Robust Target Localization in Wireless Sensor Networks

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Tasks of Wireless Sensor Networks (WSN)

- monitor physical and environmental conditions
- perform surveillance tasks (target detection, localization, and tracking)

WSN Characteristics

Sensors

- deployed over a region according to a regular pattern (e.g. grid), at random, or in an ad hoc manner
- collect information about the surrounding environment
- have fairly limited data processing capabilities
- communicate with other sensors and a remotely located (**fusion**) center

Fusion center processes data from sensors and makes a global (and more precise) situational assessment.

Target Localization: Problem Formulation

- N sensors deployed over the monitored region
- A target at location v emits a signal of strength S_0
- Sensor i located at position s_i obtains energy readings

$$E_i = S_i + \epsilon_i = S_0 C_\eta(\delta_i(v)) + \epsilon_i,$$

  where $\delta_i(v) = \|v - s_i\|$, $C_\eta(\cdot)$ is a monotone signal decay function, $C_\eta(0) = 1$.
- Assume random noise ϵ_i are i.i.d.
- Each sensor makes a decision $Y_i = \mathbf{1}(E_i \geq \tau)$
- τ is a function of an individual sensor false alarm probability γ

Related Work

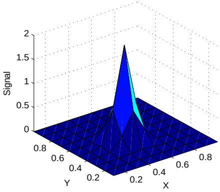
- **Overview**: surveillance monitoring (Estrin, 2006);
- **Information Fusion and Target Detection**: (Clouqueur, 2001, Katenka et al., 2006).
- **Localization**: radar systems (Abdel-Samad and Tewfik,1999); energy based methods (Kaplan,2001; Li,2002; Sheng, 2003; Blatt and Hero, 2006); binary decisions based methods (Niu and Varshney, 2004; Noel, 2006; Ermis and Saligrama, 2006).

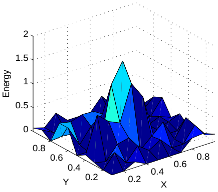
Signal Decay Models Considered

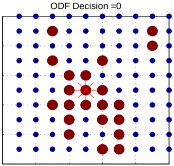
$$\text{M1: } S_i = S_0 \exp(-\delta_i(v)/\eta)^2),$$
$$\text{M2: } S_i = \frac{S_0}{1 + (\delta_i(v)/\eta)^3}.$$

where η is a tuning parameter

- **M1** is appropriate for capturing temperature attenuation patterns (example below)
- **M2** is widely used for acoustic signals







Decision Fusion for the WSN

Ordinary Decision Fusion (ODF)

- Only positive decisions Y_i are transmitted to fusion center $\Rightarrow$ low communication cost
- More robust to noise than energy-based methods (**value fusion**)

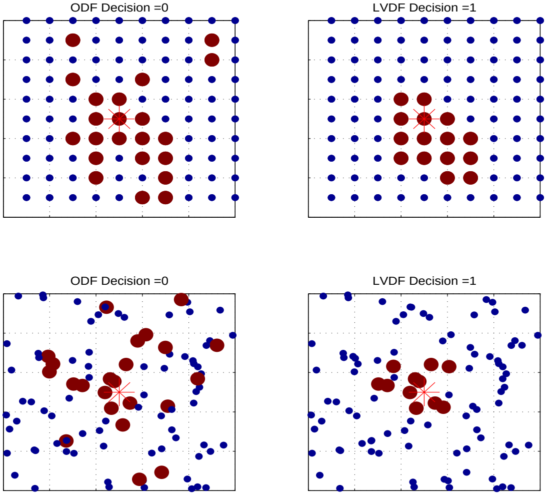
Local Vote Decision Fusion (LVDF)

1. Sensor i makes an initial decision Y_i and communicates it to all sensors in its neighborhood $U(i)$
2. Given the set of decisions $\{Y_j : j \in U(i)\}$, sensor i adjusts its initial decision according to a majority vote:

$$Z_i = \mathbf{1}\Big(\sum_{j \in U(i)} Y_j > M_i/2\Big),$$

where $M_i = |U(i)|$ is the size of the neighborhood

ODF vs. LVDF Illustration



Surveillance Tasks

Given the energy measurements E_i, or binary initial Y_i, or corrected Z_i decisions,

1. detect the presence of a target
2. identify target location $v = (v_x, v_y)$
3. estimate the strength of the signal S_0
4. with information available over time, track target trajectory through the monitoring region

Remark: In the remainder, it is assumed that all communication tasks have been resolved.

Competing Estimators

1. ML(E) Maximum mikelihood estimation based on energy readings E_i (gold standard)
2. ML(Y) Maximum likelihood estimation based on original decisions Y_i
3. ML(Z) Maximum likelihood based on corrected decisions Z_i
4. Hubrid methods that combine energy readings E_i from sensors that returned a positive decision and imputed energy readings from the remaining sensors

Target Localization from ODF

Let $\theta = (v_x, v_y, S_0, \sigma^2, \eta)$ vector of unknown parameters. Assuming Gaussian i.i.d. noise with 0 mean and variance σ^2, $E_i \sim N(S_0 C_\eta(\delta(v)), \sigma^2)$, $\{Y_i\} \sim$ Bernoulli(α_i)

$$\alpha_i(\theta) = 1 - \Phi(A_i(\theta)),$$

where $A_i(\theta) = \frac{\tau - S_0 C_\eta(\delta_i(v))}{\sigma}$.

- Direct numerical maximization of the log-likelihood function of $\{Y_i\}$:

$$\ell_Y(\theta) = \sum_{i=1}^{N} [Y_i \log \alpha_i(\theta) + (1 - Y_i) \log(1 - \alpha_i(\theta))] .$$

- Expectation-Maximization (EM) algorithm.

Target Localization from ODF

Expectation-Maximization (EM) algorithm

M-step:

$$\hat{S}_0 = \frac{\sum_{i=1}^{N} E_i C_\eta(\delta_i(v))}{\sum_{i=1}^{N} C_\eta^2(\delta_i(v))},$$

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^{N} (E_i - \hat{S}_0 C_\eta(\delta_i(v)))^2$$

E-step:

$$\hat{E}_i = \mathbb{E}[E_i | Y] = \mathbb{E}[E_i | Y_i]$$

$$\mathbb{E}[E_i | Y_i = 0] = \frac{\int_{-\infty}^{\tau} x p_{E_i}(x)\, dx}{\int_{-\infty}^{\tau} p_{E_i}(x)\, dx} =$$

$$= S_0 C_\eta(\delta_i(v)) - \frac{\sigma \exp\left(-\frac{A_i(\theta)^2}{2}\right)}{\sqrt{2\pi}\Phi(-\frac{A_i(\theta)^2}{2})}$$

Target Localization from LVDF

- Direct log-likelihood is complicated (Z_i correlated)
- Use pseudo-likelihood (assuming Z_i *independent*)
- Assume for neighbors $j \in U(i)$, $\mathbb{P}(Y_j = 1) \approx \mathbb{P}(Y_i = 1)$

Let $\beta_i(\theta) = \mathbb{P}(Z_i = 1)$,

$$\mathbb{P}(Z_i = 1) = \mathbb{P}(\sum_{j \in U(i)} Y_j \geq \frac{M}{2}) \approx \sum_{k=[M/2]}^{M} \binom{M}{k} \alpha_i^k (1-\alpha_i)^{M-k}$$

The **pseudo-loglikelihood** function for the adjusted decisions Z_i is given by:

$$\ell_Z(\theta) = \sum_{i=1}^{N} [Z_i \log \beta_i(\theta) + (1 - Z_i) \log(1 - \beta_i(\theta))] .$$

Expectation-Maximization Algorithm

- **M-step** is the same as for ODF
- **E-step** requires calculating $\mathbb{E}[E_i|Z]$:

$$\mathbb{P}[E_i|Z] = \frac{1}{\mathbb{P}(Z)} \sum_{k=0,1} \mathbb{P}(E_i, Z|Y_i = k)\mathbb{P}(Y_i = k) =$$

$$= \frac{1}{\mathbb{P}(Z)} \sum_{k=0,1} \mathbb{P}(E_i|Y_i = k)\mathbb{P}(Z|Y_i = k)\mathbb{P}(Y_i = k)$$

$$\mathbb{E}[E_i|Z] = \sum_{k=0,1} \mathbb{E}(E_i|Y_i = k)\mathbb{P}(Y_i = k|Z)$$

Expectation-Maximization Algorithm

$$\mathbb{E}[E_i|Z] = \sum_{k=0,1} \mathbb{E}(E_i|Y_i = k)\mathbb{P}(Y_i = k|Z)$$

- $\mathbb{E}[E_i|Y_i]$ obtained in the E-step for ODF
- $\mathbb{P}(Y_i = k|Z)$ to be calculated as:

$$\mathbb{P}(Y_i = 1|Z) = \frac{\mathbb{P}(Y_i = 1)\mathbb{P}(Z|Y_i = 1)}{\mathbb{P}(Z)}$$

$$= \alpha_i \prod_{j:i \in U(j)} \frac{\mathbb{P}(Z_j|Y_i = 1)}{\mathbb{P}(Z_j)}$$

Hybrid Localization Methods

Using the energy readings from $Y_i = 1$ or $Z_i = 1$

- Reduces communication cost relative to ML(E)
- Improves localization relative to ML(Y) or ML(Z)

Impute remaining energy readings by:

- Hybrid Expectation Maximization (HEM): model $\hat{E}_i = \mathbb{E}[E_i | Y_i = 0]$ for ODF and $\hat{E}_i = \mathbb{E}[E_i | Z], \ i \in \{Z_i = 0\}$
- Hybrid Maximum Likelihood (HML): set $\hat{E}_i = \tau$ for $i \in \{Y_i = 0\}$ ($i \in \{Z_i = 0\}$)

Simulation Study Setup

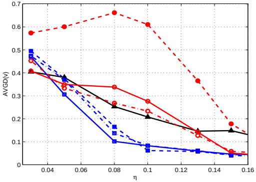
- Focuse on the target location v and the signal amplitude S_0 estimation
- Assume the parameters η and σ^2 are known
- Gaussian noise with mean zero, variance σ^2
- WSN deployed on a 20×20 grid in the unit square, true target location $v = (1/4, 1/4)$, $S_0 = 2$, the individual sensor's false alarm $\gamma = 0.1$, and the network's false alarm $F = 0.1$.
- signal-to-noise ratio SNR = S_0/σ
- *all* the algorithms detected the target according to their respective detection algorithm

AVERAGE DISTANCE (AVGD) FROM THE TRUE LOCATION v AS A FUNCTION OF SNR

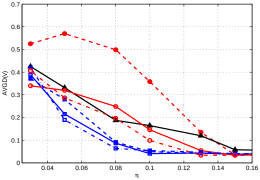
SNR	ML(E)	ML(Y)	EM(Y)	HML(Y)	HEM(Y)
	Model M1, $\eta = 0.1$				
2	0.208	0.576	0.223	0.329	0.236
5	0.011	0.502	0.113	0.275	0.066
10	0.005	0.421	0.028	0.221	0.006
	Model M2, $\eta = 0.1$				
2	0.164	0.388	0.119	0.226	0.111
5	0.011	0.101	0.018	0.031	0.012
10	0.005	0.064	0.017	0.021	0.005

SNR	ML(E)	ML(Z)	EM(Z)	HML(Z)	HEM(Z)
	Model M1, $\eta = 0.1$				
2	0.208	0.077	0.056	0.075	0.089
5	0.011	0.019	0.020	0.012	0.012
10	0.005	0.019	0.016	0.010	0.006
	Model M2, $\eta = 0.1$				
2	0.164	0.066	0.050	0.093	0.049
5	0.011	0.022	0.021	0.012	0.012
10	0.005	0.020	0.020	0.006	0.005

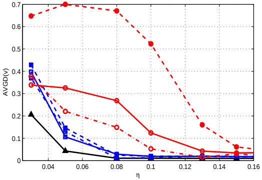
Average distance from true target location V as a function of η (controls the size of the target)



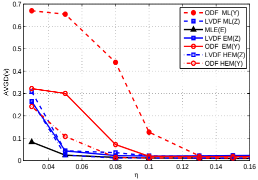
SNR=2 (M1)



SNR=2 (M1)

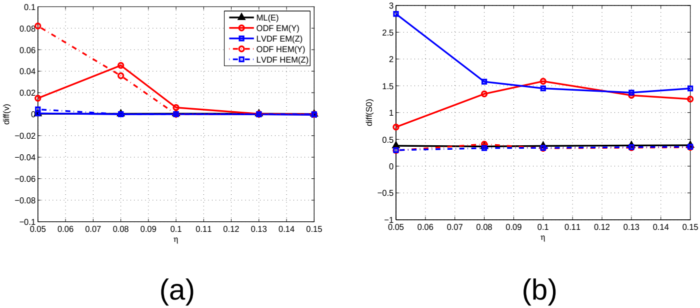


SNR=5 (M1)



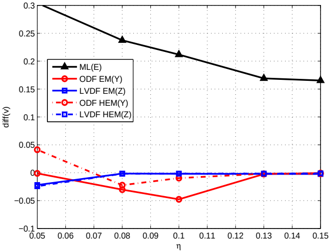
SNR=5 (M1)

TRUE MODEL M2 MISSPECIFIED AS M1 WITH SNR=5

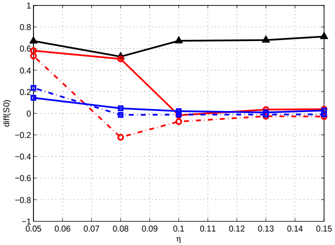


- **(a)** the difference between average distances from true v for misspecified and true models
- **(b)** the difference between RMSE of $\hat{S}_0$ for misspecified and true models

TRUE NOISE DISTRIBUTION t_3 MISSPECIFIED AS GAUSSIAN, WITH SNR=5



(a)



(b)

- **(a)** the difference between average distances from true v for misspecified and true models
- **(b)** the difference between RMSE of $\hat{S}_0$ for misspecified and true models

Summary of simulation results

- in the low SNR (2) regime, LVDF clearly outperforms the "gold standard"
- for the medium and high SNR regimes, LVDF exhibits a competitive performance (HEM(Z) essentially the same as the "gold standard")
- for the ODF based algorithms, the EM version significantly outperforms ML(Y)
- for larger values of SNR the accuracy of all the algorithms improves
- the ML(E) algorithm, together with both variants of the LVDF algorithms approx. achieve the nominal level of 95% for all SNRs examined, whereas the ODF ones fall short
- LVDF proves to be robust to signal model and to the noise distribution misspecification

Tracking of Moving Targets

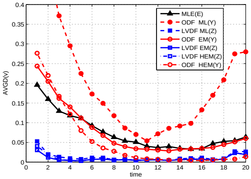
- Examine the performance of the various algorithms when it comes to tracking the position of a target moving through the monitoring region
- Assume sensors record energy readings at time slots $t = 1, 2, \ldots$ and make decisions at each time slot
- Combine the results for target location and signal magnitude over time using an exponentially weighted moving average scheme (EWMA):

$$v_t = \lambda \hat{v}_t + (1-\lambda) v_{t-1}, \; t = 1, 2, \cdots$$
$$S_{0,t} = \lambda \hat{S}_{0,t} + (1-\lambda) S_{0,(t-1)}, \; t = 1, 2, \cdots$$

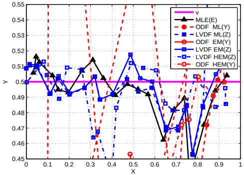
- use $\tilde{v}_{t-1}$ as a starting value for localization at time t

Target Tracking

- Scenario: Target moving from west to east through the middle of the monitoring region; signal is stationary $S_{0,t} = 2$ with SNR=2



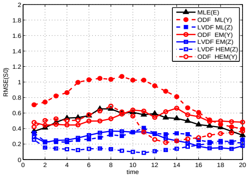
Scenario 1:AVGD



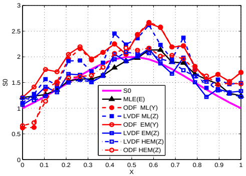
Scenario 1:Trajectory

Signal Diagnostics

- Scenario 2: Target is stationary at $v = (0.25, 0.25)$; the signal amplitude evolves over time, $S_0(t) = 2/(1 + 0.01(t - 10)^2)$.



Scenario 2:RMSE



Scenario 2:Diagnostics

Multiple Targets: Problem Formulation

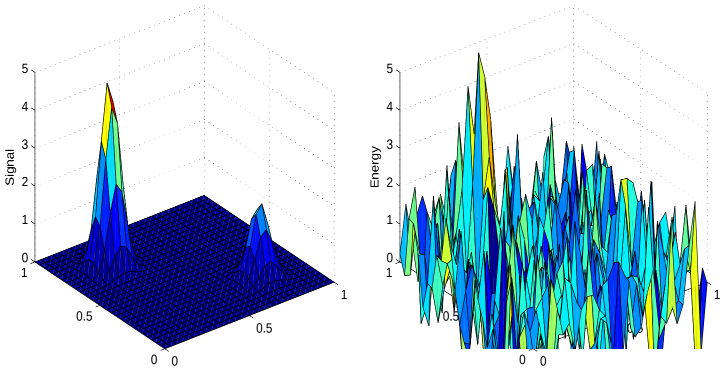
There are p possible targets in the monitored region, and each target at location $v_j \in R$ emits a signal $S_i^j \equiv S_i(v_j)$ measured at sensor location s_i. The energy measured by the i-th sensor is:

$$E_i = S_i + \epsilon_i = \sum_{j=1}^{p} S_i(v_j) + \epsilon_i,$$

where $S_i(v_j) = S_0^j C_{\eta_j}(||s_i - v_j||, \eta_j)$ is the signal model for the target j, $j = 1, .., p$.

- The detection techniques are extended to the multiple target detection problem.
- The multiple target localization is more challenging, when the number of targets present is unknown.

The two-target signal and measured energies generated by model M1



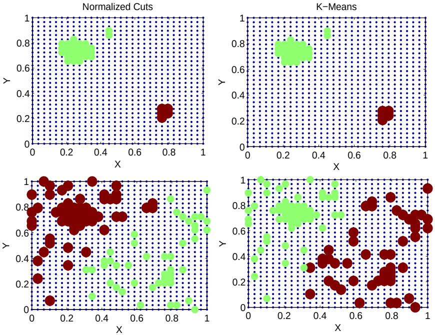
$S_0^{(1)} = 5$, $v_1 = (0.25, 0.75)$, $S_0^{(2)} = 2$, $v_2 = (0.75, 0.25)$,
$\eta_1 = \eta_2 = 0.07$; the noise with $\sigma = 1$.

Heuristic algorithm for Multiple Target Localization

1. Obtain initial (ODF) or corrected (LVDF) decisions.
2. Apply a clustering technique to the locations of positive decisions. The number of targets may be assumed known or the clustering technique can be used to estimate the number of targets.
3. Apply any of the localization and signal estimation techniques described before to each of the clusters separately to obtain location and signal amplitude estimates for each target.

THE RESULTS OF THREE CLUSTERING METHODS

LVDF (top row) and ODF (bottom row)



Concluding Remarks

The LVDF algorithms prove to be:

- highly accurate in terms of location estimation
- robust to the presence of high levels of noise
- robust to signal model and noise distribution misspecifications

De-noising LVDF property helps:

- estimating number of multiple targets and their location
- scheduling sensor sampling regime