CT-TH Curve Fitting

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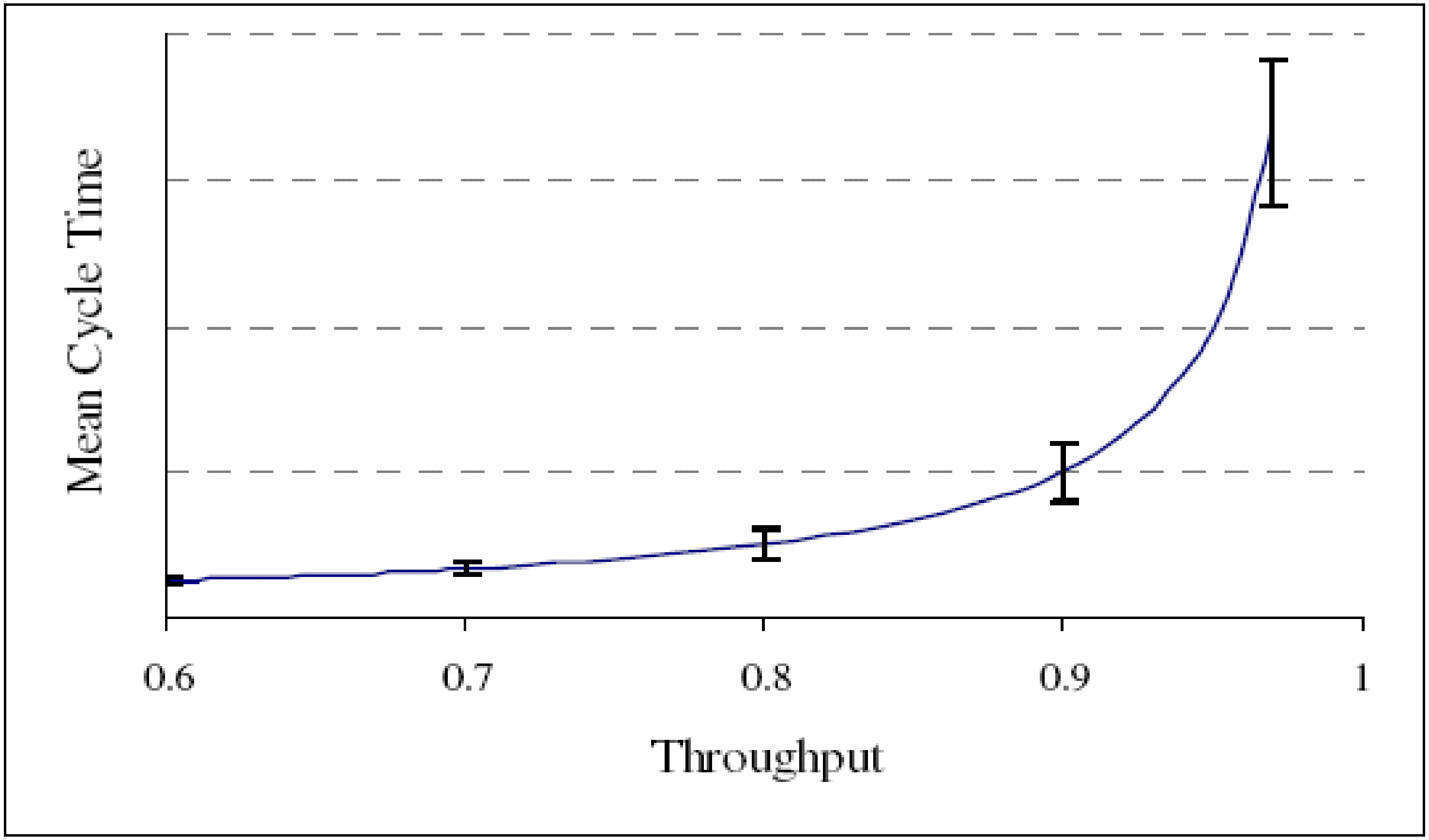
Problem Description

- There is a relationship between the cycle time (CT) and the throughput (TH) of a production system
- TH is often scaled to throughput rate, which ranges from 0 to 1
  - Represents the percent of theoretical capacity in use
  - CT goes to ∞ (exponentially) as TH goes to 1

CT – TH Curve



History

- Cheng and Kleijnen proposed a metamodel to represent CT-TH curves

$$y = \frac{\sum_{i=0}^{n} c_i * x^i}{(1-x)^p}$$

- Yang, Ankenman, and Nelson used this model to fit a CT-TH curve for MASM Data Set 1

Proposed Improvement

- Cheng and Kleijnen, and Yang, *et al.* use unequal allocation of simulation runs to attempt to stabilize the variance of the input points
- This means allocating huge effort to the high TH portions of the curve
- We propose fitting a different model
  - Using weights to allow for unequal variance of the data points
  - Using individual replications as opposed to average values

JRM Model

- CT-TH curves are exponential growth curves
- Combining growth models with the Cheng and Kleijnen model, we obtain:

$$y = \beta_0 + \frac{(\beta_1 * \exp(\beta_2 * x))}{(1 - x)^{\beta_3}}$$

E[CT]

Growth Curve

Throughput Rate

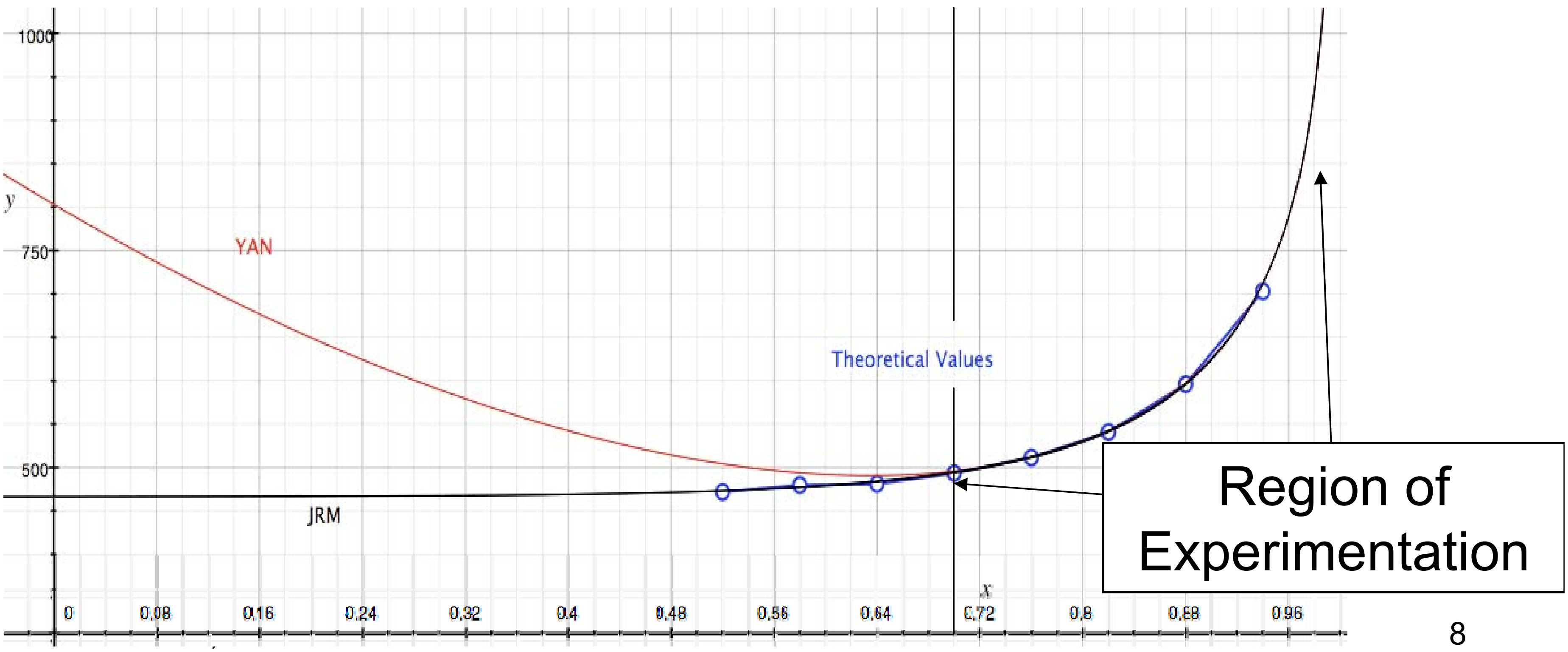
Modeling Data

- Both YAN and JRM models fit from the following data table
- Unequal variance is clearly present

Throughput rate	Cycle Time	Std Dev
0.7	493.4	1.47
0.76	511.44	3.75
0.82	540.82	4.7
0.88	595.92	4.93
0.92	661.16	6.79
0.95	743.19	9.94
0.96	786.48	11.22
0.97	851.05	17.29
0.98	929.97	22.58

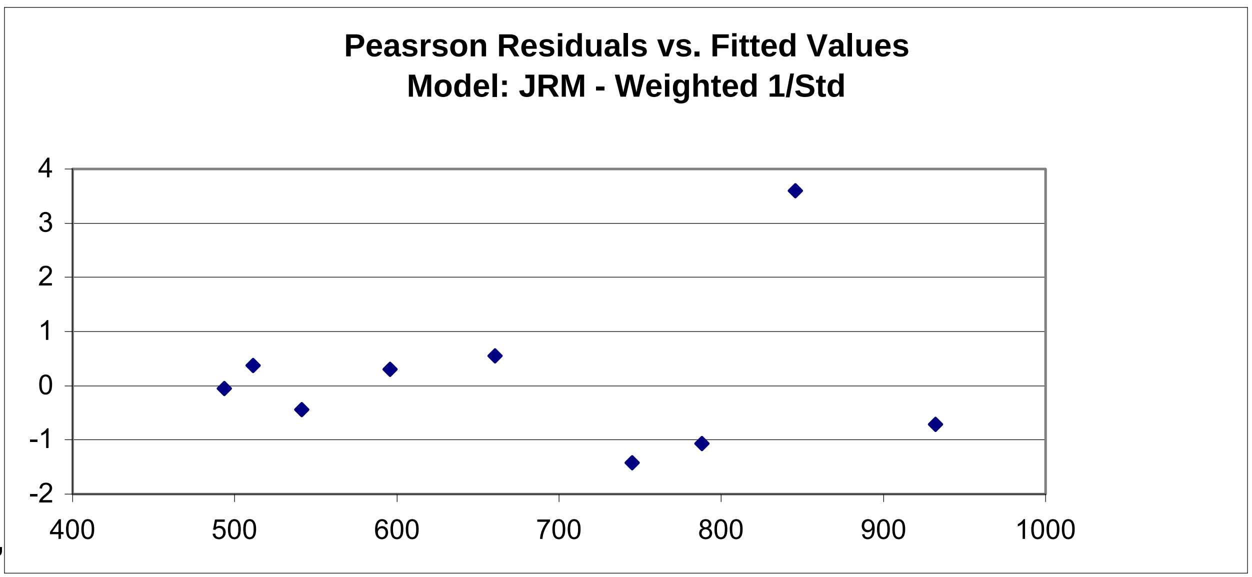
Comparing Models

- JRM model provides much better extrapolation and slightly better fit in the region of experimentation



JRM Model Adequacy

- The Pearson residuals vs. the fitted values showed one outlier (.97 TH)
- Removing the outlier and refitting did not significantly alter the coefficient estimates



Confidence Limits

- If an unweighted model is fit to the data, the confidence intervals jump around unpredictably
- A weighted model fit gives monotonically increasing CI widths as TH (and variability) increase
- This is a much more reliable result given the underlying variability of the data

Confidence Interval Ranges

Design Points	Non-Weighted CI Range	Weighted CI Range
70	12.1	4.21
76	8.1	3.59
82	8.53	5.02
88	9.33	5.27
92	7.92	5.44
95	8.29	6.8
96	8.42	6.97
97	7.99	7.72
98	13.57	15.77

Individual Values vs. Averages

- Using the individual points as opposed to the averages gives a better estimate of error at those points
- The resulting prediction intervals reflect more realistic results

Prediction Intervals

Design Points	Averages PI Range	Individuals PI Range
70	6.13	15.03
76	7.98	23.33
82	9.43	26.28
88	9.73	26.95
92	11.03	31.5
95	13.45	38.13
96	14.17	40.46
97	17.15	50.08
98	23.56	58.76

Conclusions / Future Work

- The JRM model outperforms the YAN model, especially in extrapolation performance
- This model allows for unequal variance in the observations
- Question: What is the optimal design for fitting the JRM (nonlinear, unequal variance) model?
  - D-optimality
  - I-optimality

References

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Questions ?

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