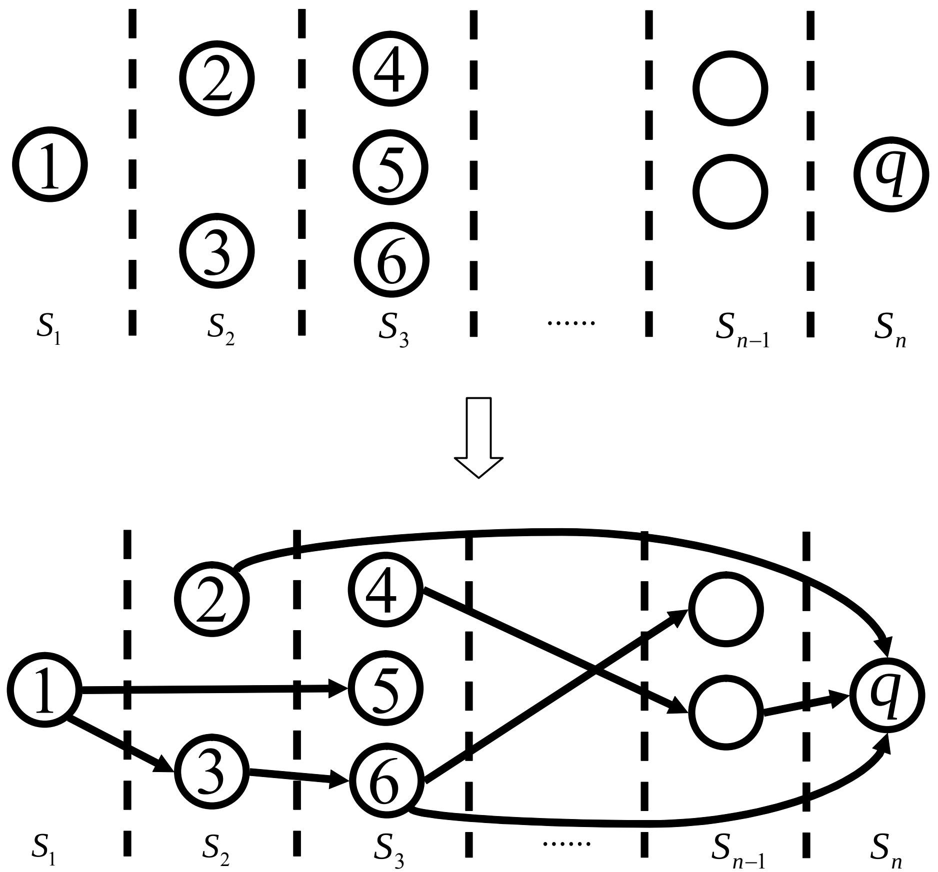
Problem Formulation



- **Assumptions**
  - (A1) KPCs at the same stage do not influence each other
  - (A2) Var(X_j)=Local Variation + Propagated Variation (X_i, $i \in \mathcal{P}_j$)
- **The problem**
  - Identify which preceding KPCs contribute variation to KPC j

j=1,…, q

Objective



Process layout

A graph representing direct influences

Definition of "Direct Influence"

Direct Influence

If X_i uniquely contributes to the variation of X_j, then we claim X_i directly influences X_j .

$$\mathrm{var}(\hat{X}_j(X_{\mathcal{P}_j})) = \mathrm{var}(\hat{X}_j(X_{\mathcal{P}_j} \setminus \{X_i\})) + \mathrm{var}(\hat{X}_j(X_i - \hat{X}_i(X_{\mathcal{P}_j} \setminus \{X_i\})))$$

- Variation of X_j explained by all variables in $\mathcal{P}_j$
- Variation of X_j explained by all variables in $\mathcal{P}_j$ except X_i
- Variation of X_j explained by X_i adjusted for other variables

Unique contribution of X_i

$=0 \Rightarrow X_i$ has no direct influence on X_j

$\neq 0 \Rightarrow X_i$ has direct influence on X_j

Connection with Graphical Models

i has no direct influence on *j*

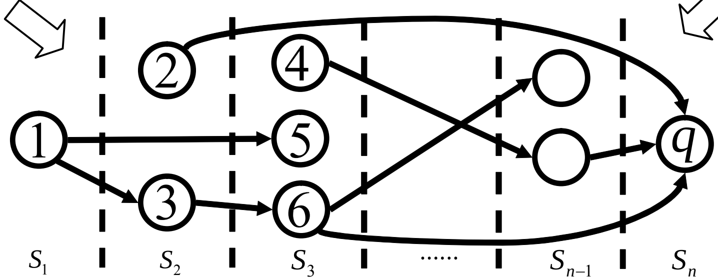
Least Squares Regression

Normal Assumption

$i \perp\!\!\!\perp j \mid \mathcal{P}_j \backslash \{i\}$ or $\mathrm{corr}(i, j \mid \mathcal{P}_j \backslash \{i\}) = 0$

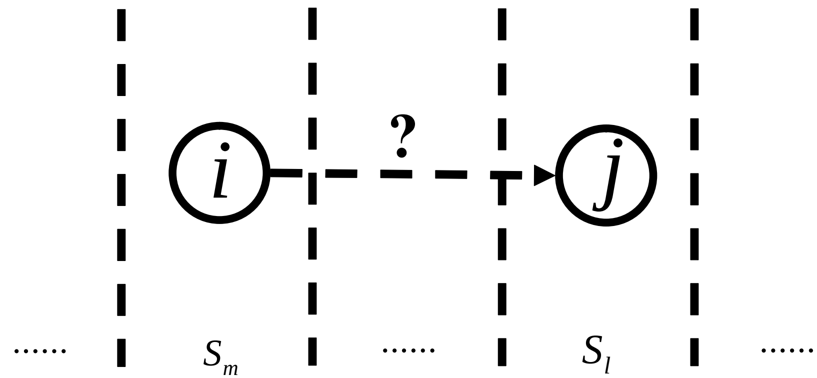
The graph representing direct influences

Chain Graph



A typical problem in Graphical Models: build a Chain Graph

Conventional Method



- **Test**

$$H_{ij} : \text{corr}(i, j \mid \mathcal{P}_j \backslash \{i\}) = 0 \quad \text{vs.} \quad K_{ij} : \text{corr}(i, j \mid \mathcal{P}_j \backslash \{i\}) \neq 0, \quad i \in \mathcal{P}_j$$

Conditioning set

- **Decision rule**

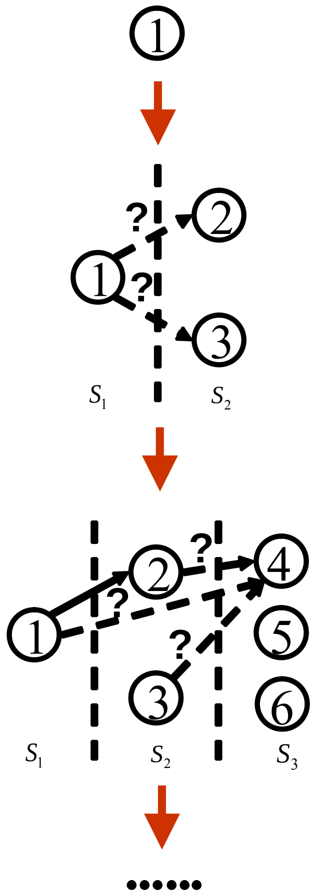
H_{ij} is rejected $\Rightarrow$ X_i has direct influence on X_j

- **Drawback**

Larger conditioning set ⇒ More variables involved; Lower detection power

Proposed Methodology

Iterative CG building technique

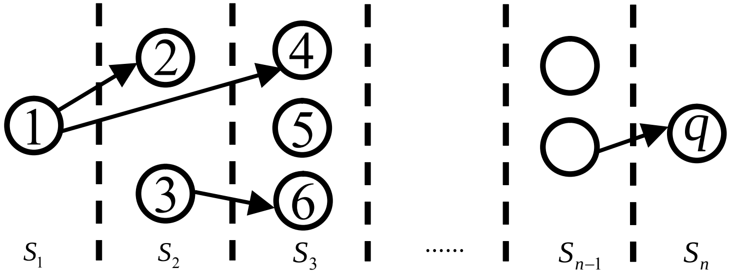


Existing test results

Smaller conditioning set

$\mathcal{P}_j \backslash \{i\} \Rightarrow \mathcal{R}(i,j)$

Chain Graph



Comparison with Other Effort

Conventional method

$$H_{ij} : \mathrm{corr}(i, j | \mathcal{P}_j \backslash \{i\}) = 0 \quad \text{vs.} \quad K_{ij} : \mathrm{corr}(i, j | \mathcal{P}_j \backslash \{i\}) \neq 0$$

⇓ **Reduce conditioning set**

- **Available effort**

$$\mathcal{P}'(i,j) \subseteq \mathcal{P}_j \backslash \{i\}$$

s.t.

$$\mathbf{corr}(i, j | \mathcal{P}_j \backslash \{i\}) = \mathbf{corr}(i, j | \mathcal{P}'(i,j))$$

⇓

$$H_{ij} : \mathrm{corr}(i, j | \mathcal{P}'(i,j)) = 0$$

- **Our effort**

$$\mathcal{R}(i,j) \subseteq \mathcal{P}_j \backslash \{i\}$$

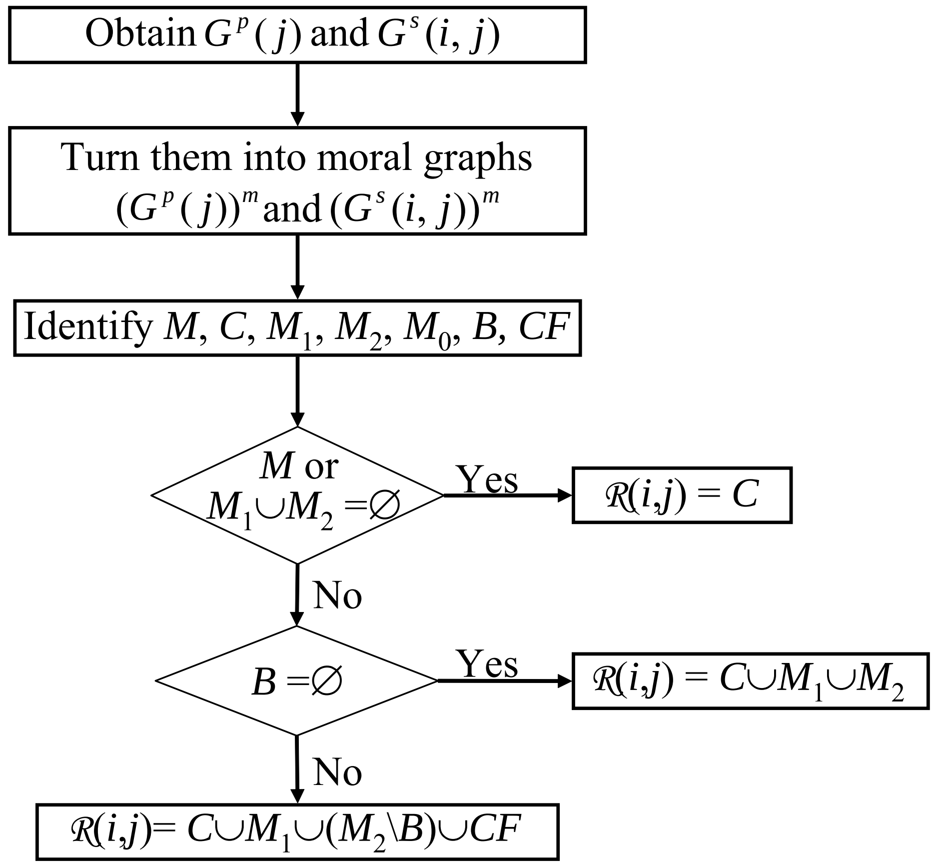
s.t.

$$\mathbf{corr}(i, j | \mathcal{P}_j \backslash \{i\}) = 0 \Leftrightarrow \mathbf{corr}(i, j | \mathcal{R}(i,j)) = 0$$

⇓

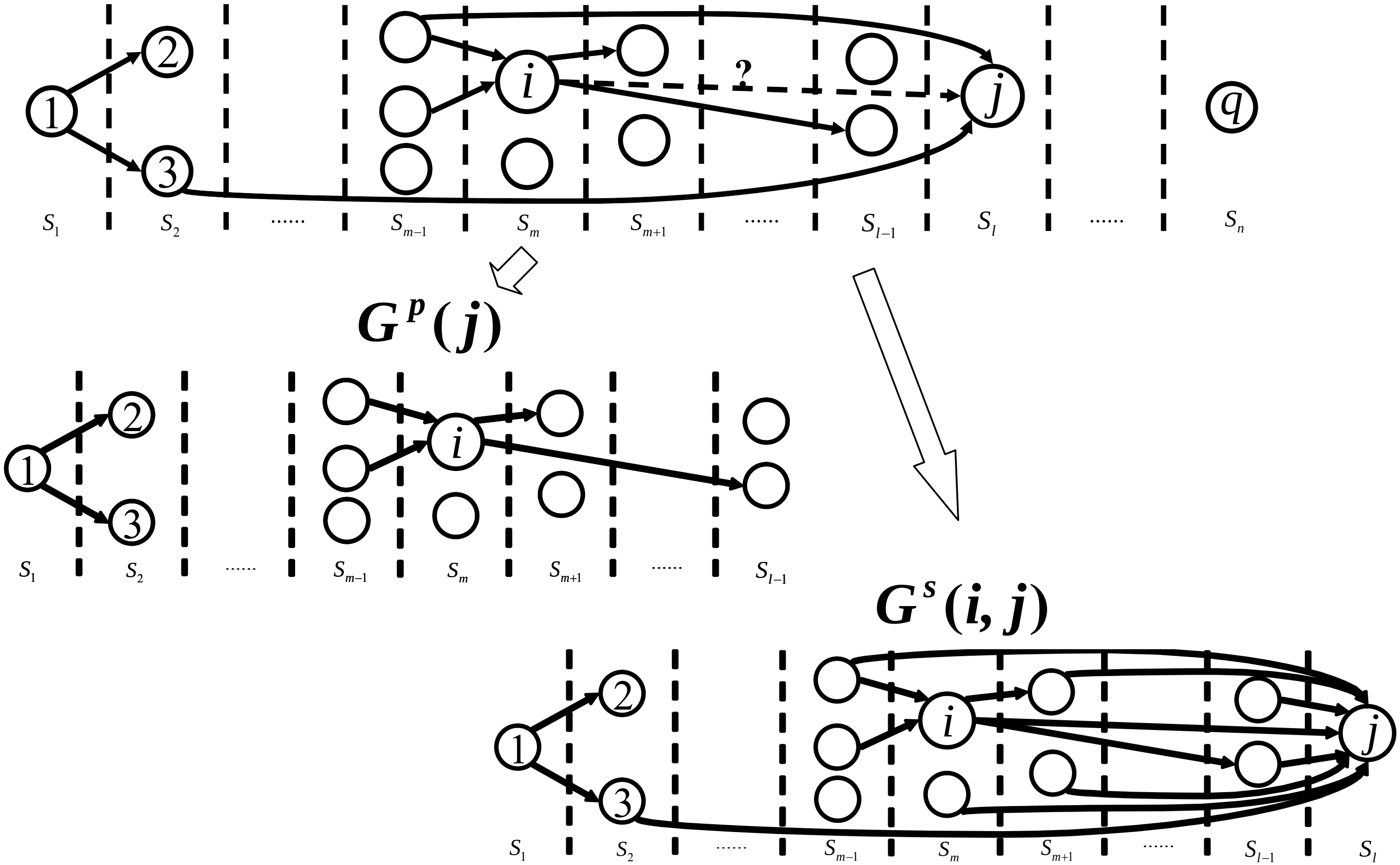
$$H_{ij} : \mathrm{corr}(i, j | \mathcal{R}(i,j)) = 0$$

Procedure to Identify $\mathcal{R}(i,j)$

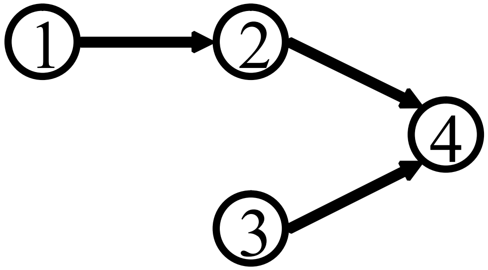


Step 1: Identify Two Subgraphs

Available relationships

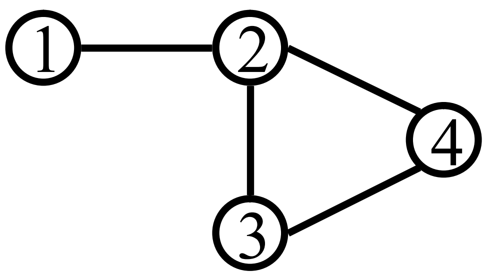


Definition of Moral Graph



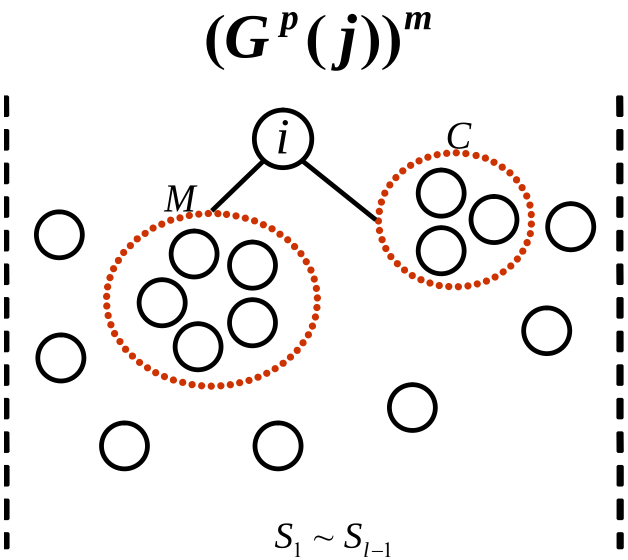
Directed Independence Graph ***D***

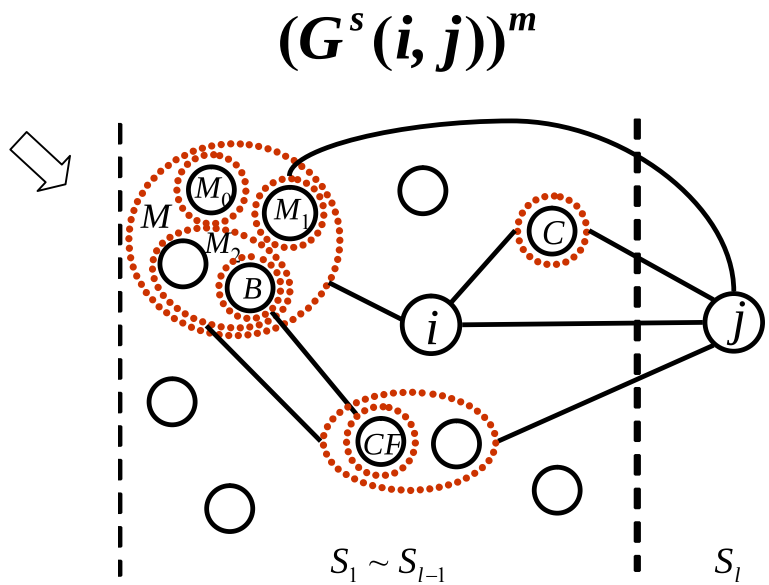
I. Join viables with common children by undirected edges
II. Replace each direct edge with an undirected one



Associated Moral Graph D^m

Step 3: Identify Important Subsets

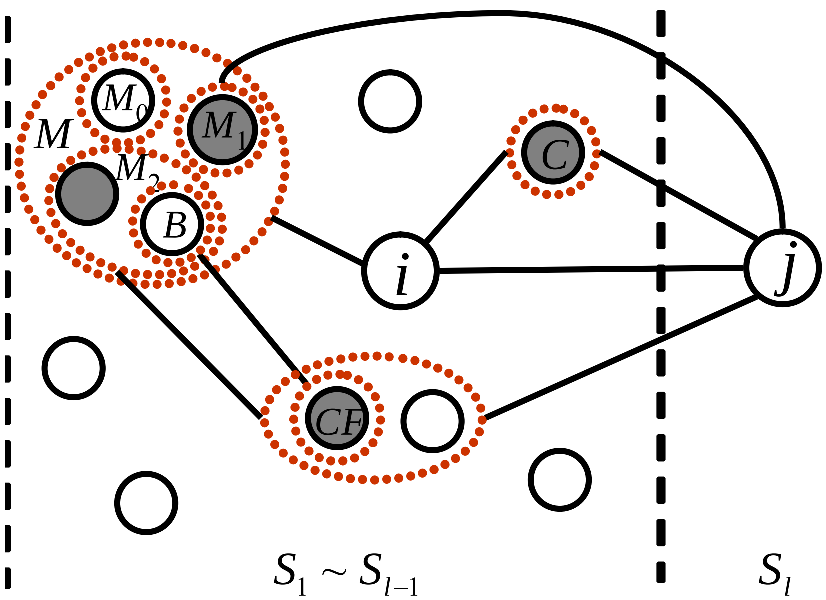




$\mathcal{R}(i,j)$ Identified

General expression

$$\mathcal{R}(i,j) = C \cup M_1 \cup (M_2 \setminus B) \cup CF$$



Overview of Proof

- **Goal** $\boldsymbol{i \perp\!\!\!\perp j \mid \mathcal{P}_j \setminus \{i\} \Leftrightarrow i \perp\!\!\!\perp j \mid \mathcal{R}(i,j)}$
- **Steps of proof**

Property: If (1) $T \perp\!\!\!\perp Y \mid X$ or $T \perp\!\!\!\perp Z \mid X$ (2) $T \perp\!\!\!\perp Y \mid X,Z$ or $T \perp\!\!\!\perp Z \mid X,Y$

Then $Y \perp\!\!\!\perp Z \mid X,T \Leftrightarrow Y \perp\!\!\!\perp Z \mid X$

$\Downarrow$

Lemma 1: $i \perp\!\!\!\perp j \mid \mathcal{P}_j \setminus \{i\} \Leftrightarrow i \perp\!\!\!\perp j \mid C \cup M$

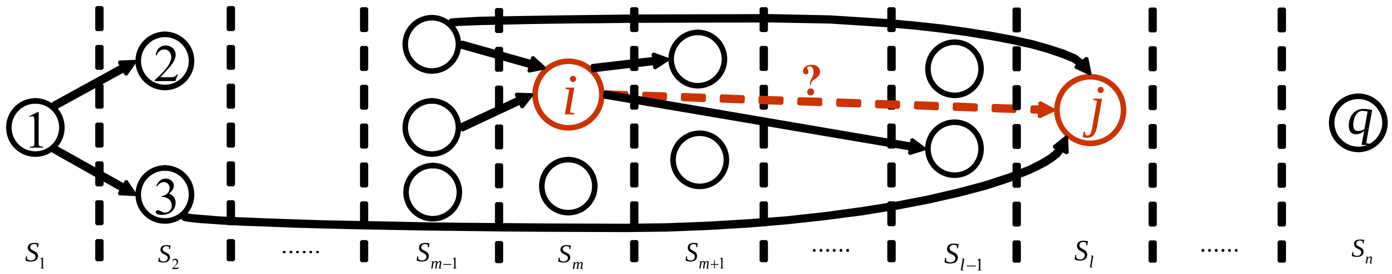
$\Downarrow$

Lemma 2: $i \perp\!\!\!\perp j \mid \mathcal{P}_j \setminus \{i\} \Leftrightarrow i \perp\!\!\!\perp j \mid C \cup M_1 \cup M_2$

$\Downarrow$

Goal: $i \perp\!\!\!\perp j \mid \mathcal{P}_j \setminus \{i\} \Leftrightarrow i \perp\!\!\!\perp j \mid C \cup M_1 \cup (M_2 \setminus B) \cup CF$

Statistical Testing Procedure



- **Procedure to identify $\mathcal{R}(i,j)$**

- **Test**

$$H_{ij} : \mathrm{corr}(i, j \mid \mathcal{R}(i,j)) = 0 \qquad \text{vs.} \qquad K_{ij} : \mathrm{corr}(i, j \mid \mathcal{R}(i,j)) \neq 0$$

- **Decision rule**

H_{ij} is rejected $\Rightarrow$ X_i has direct influence on X_j

Partial Correlation Test

Let

$$z_{ij|\mathcal{R}(i,j)} = \frac{1}{2}\log\frac{1+r_{ij|\mathcal{R}(i,j)}}{1-r_{ij|\mathcal{R}(i,j)}}$$

$$\sqrt{N-3-k(i,j)}\,z_{ij|\mathcal{R}(i,j)} \sim N(0,1), \quad N \to \infty$$

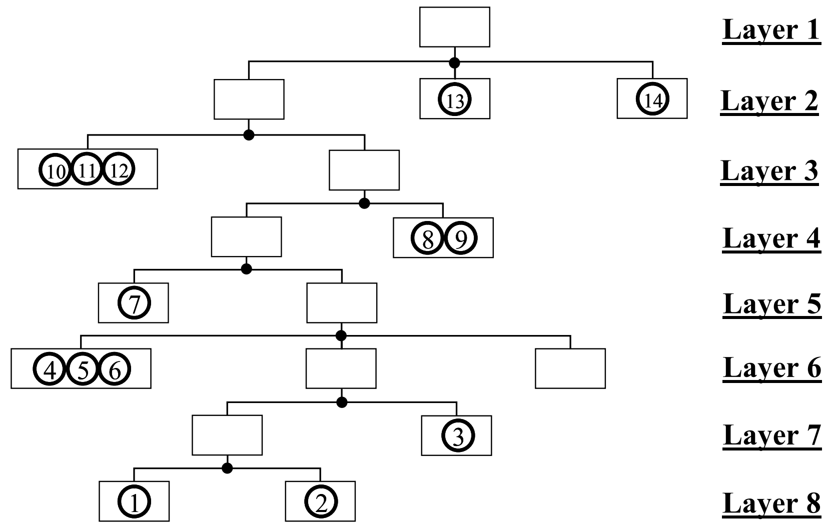
$r_{ij|\mathcal{R}(i,j)}$---- sample partial correlation

N-----sample size

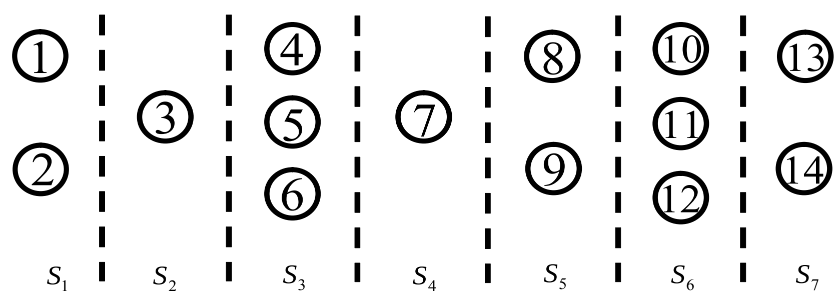
$k(i,j)$-----number of variables in $\mathcal{R}(i,j)$

Case Study

- Selected KPCs

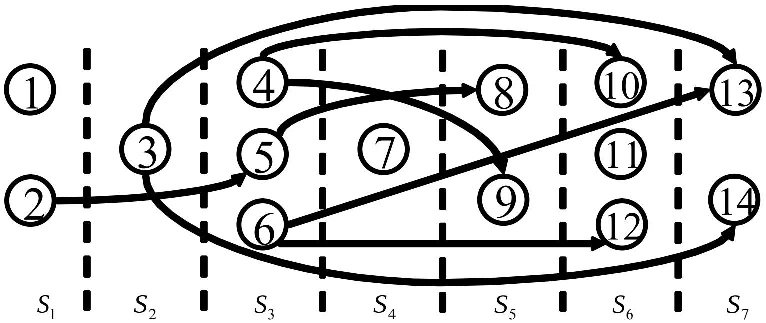


- Process layout



Identified Graph and Test Results

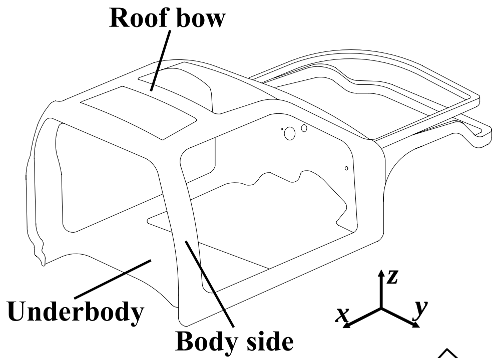
- **Chain graph constructed**



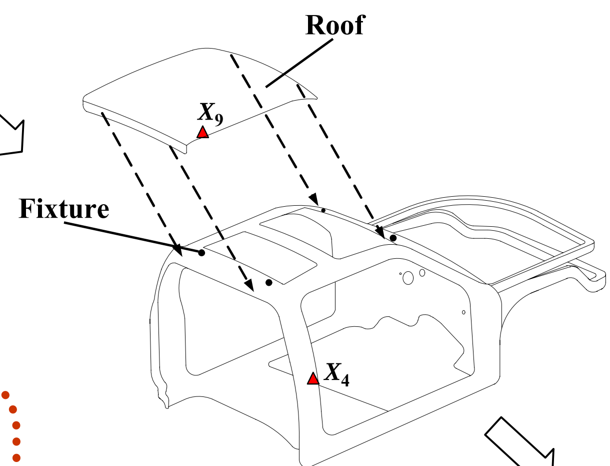
- **Test results of identified direct influences (C=2.807)**

Test #	Partial Correlation	Statistic
1	corr(2,5)	-3.0717
2	corr(5,8)	-3.3176
3	corr(4,9)	7.8978
4	corr(4,10|9)	8.1556
5	corr(6,12)	4.6126
6	corr(3,13)	7.2648
7	corr(6,13|12)	-3.3438
8	corr(3,14)	3.7819

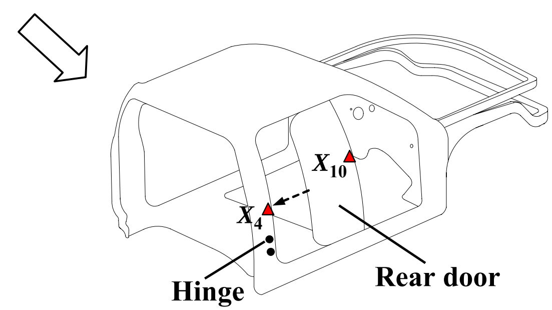
Interpretation



Car frame



Roofing



Rear door installation

Test results

(4) → (9)

(4) → (10)

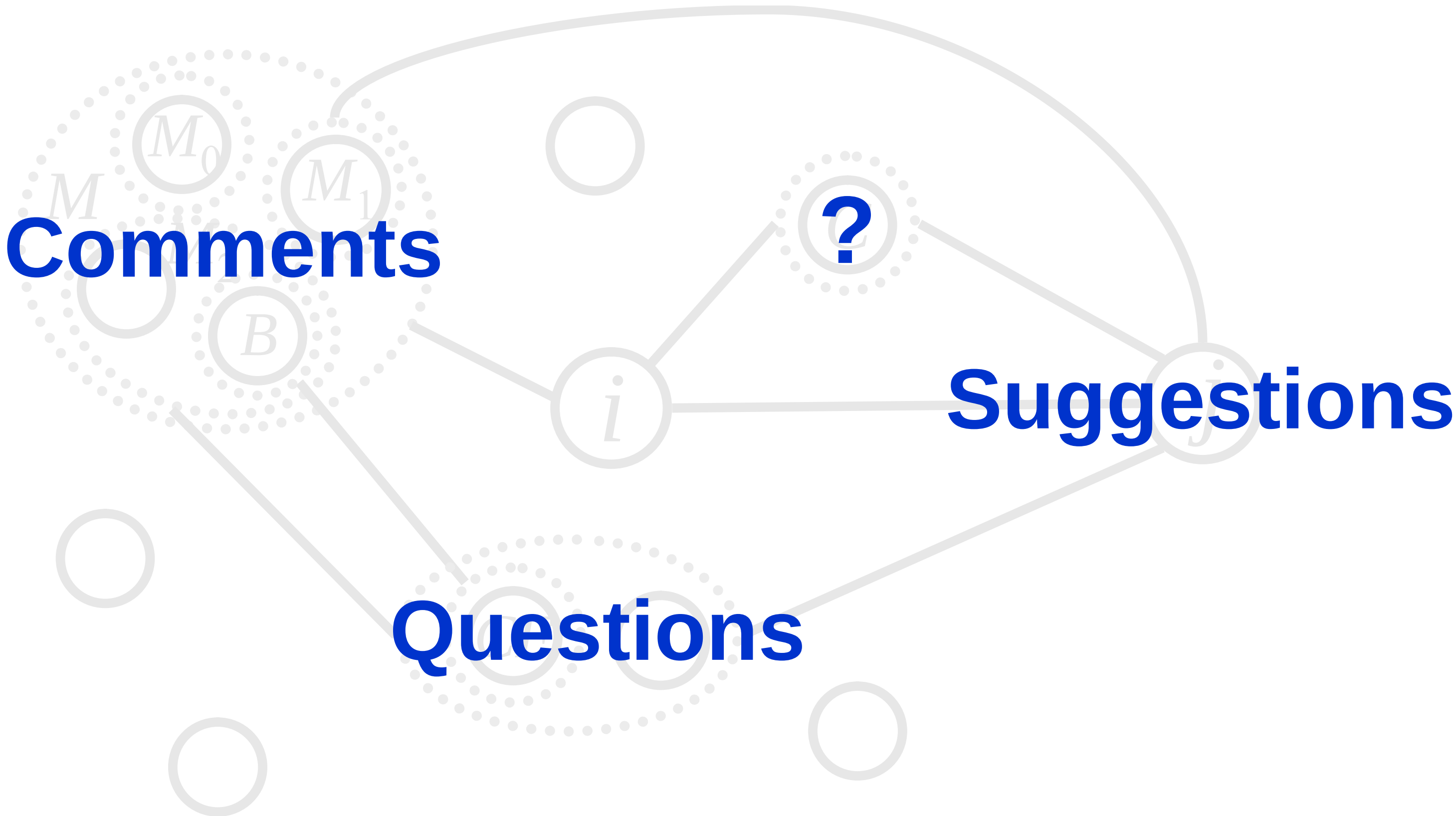
(9) ↛ (10)

Summary and Future Work

- A new and efficient methodology to conquer inter-stage complexity is presented and validated by case study

- A statistical testing procedure which can greatly reduce the redundancy in the testing is developed to build the chain graph of KPCs in a process

- Future work
  - **Extend the procedure to cases where (A1) is not satisfied**
  - **Efficient algorithm to identify B and CF**
  - **Study on overall errors of the procedure**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