An Introduction to a New Methodology of Random Sequential Probability Ratio Test with Data Analysis

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Abstract

Wald's (1947, *Sequential Analysis*, New York: Wiley) *sequential probability ratio test* (SPRT) remains relevant in addressing a wide range of practical problems. Clinical trials owe a great deal of debt to this methodology. There has been a recent surge of applications in many areas including sonar detection, tracking of signals, detection of signal changes, computer simulations, agriculture, pest management, educational testing, economics, and finance.

Obviously there are circumstances where sampling one observation at-a-time may not be practical. In contexts of continuously monitoring, for example, inventory, queues or quality assurance, the data may appear *sequentially* in groups where the group sizes may be ideally treated as random variables themselves. For example, one may sequentially record the number of stopped cars (M_i) and the number of cars ($\Sigma_{j=1}^{M_i} X_{ij}$) without "working brake lights" when a traffic signal changes from green to red, $i = 1, 2, \ldots$. This can be easily accomplished since every "working brake-light" must glow bright red when a driver applies brakes. One notes that (i) it may be reasonable to model M_i's random, but (ii) it would appear impossible to record data sequentially one-by-one on the status of brake lights (that is, $X_{ij} = 0$ or 1) individually for car #1, and then for car #2, and so on!

In order to address these situations, we start with an analog of the concept of a best "fixed-sample-size" test based on data $\{M_i, X_{i1}, ..., X_{iM_i}, i = 1, ..., k\}$. Then, a *random sequential probability ratio test* (RSPRT) is developed for deciding between a simple null and a simple alternative hypotheses with preassigned Type I and II errors α, β. The RSPRT and the best "fixed-sample-size" test with $k \equiv k_{\min}$ associated with same errors α, β are compared. An illustration of RSPRT will include a binomial distribution with substantive computer simulations. A real application will be highlighted.

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