RESIDUALS AND THEIR ANALYSES FOR ACCELERATED LIFE TESTS WITH STEP OR VARYING STRESS

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PURPOSE: Define and plot suitable residuals to evaluate the model and data.

OVERVIEW

- STEP-STRESS TEST
- DATA
- CONSTANT-STRESS MODEL
- CUMULATIVE EXPOSURE/DAMAGE MODEL
- MAXIMUM LIKELIHOOD FIT
- RESIDUAL DEFINITION
- PLOTS OF RESIDUALS
- CONCLUDING REMARKS

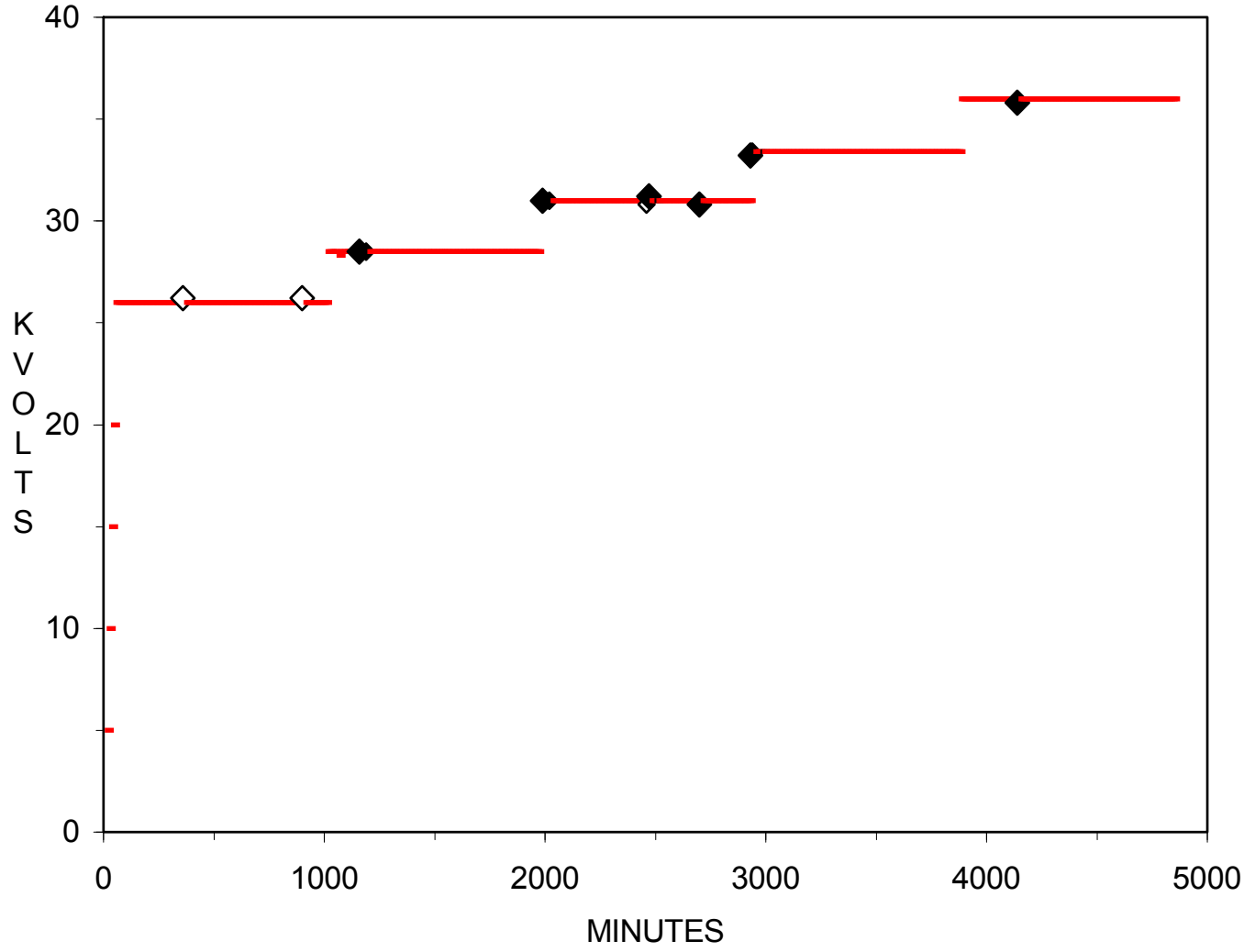
STEP-STRESS TEST

Step:	1	2	3	4	5	6	7	8	9	10	…
KVolts:	5.0	10.0	15.0	20.0	26.0	28.5	31.0	33.4	36.0	38.5	…
Hold: Min.	10	10	10	10	Δ	Δ	Δ	Δ	Δ	Δ	…

where Δ = 15, 60, 240, or 960 minutes.

Estimate the life dist. at 400 V/mil, the 1% point $t_1(400)$.

STEP-STRESS AND DATA ♦ failed, ◊ censored



CRYOGENIC CABLE INSULATION DATA

Spec-imen	Thick mils	Hold (min.)	Failure Step	Time (min.)	Residual
1	27	15	9	102	0.136
2	27	15	9	113	0.373
3	27	15	9	113	0.373
4	29.5	60	10	370+	0.706+
5	29.5	60	10	345+	0.355+
6	28	60	10	345	1.00
7	29	240	10	1333	3.44
8	29	240	10	1249	1.78
9	29	240	10	1333+	3.44+
10	29	240	9	1106.4	0.907
11	30	240	10	1250.8	0.922
12	29	240	9	1097.9	0.863
13	30	960	7	2460.9+	0.0947+
14	30	960	7	2460.9	0.0947
15	30	960	7	2703.4	0.127
16	30	960	8	2923.9	0.158
17	30	960	6	1160.0	0.00784
18	30	960	7	1962.9	0.0282
19	30	960	5	363.9+	0.00130+
20	30	960	5	898.4+	0.00344+
21	30	960	9	4142.1	1.41

CONSTANT-STRESS MODEL (POWER-WEIBULL)

$$F(t) = 1 - \exp\{-[t(V/V_0)^p]^\beta\}$$

where V is the voltage stress (voltage/thickness),

β Is the Weibull shape parameter (to be estimated),

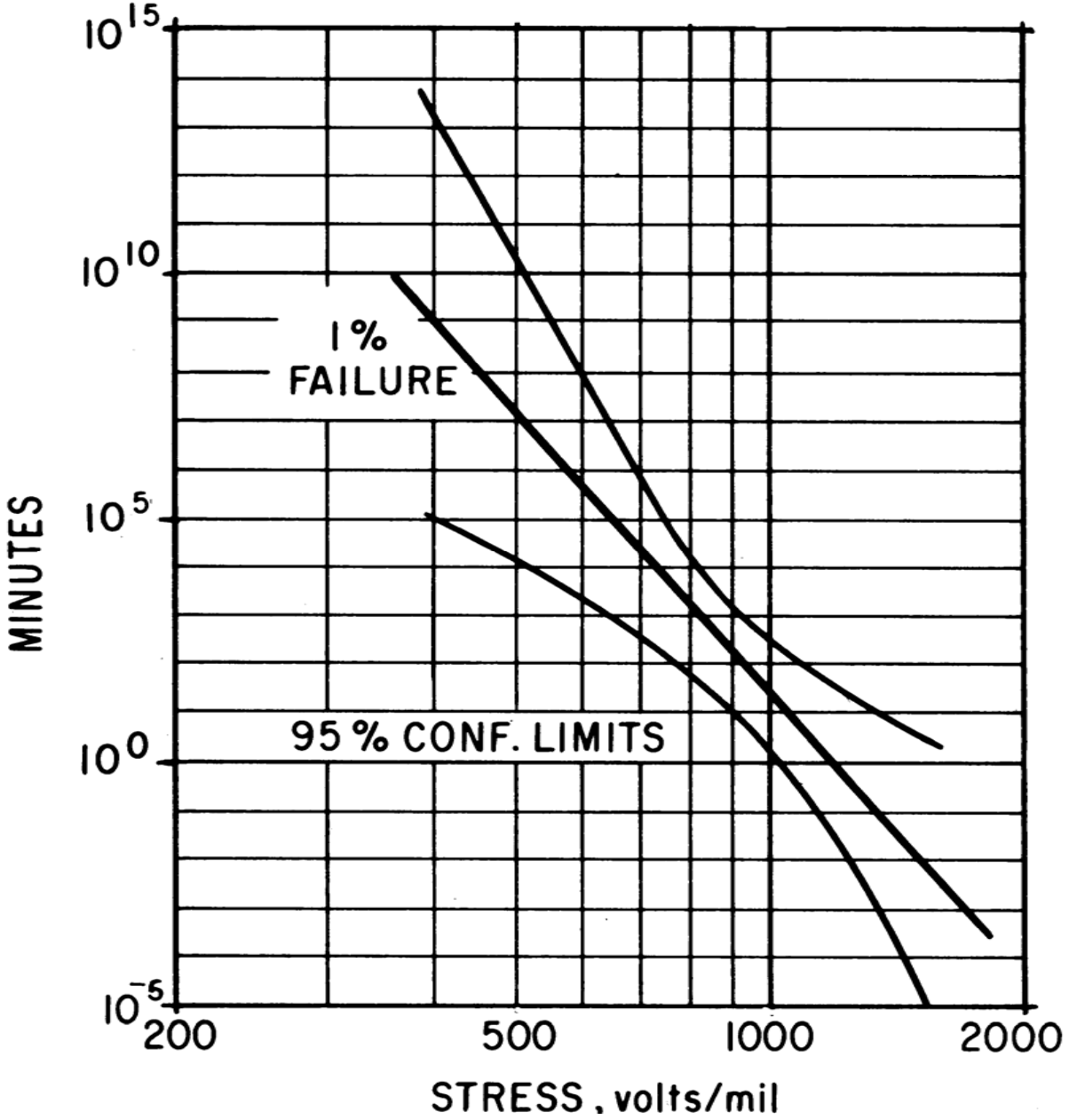
$\alpha(V) = (V_0/V)^p$ is the Weibull scale parameter,

and p and V_0 are parameters to be estimated.

The F-th percentile at constant stress V' is

$$t_F(V') = -(V_0/V')^p \{\ln[1-(F/100)]\}^{1/\beta}.$$

INVERSE POWER RELATIONSHIP



CUMULATIVE DAMAGE/EXPOSURE MODEL

Under a time-varying stress $V(t)$, the population distribution of time t to failure is

$$F[t;V(t)] = 1 - \exp\{-[\varepsilon(t)]^{\beta}\}$$

where the *cumulative exposure* is

$$\varepsilon(t) \equiv \int_0^t dt \,/\, \alpha[V(t);V_0,p].$$

$\varepsilon(t)$ has a Weibull distr. with shape β and $\alpha = 1$.

For steps (V_i,τ_i), $\tau_{I-1} < t \le \tau_I$ and where $\alpha_i = (V_0/V_i)^p$,

$$\varepsilon(t) = [(\tau_1 - 0)/\alpha_1] + [(\tau_2 - \tau_1)/\alpha_2] + \cdots + [(t - \tau_{i-1})/\alpha_I].$$

MAXIMUM LIKELIHOOD FIT

Estimates and 95% limits (normal approx. and LR)

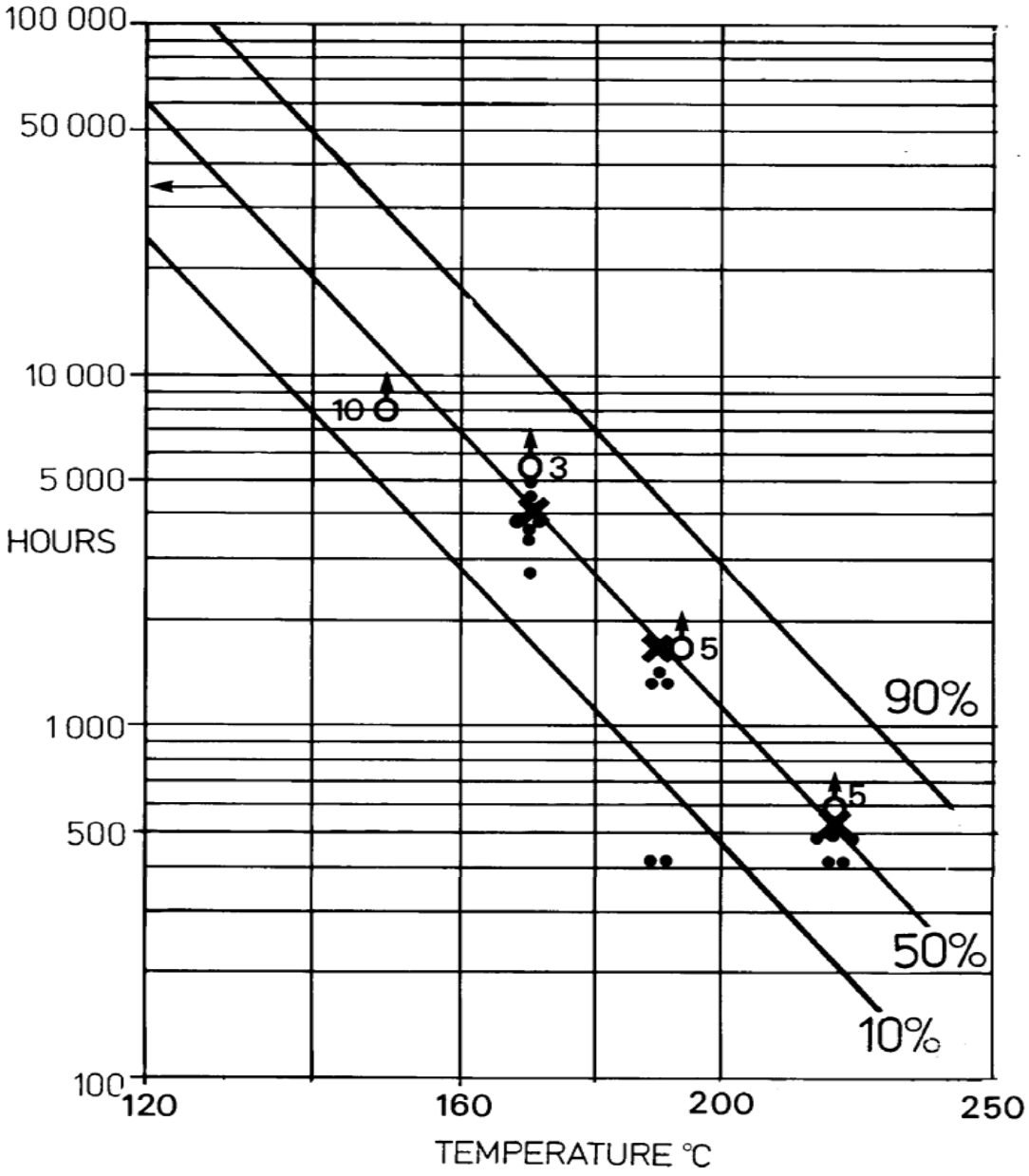
$\beta^* = 0.756$ (0.18, 1.33) (0.27, 1.39)

$p^* = 19.9$ (6.2, 33.7) (11.0, 47.2)

$V_0^* = 1616$ (1291, 1941)

$t_1^*(400) = 2.8\times10^9$ (2.65×10^4, 2.98×10^{14})

CONSTANT-STRESS RESIDUALS



RESIDUAL DEFINITION FOR VARYING STRESS

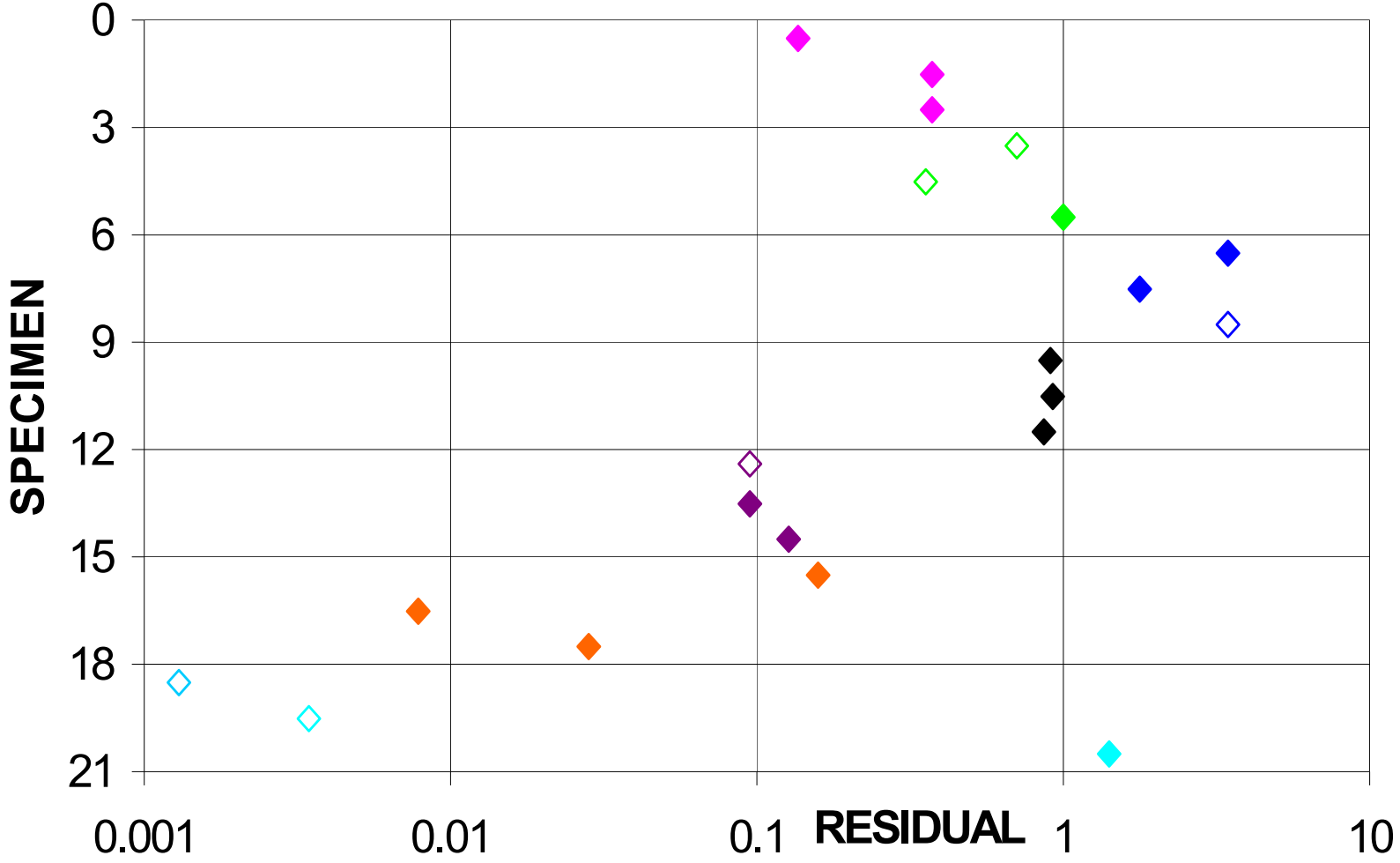
For a failure or censoring time t_i, the corresponding observed or censored *residual* is the cumulative exposure

$$e_i \equiv \varepsilon^*(t_i) \equiv \int_0^{t_i} \mathrm{d}t \,/\, \alpha[V(t); V_0^*, p^*] \,.$$

When the constant-stress and cumulative exposure models are correct, these residuals have a Weibull distribution with shape β and $\alpha = 1$.

They can be plotted to assess the validity of the model and data.

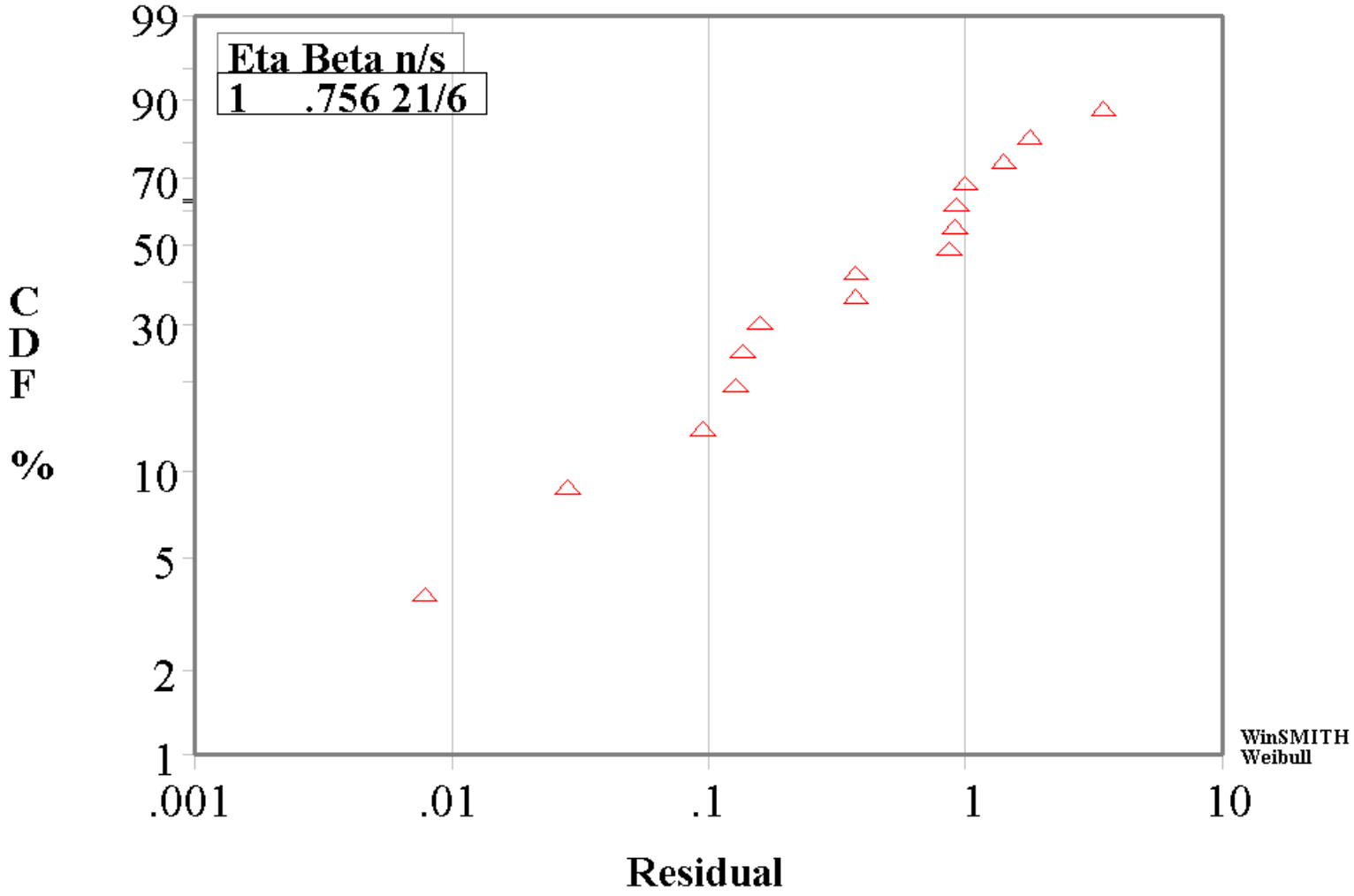
RESIDUALS VERSUS SPECIMEN NUMBER



♦ failed ◊ censored

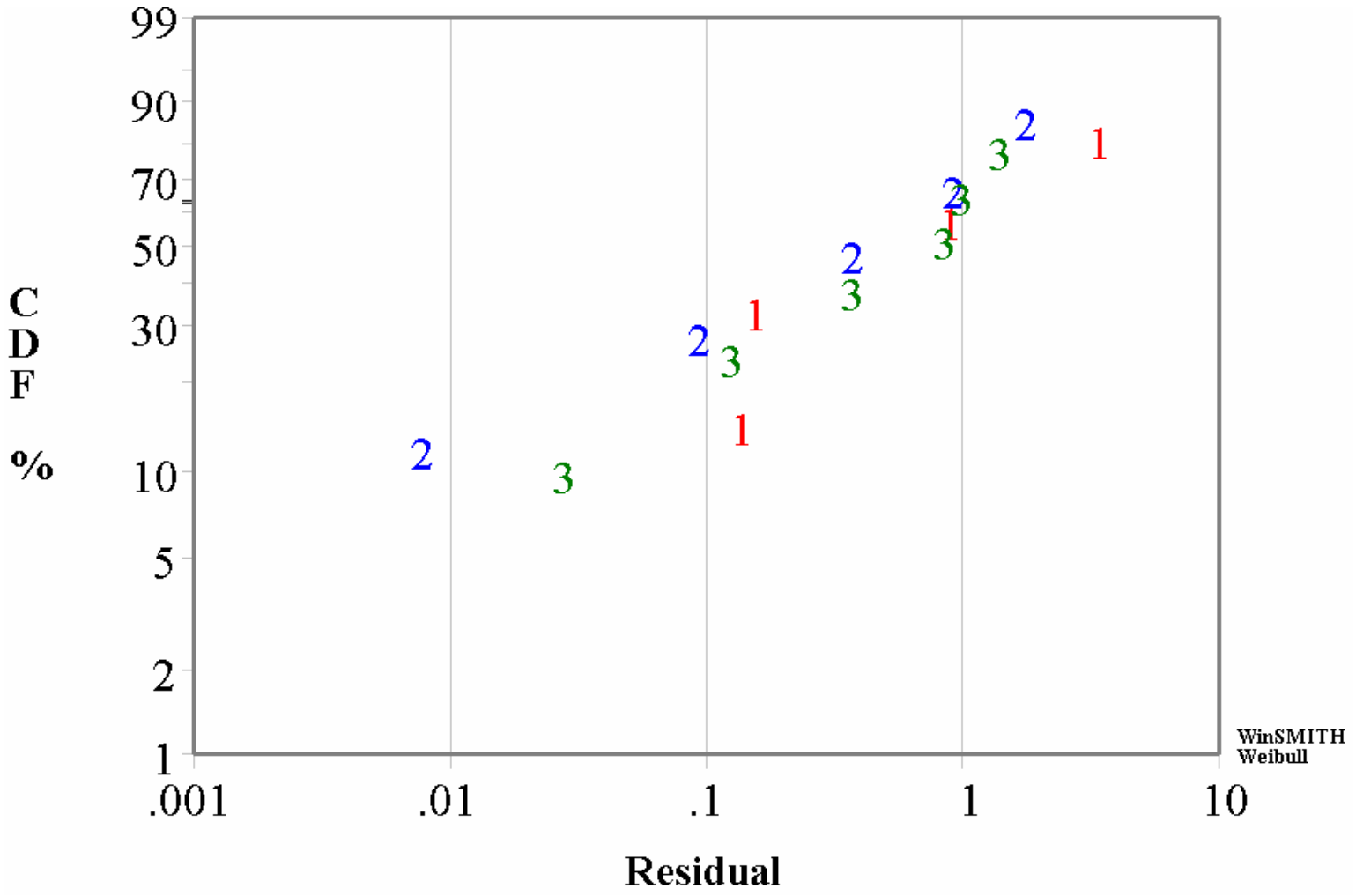
This plot suggests that groups of three specimens differ.

WEIBULL PLOT OF POOLED RESIDUALS



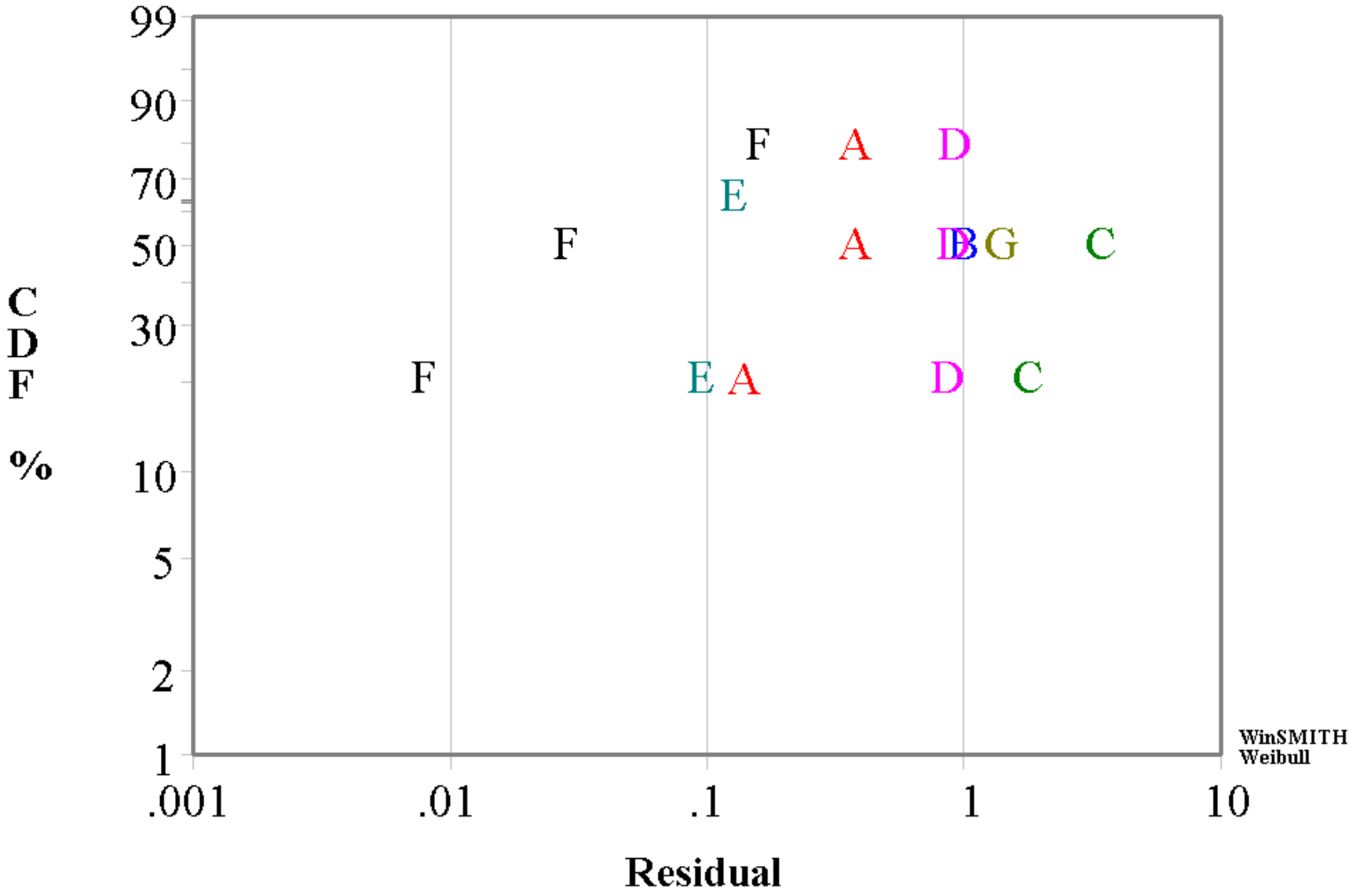
Reasonably straight plot supports the Weibull dist.

WEIBULL PLOT FOR TEST POSITIONS 1, 2, 3



Reasonably straight plots support a Weibull distribution.
Superimposed plots indicate no position effect.

WEIBULL PLOT FOR SEVEN GROUPS A, B, ···, G



Clear group effect. Within group $\beta^* \cong 2.5$, higher. Cause?

CONCLUDING REMARKS

- The plots are informative.

- They should be supplemented by analytic methods.

- The residuals and plots extend to

- other distributions (e.g., lognormal) and other relationships where the scale parameter is a function of stress and all other parameters are not,

- K stresses $V_1(t)$, ···, $V_K(t)$,

- field data where each unit has a different stress history.

REFERENCES

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–– (1990), *Accelerated Testing: Statistical Model, Test Plans, and Data Analyses*, Wiley, New York.

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