An Assessment of Methods for the Statistical Monitoring of Autocorrelated Data

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Outline

- The environment
- Our problem
- Methods evaluated
- Evaluation criteria
- Method details
- Method assessment
- Conclusions

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The Environment

- Ecolab has 24 plants in 8 countries
- Plants have 6 to 12 lines and manufacture dozens of products at high volumes
- Multiple product characteristics are monitored on each line

The Problem

- Many of the data streams have ***autocorrelation*** and ***wandering means*** *(non-stationary)*
  - Most commonly first or second order autoregressive series
- Plants do not have statisticians
- Impact:
  - High false alarm rates, increased costs
  - Operators and management … “SPC does not work”
  - Control limits manually adjusted

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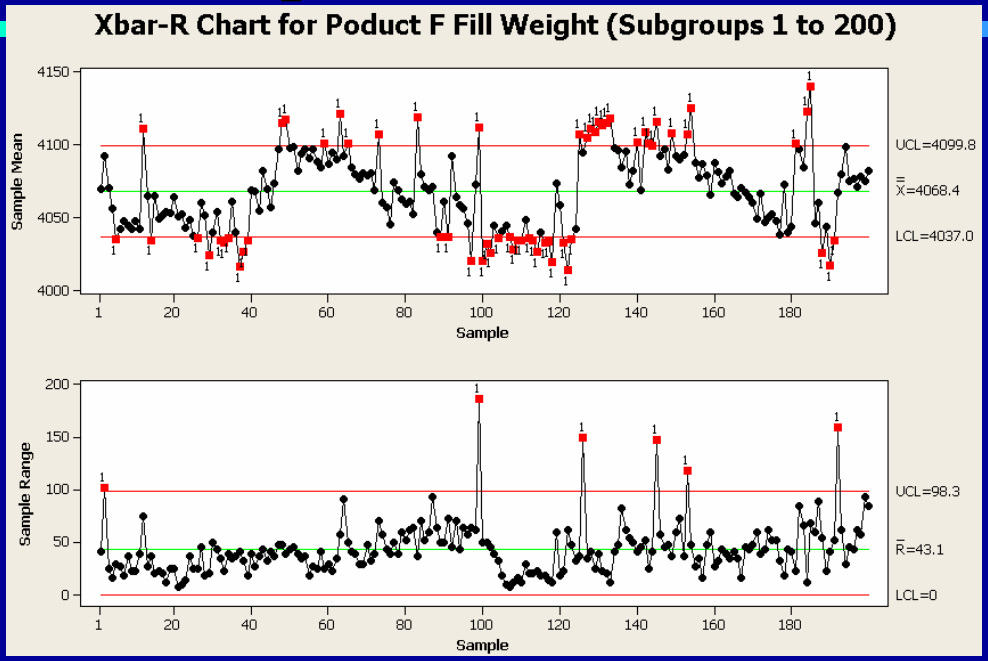
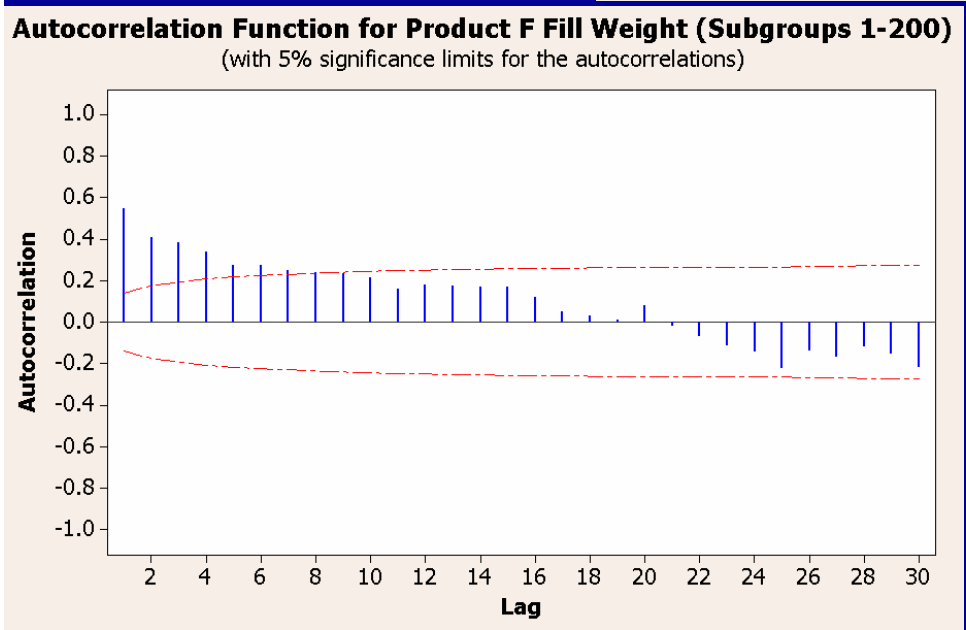
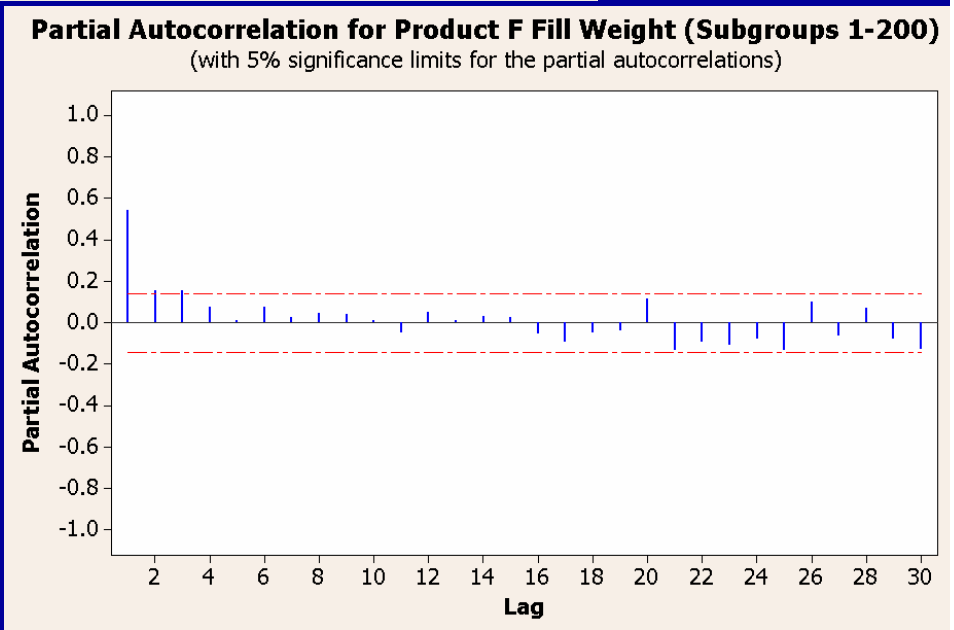
Data Example

- Subgroup size=4
- Sampled at 15 minute intervals







Methods Reviewed

- **Shewhart charts**
- **Shewhart charts but decrease sampling frequency**
- **Fit time series model and control chart residuals**
  - Montgomery, D.C. & Mastrangelo, C.M. (1991). Some statistical process control methods for autocorrelated data. *Journal of Quality Technology*, 23, 179-193.
  - Lu, C. & Reynolds, M.R. (1999). EWMA control charts for monitoring the mean of autocorrelated processes. *Journal of Quality Technology*, 31, 166-188.
- **Modified control limits**
  - Vasilopoulos, A.V. & Stamboulis, A.P. (1978). Modifications of control limits in the presence of data correlation. *Journal of Quality Technology*, 10, 20-30.
- **Box-Jenkins Bounded Manual Adjustment Charts**
  - Box, G.E.P. (1991) Bounded adjustment charts. *Quality Engineering*, 4, 333-340.
  - Box, G.E.P & Paniagua-Quinones, C. (2007). Two charts: not one. *Quality Engineering*, 19, 93-100.
  - Hunter, J. S. (1998). The Box-Jenkins bounded manual adjustment chart: a graphical tool designed for use on the production floor. *Quality Progress*, 129-137.

Method Evaluation Criteria

Evaluation Criteria	Weight
Deals with autocorrelation	0.20
Deals with wandering means	0.25
Limited statistician involvement	0.15
Low level of effort to implement	0.20
Supported by available software	0.05
Operators can understand charts	0.10
Feedback on how to adjust process	0.05

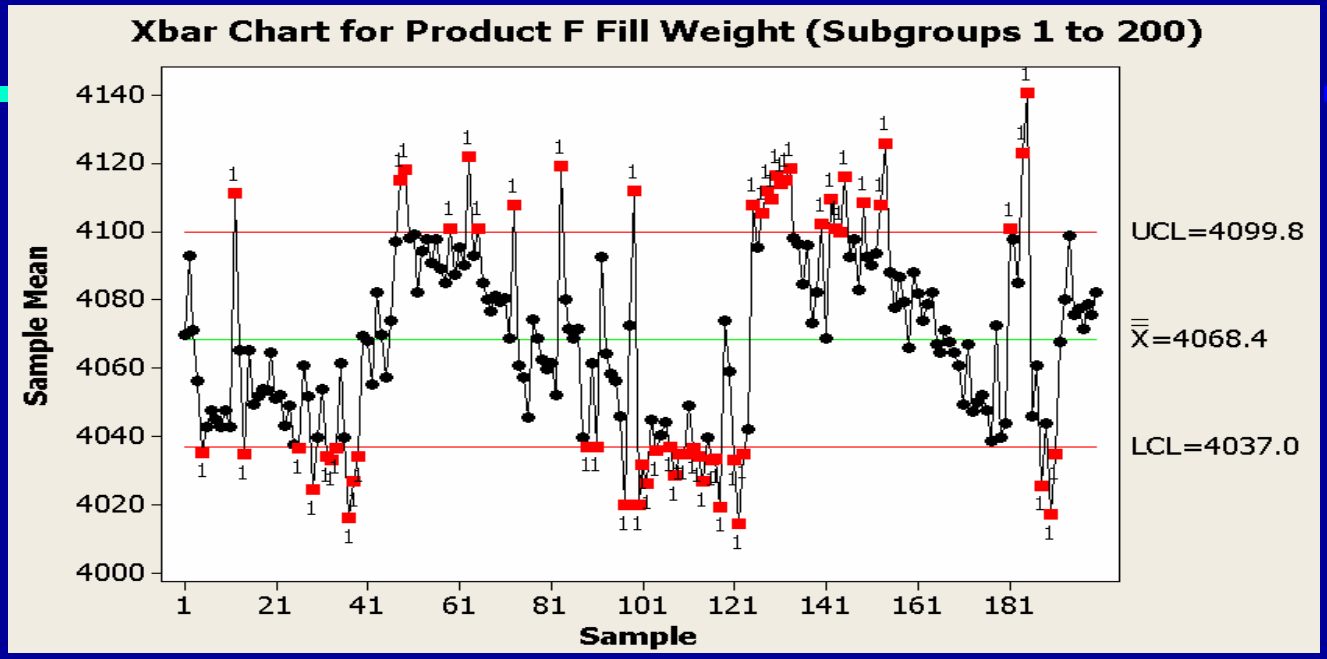
Montgomery & Mastrangelo (1991)

Tasks

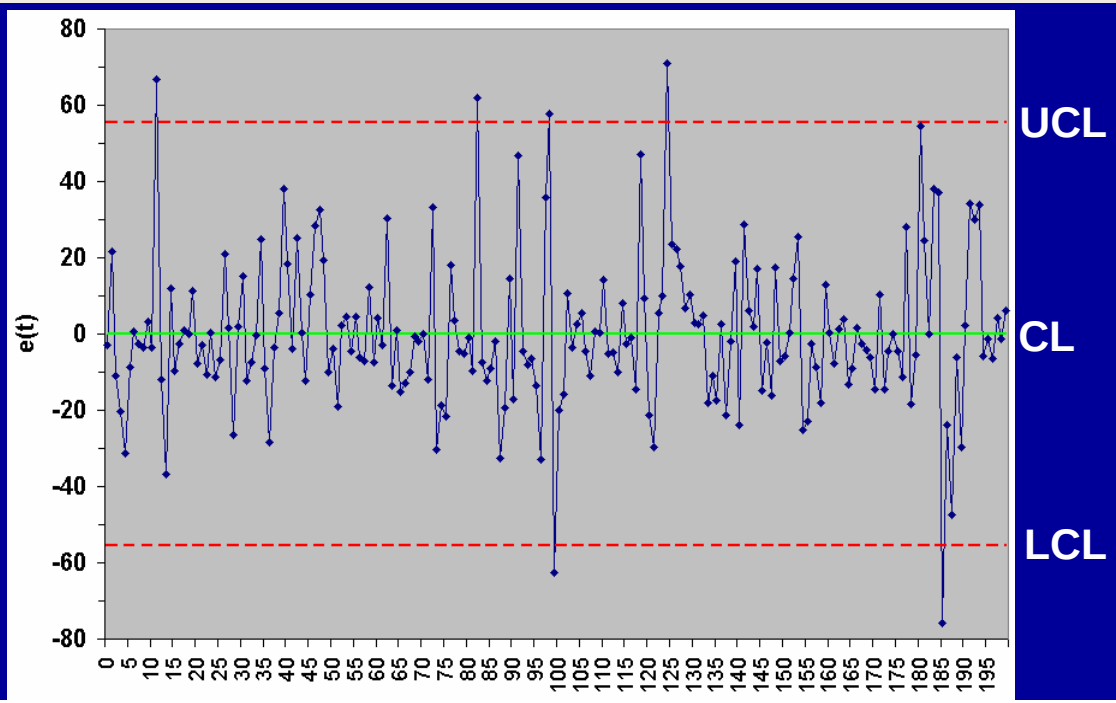
- Analyze data to confirm meet application conditions of method
- Estimate optimal smoothing constant (λ)
- Estimate standard deviation of forecast errors (σ_p)
  - Needed to calculate control limits

Montgomery & Mastrangelo (1991)

Shewhart X-bar chart



EWMA of Model Residuals



Montgomery & Mastrangelo (1991)

Observations

- Good performance handling autocorrelation
- Wandering mean issues…
  - M & M (1991, p. 182): "…if the observations from the process are positively autocorrelated and the process mean does not drift too quickly…"
- Relatively easy to implement
- What data to include (exclude) when estimating σ_p
- Requires statistician to perform time-series analysis
- Operator acceptance

Vasilopoulis & Stamboulis (1978)

Tasks

- Estimate autoregressive parameters α_1, α_2
- Get A(0,0,n) term
- Get $\lambda^{\frac{1}{2}}(\alpha_1, \alpha_2, n)$ term using figure lookup
- Calculate modified control limits

$$\bar{\bar{x}} \pm A(0,0,n)\lambda^{\frac{1}{2}}(\alpha_1, \alpha_2, n)\sigma$$

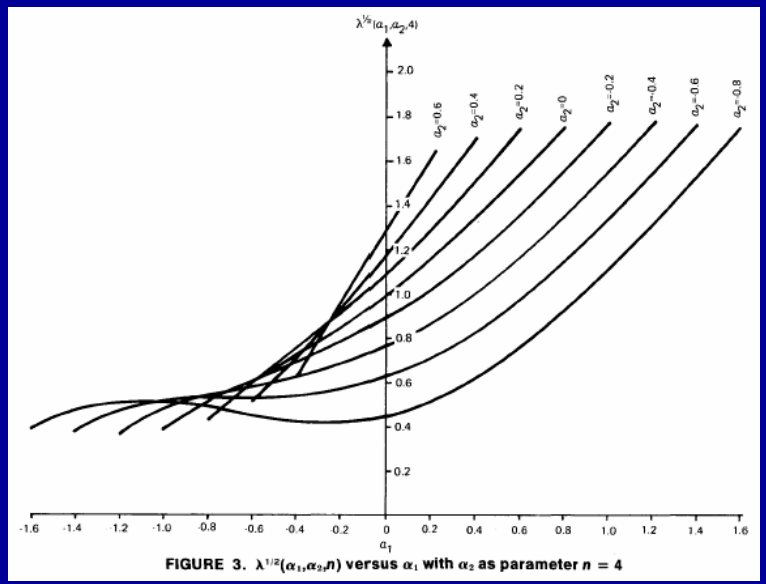


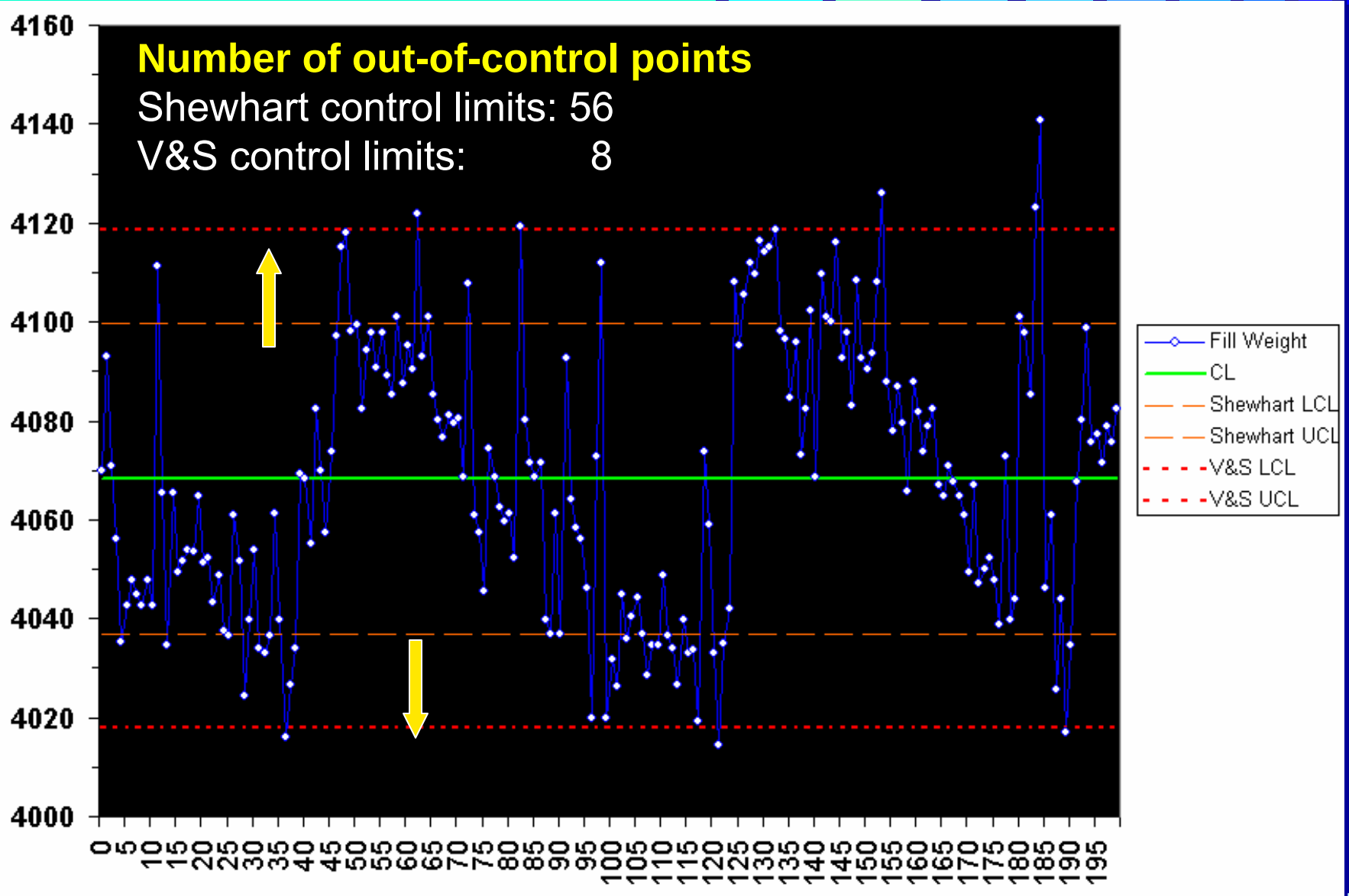
FIGURE 3. $\lambda^{1/2}(\alpha_1,\alpha_2,n)$ versus α_1 with α_2 as parameter $n = 4$

4068.4 ± (1.5) (1.65) 20.35

LCL = 4018.0 *(Shewhart = 4037.0)*

UCL = 4118.8 *(Shewhart = 4099.8)*

Vasilopoulis & Stamboulis (1978)



Vasilopoulis & Stamboulis (1978)

Observations

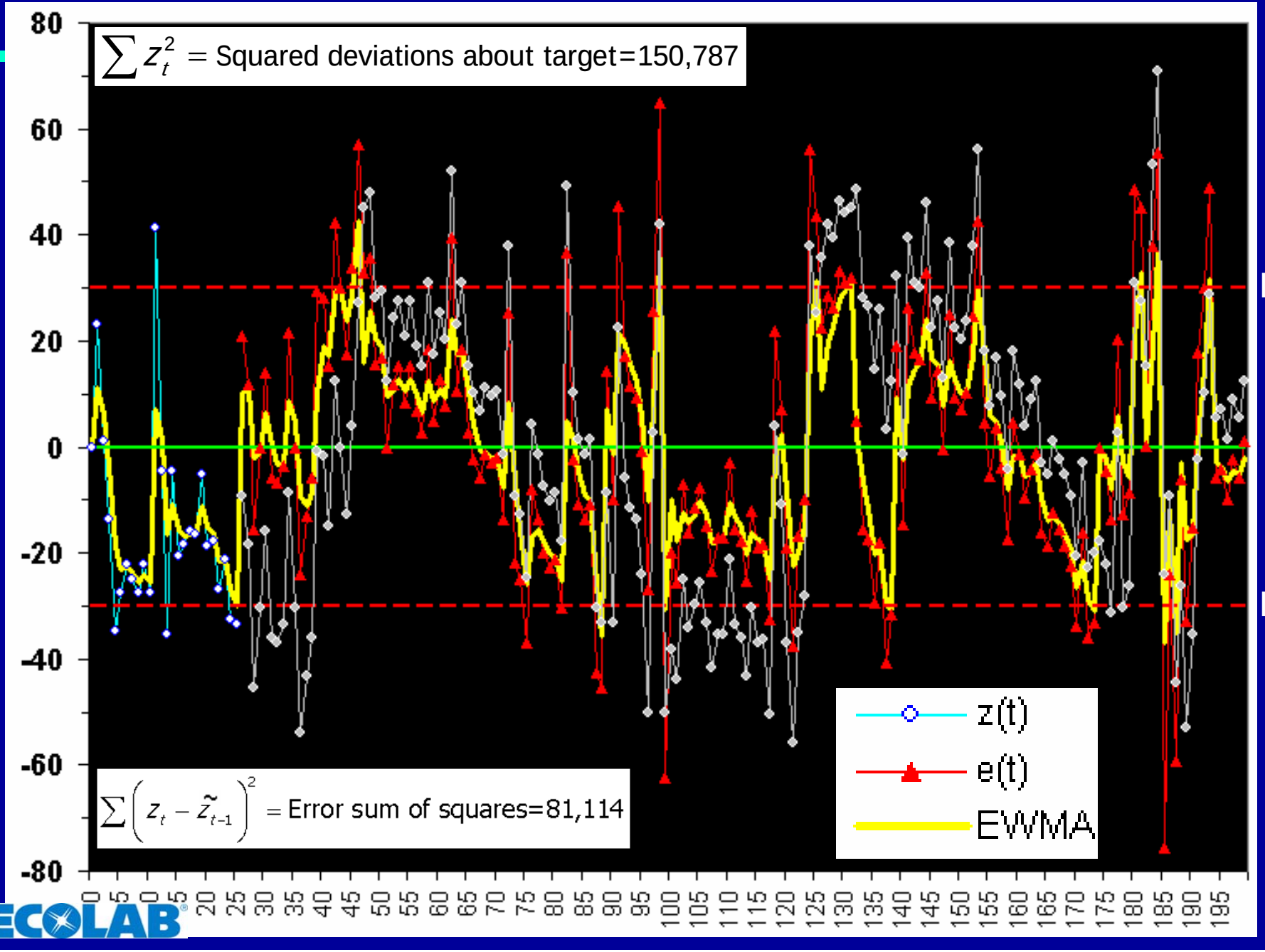
- Labor intensive, manual process
- Good performance handling autoregressive data
- Ignores wandering means *(stationarity assumed)*
- Not supported by available software
- Operator acceptance

Box-Jenkins Bounded Adjustment Chart

Tasks

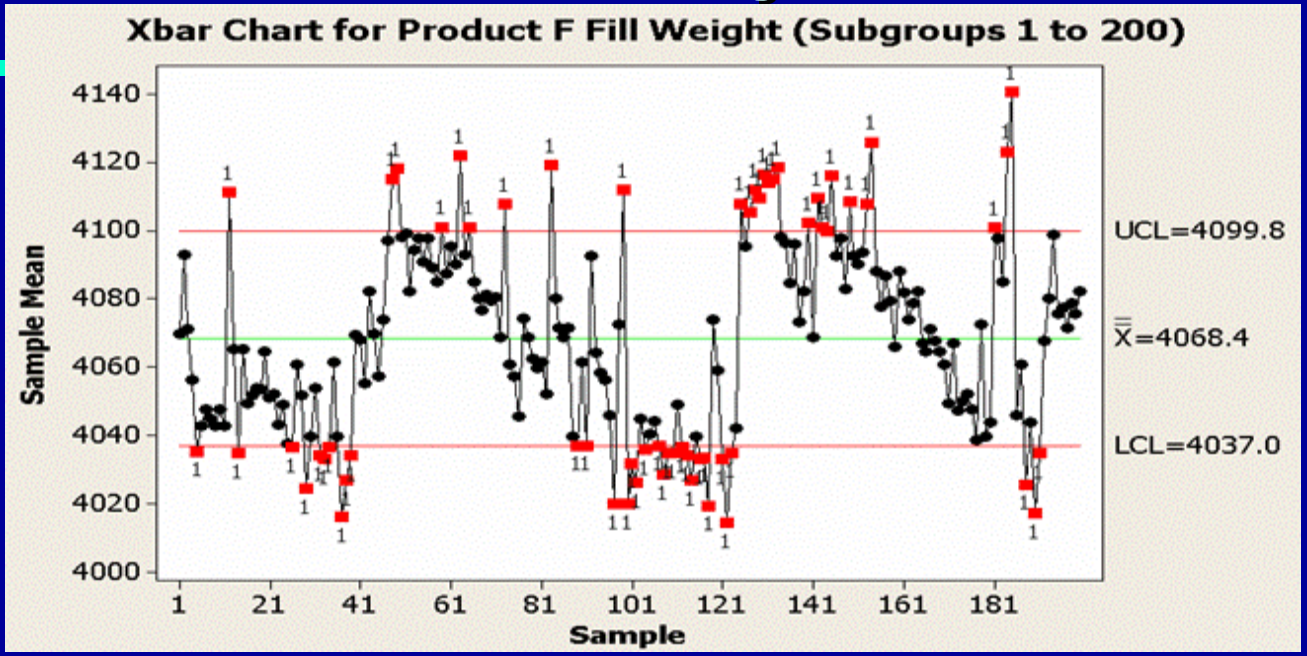
- Calculate offset from target z(t)
- Fit EWMA to z(t)
- Select adjustment limits (L)
- Select model parameters (g, G)
- Identify optimal smoothing constant (λ)
- Estimate forecast errors
- Calculate optimal process adjustment
- Adjust process
- Plot monitoring chart
- Reset model terms and continue

Box-Jenkins Bounded Adjustment Chart

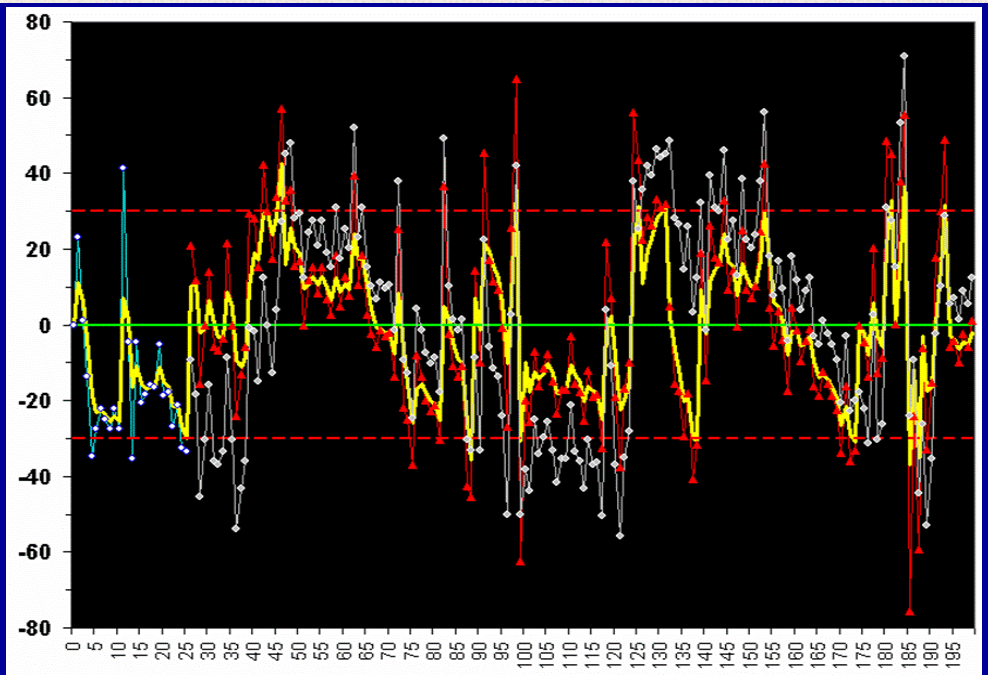


Box-Jenkins Bounded Adjustment Chart

Shewhart X-bar chart

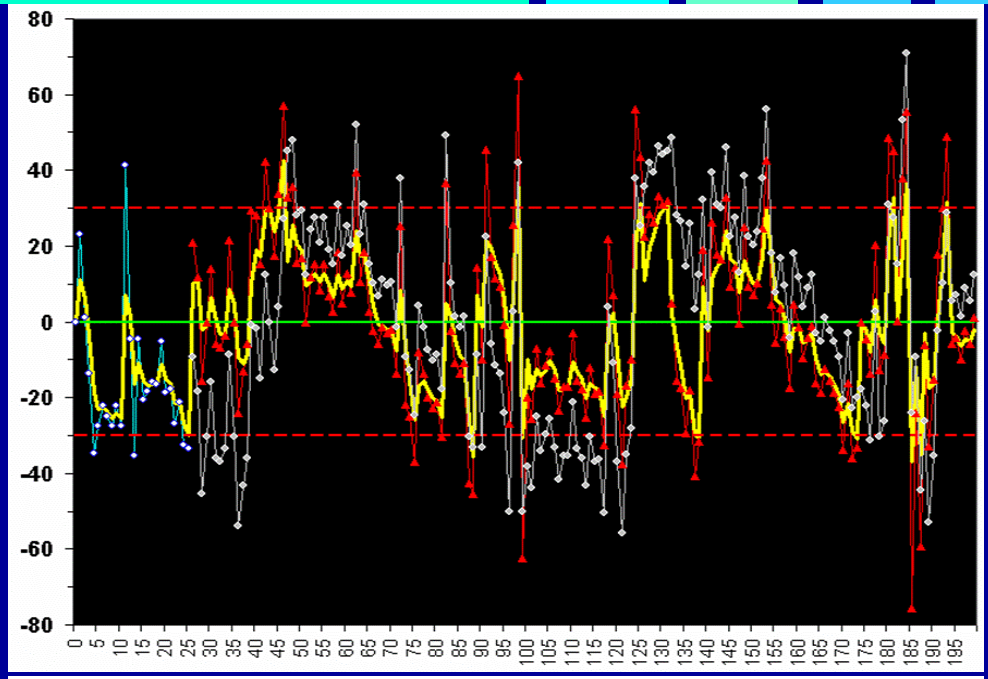


Bounded Adjustment Chart

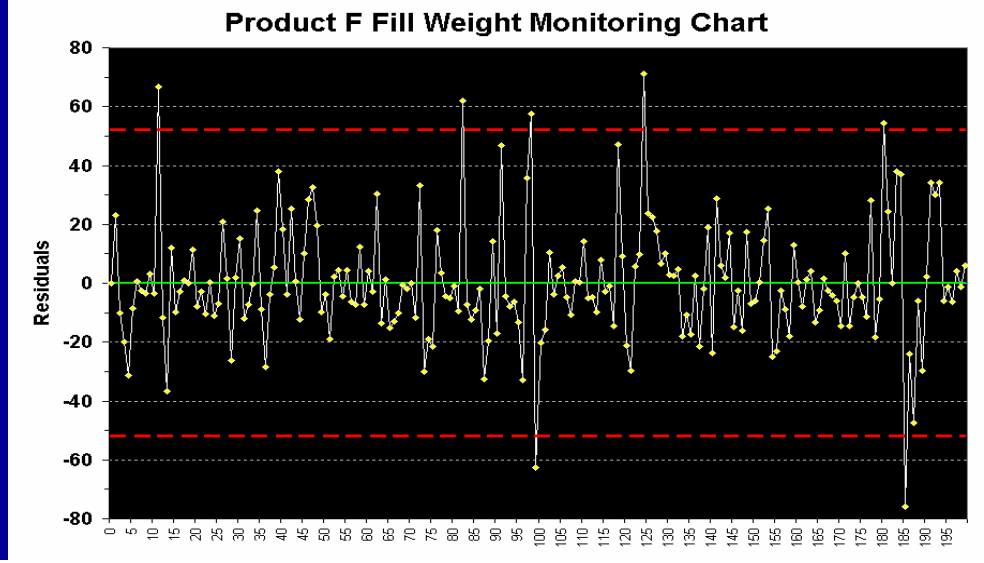


Monitoring Chart (Box & Paniagua-Quinones, 2007)

Bounded Adjustment Chart



Monitoring Chart



Error = e(t) - forecast

Box-Jenkins Bounded Adjustment Chart & Monitoring Chart

Observations

- Handles the wandering means cases
- Handles autocorrelation
- Provides information on how to adjust the process
- Operator training required for chart interpretation

Method Assessment

Evaluation Criteria	Weight	Methods: Shewhart charts	Decrease sampling frequency	Time series model--chart residuals	Modified control limits	Adjustment Charts
Deals with autocorrelation	0.20	2	1	5	3	4
Deals with wandering means	0.25	2	1	4	3	5
Limited statistician involvement	0.15	4	5	2	1	3
Low level of effort to implement	0.20	4	5	2	1	3
Supported by available software	0.05	5	4	3	1	2
Operators can understand charts	0.10	5	4	2	1	3
Feedback on how to adjust process	0.05	1	2	4	3	5
Weighted score		3.10	2.90	3.25	2.00	3.75
Rank		3	2	4	1	5

- Evaluation criteria weighted by project team
- Methods ranked from 1 to 5, with 5 reflecting a high rating on the criteria
- Weighted score = $\sum_{1}^{k}\left(w_i r_i\right)$ higher scores are better
- **Overall, the Box-Jenkins Manual Adjustment charts combined with monitoring charts best address our data**

Conclusions

- There is no easy solution to our problem
- May have to compromise technical performance to get method that is feasible and gives better performance than standard Shewhart charts
- Our results dependent on high incidence of wandering means

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