The Combined Chart Controversy

Revisiting insights by Zachary Stoumbos

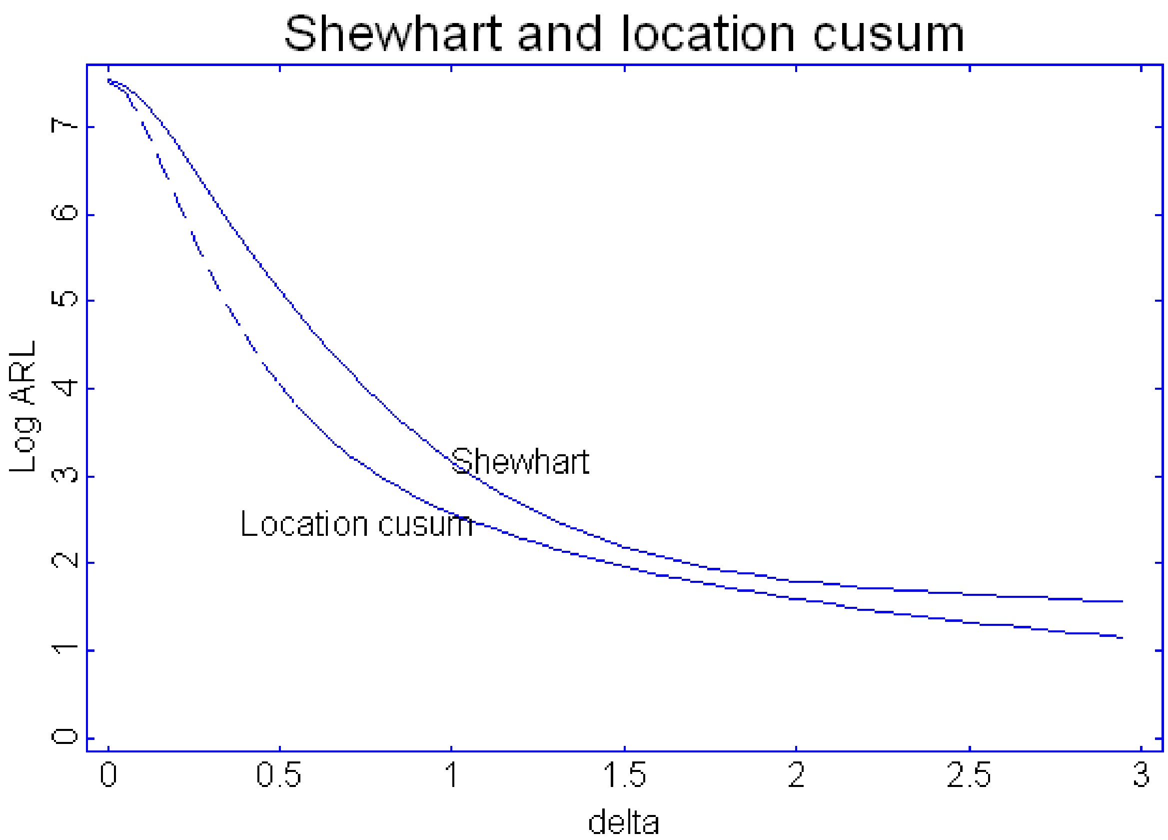
Douglas M Hawkins

Conventional Thinking

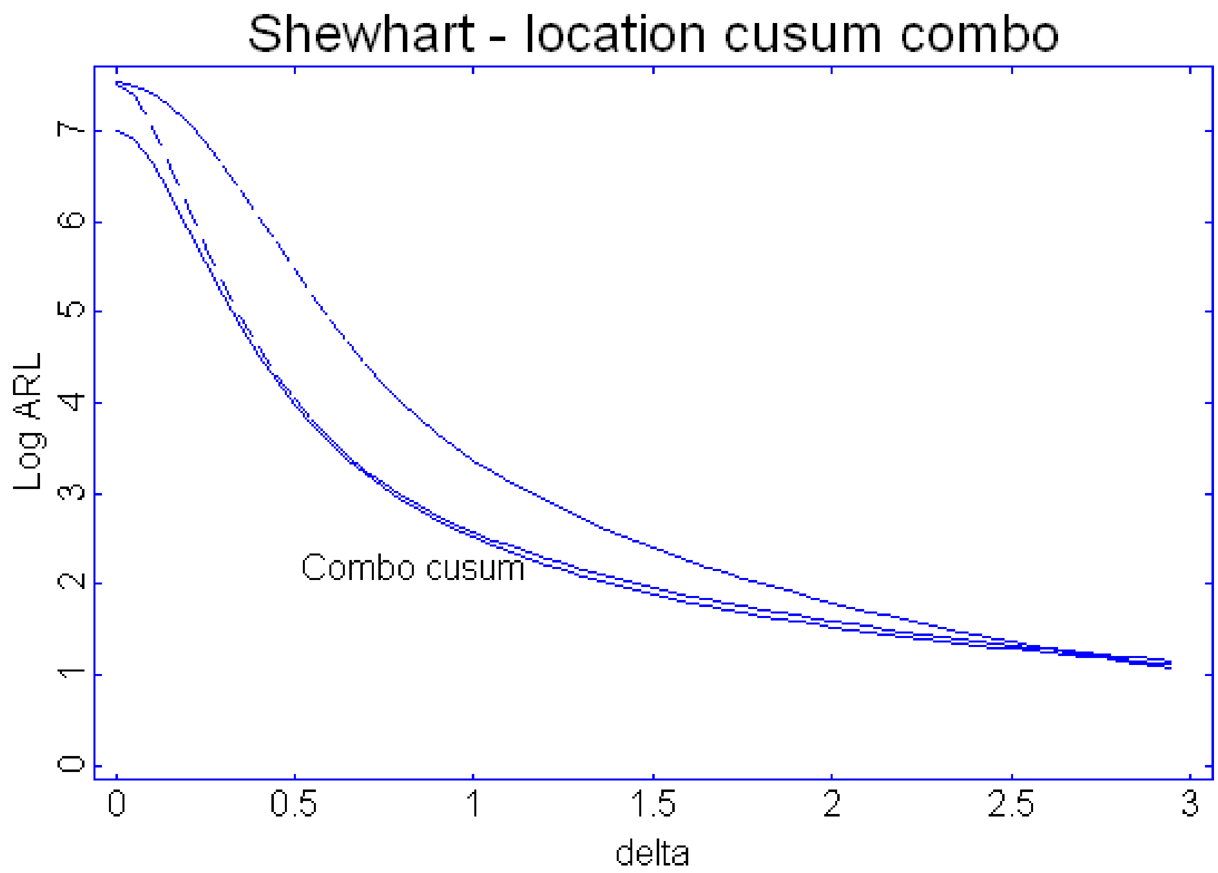
- Shewhart charts are fine for large shifts, not for small.
- Cusums and EWMAs are great for small shifts.
- So to cover all bases, combine them. You then get both small and large shifts

Illustration

- Monitor 8-hour shifts with one reading every 96 minutes.
- Use Shewhart with n=5, getting one point per shift.
- Use cusum with reference value 0.5, decision interval 6.369, getting matching IC ARL of 1850 readings.



Shewhart and location cusum
Log ARL
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
3
delta
Shewhart
Location cusum


Shewhart - location cusum combo
Log ARL
delta
Combo cusum
0
1
2
3
4
5
6
7
0.5
1.5
2.5

Surprise! (Maybe)

- The cusum beats the Shewhart at both small and large shifts. Reason is cusum can react every 96 minutes (and does so with large shifts); Shewhart has to close out rational group

Reynolds and Stoumbos 2005

- "Should exponentially weighted moving average and cumulative sum charts be used with Shewhart limits?" *Technometrics*, 47, 409-424

  replaced Shewhart with scale chart. *Note* that they used a Shewhart I chart, not a common choice.

Does this work?

- Notation. IC, data are N(μ,σ^2). Location, scale cusums are

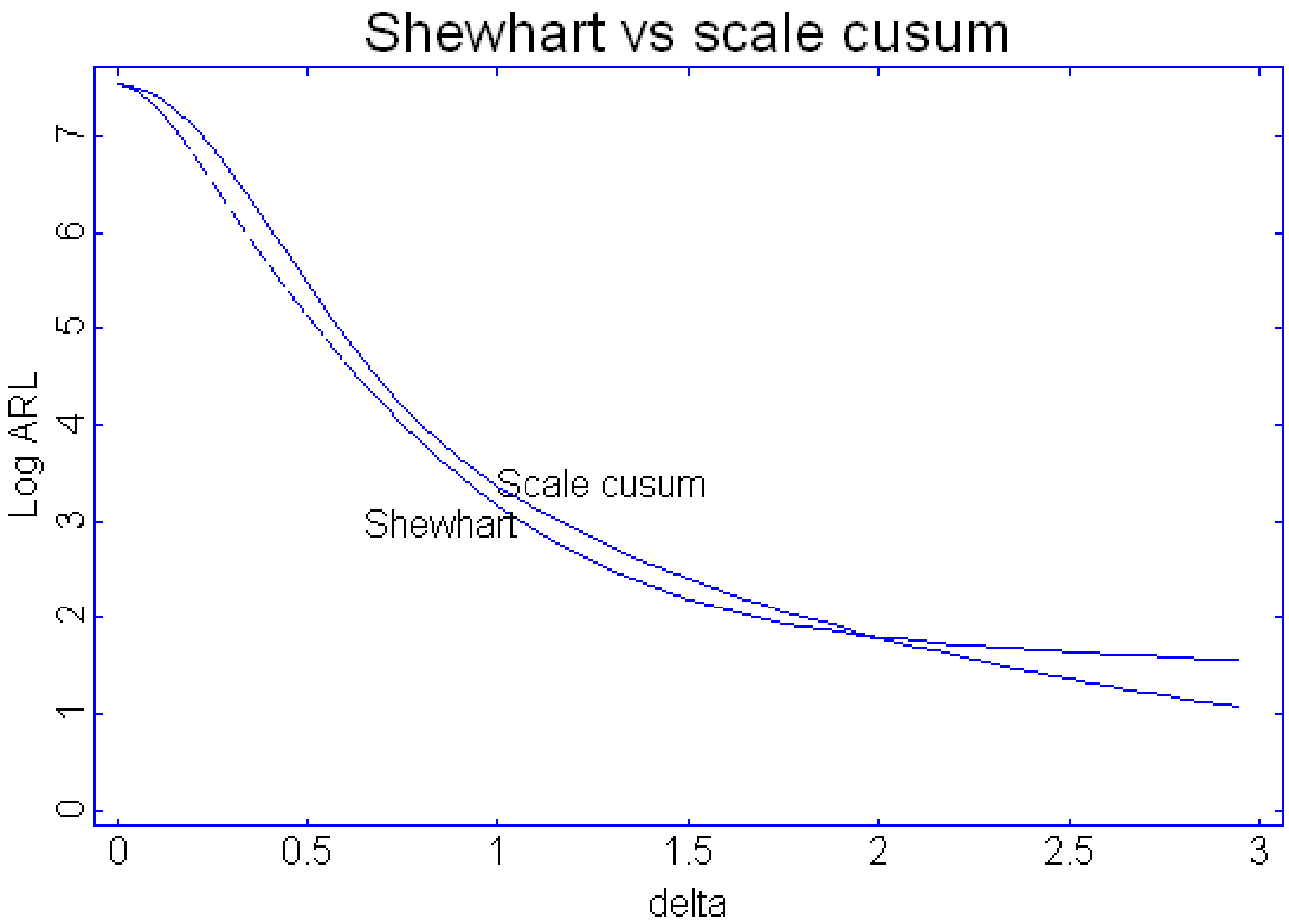
$$L_n^+ = \max(0, L_{n-1}^+ + X_n - \mu - K_L)$$

$$L_n^- = \min(0, L_{n-1}^- + X_n - \mu + K_L)$$

$$S_n = \max(0, S_{n-1} + (X_n - \mu)^2 - K_\sigma)$$

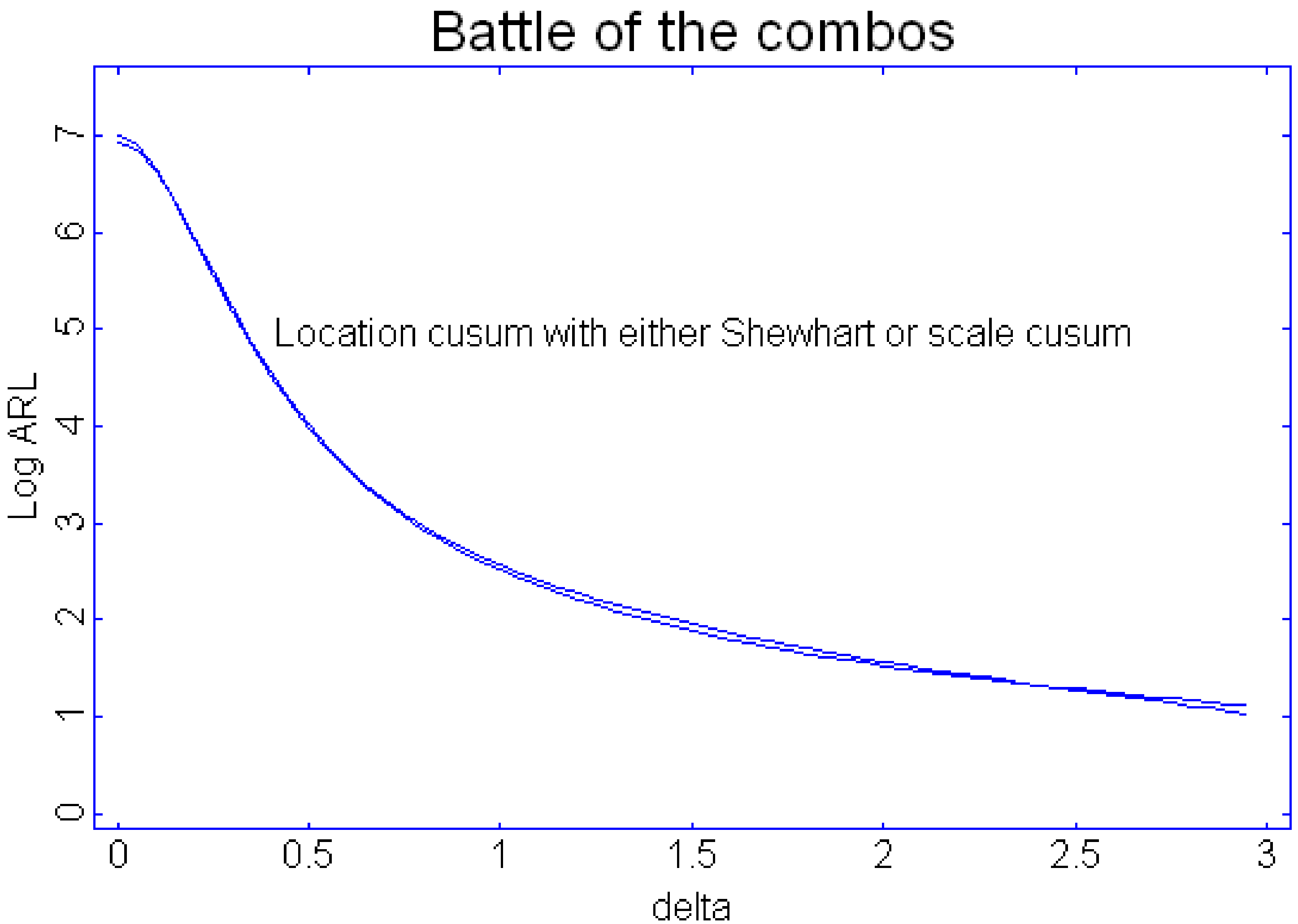
Tuning

- Tune location cusums for one standard deviation shift, so $K_L = 0.5\sigma$
- Tune scale cusum for a doubling of variance, so $K_\sigma = 1.386\ \sigma^2$
- Set decision intervals to match Shewhart IC ARL, so 1850 for the scale cusum.


Shewhart vs scale cusum
Log ARL
delta
Scale cusum
Shewhart
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
3

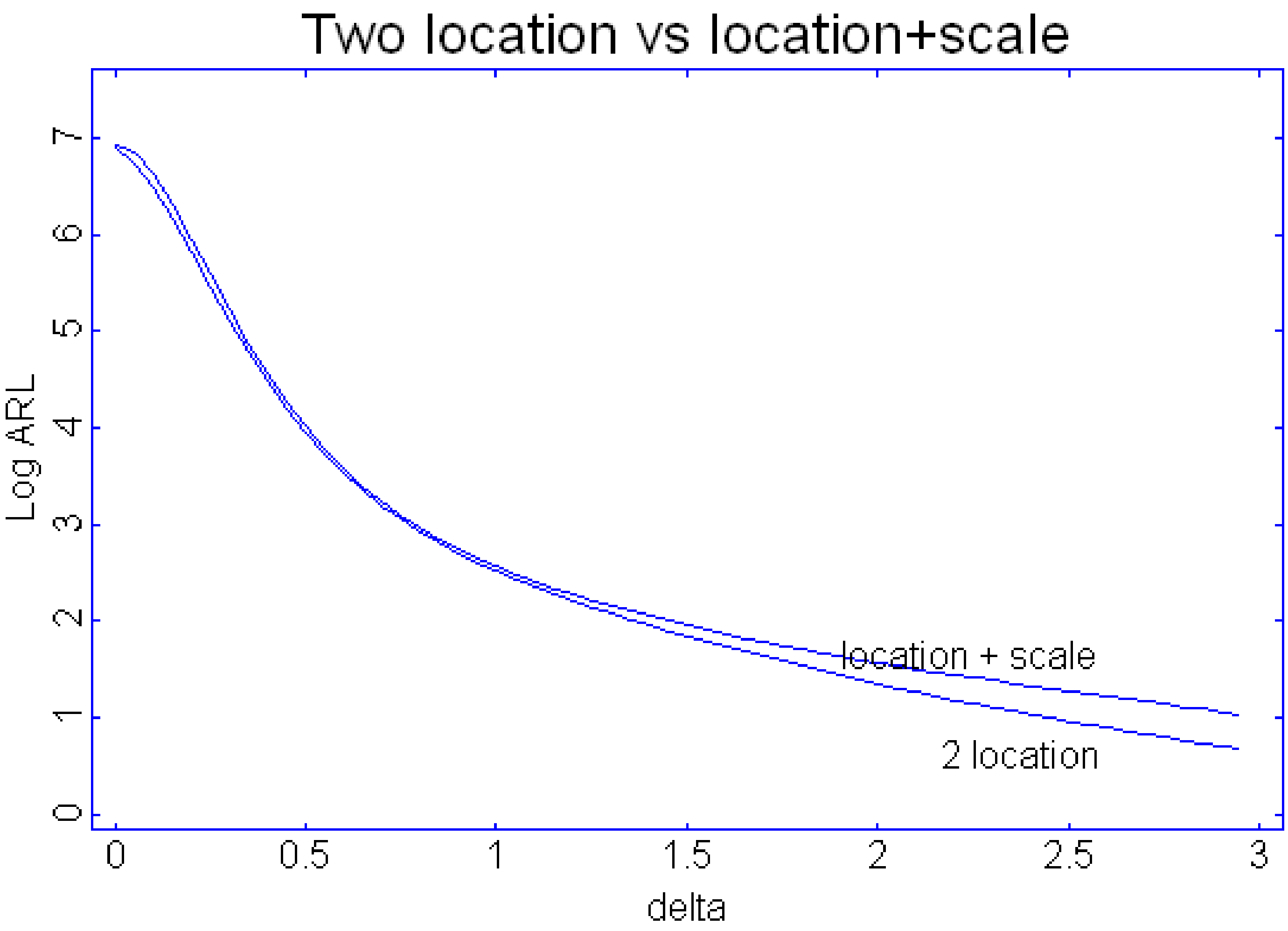
Comparison

- Shewhart Xbar and scale cusum match reasonably over most of range, but scale cusum much better for large shifts.
- Supports Reynolds-Stoumbos suggestion of using scale cusum for larger shifts.


Battle of the combos
Location cusum with either Shewhart or scale cusum
Log ARL
delta
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
3

Other possibilities

- 1982 proposal "snub-nosed V-mask" runs two location cusums – one tuned for small shifts, one tuned for large. Signals if either exceeds control limit.
- Try adding second location cusum K_L = 1.5σ, instead of Shewhart


Two location vs location+scale
Log ARL
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
3
delta
location + scale
2 location

Comparison

- Two-location-cusum approach beats location+scale. But as you want scale cusum anyway to monitor variance, not clear that this approach pays its way.

Shewhart I again

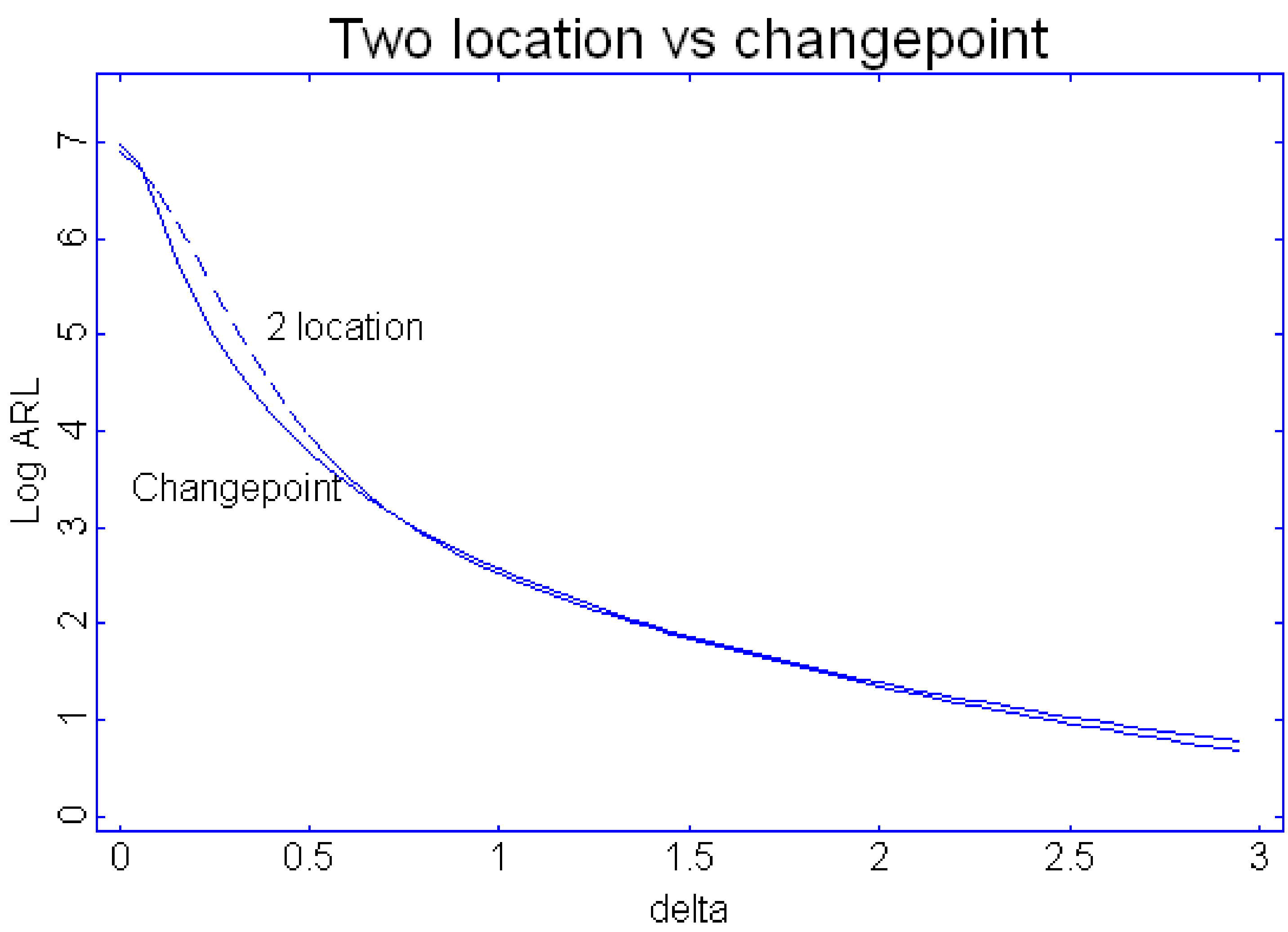
- Shewhart I chart (used in Reynolds and Stoumbos) is itself a cusum with $K=3\sigma$ and decision interval=0_+

  Thus is optimal for a shift of 6σ
- As shifts this big are obvious to naked eye, not surprising that Shewhart I does not help in finding persistent shifts.

Changepoint formulation

- Model – X_n is N(μ_0 , σ^2) $n \leq \tau$,
  N(μ_1 , σ^2) $n > \tau$.
  μ_0, σ known, τ, μ_1 unknown.
- For each n, calculate maximum Z statistic; signal if this exceeds control limit

$$Z_{\max} = \max_j |[\sum_{k=j+1}^{n} (X_k - \mu_0)/\sigma]/\sqrt{n-j}|$$



Two location vs changepoint
2 location
Changepoint
Log ARL
delta
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
3

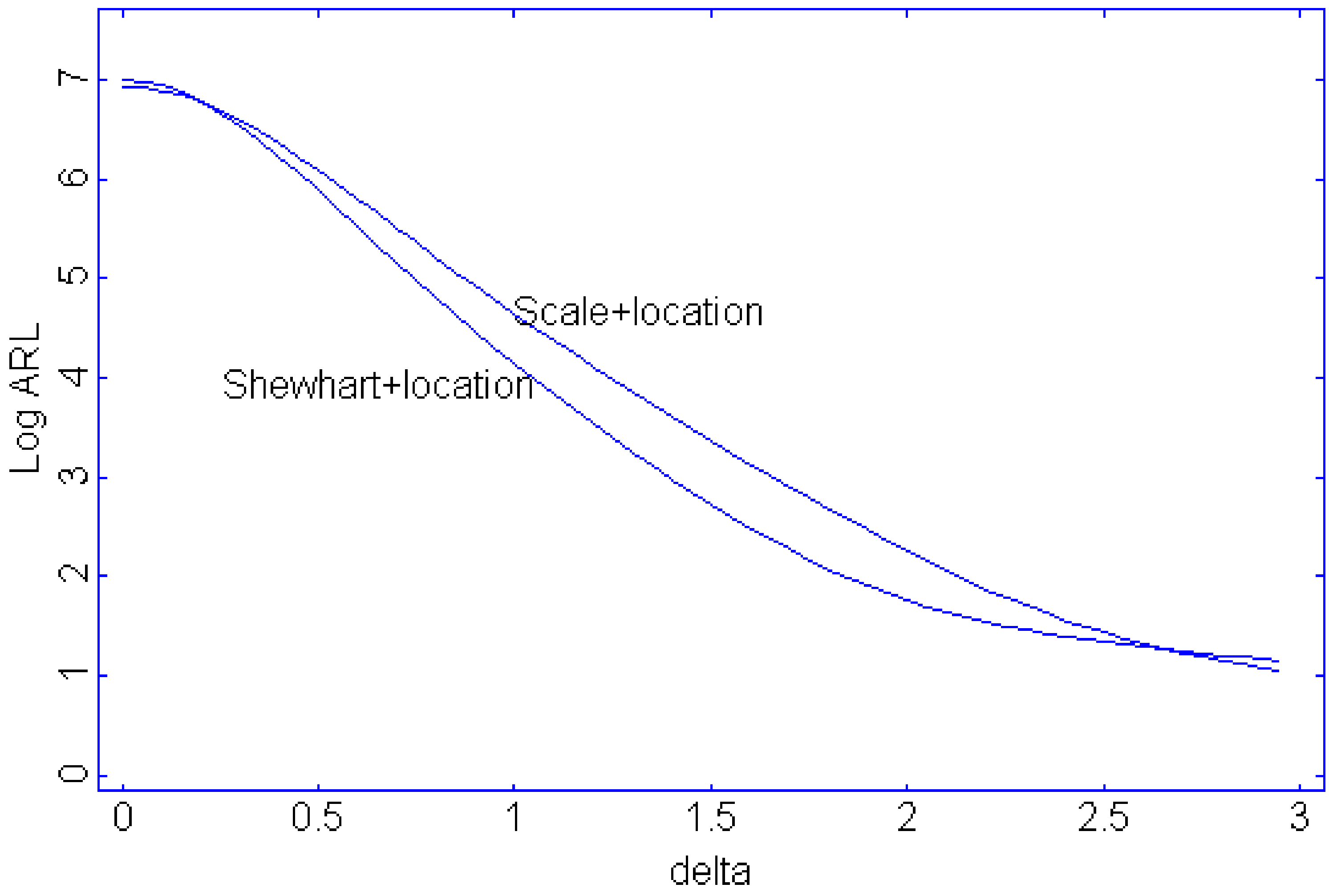
Comparison

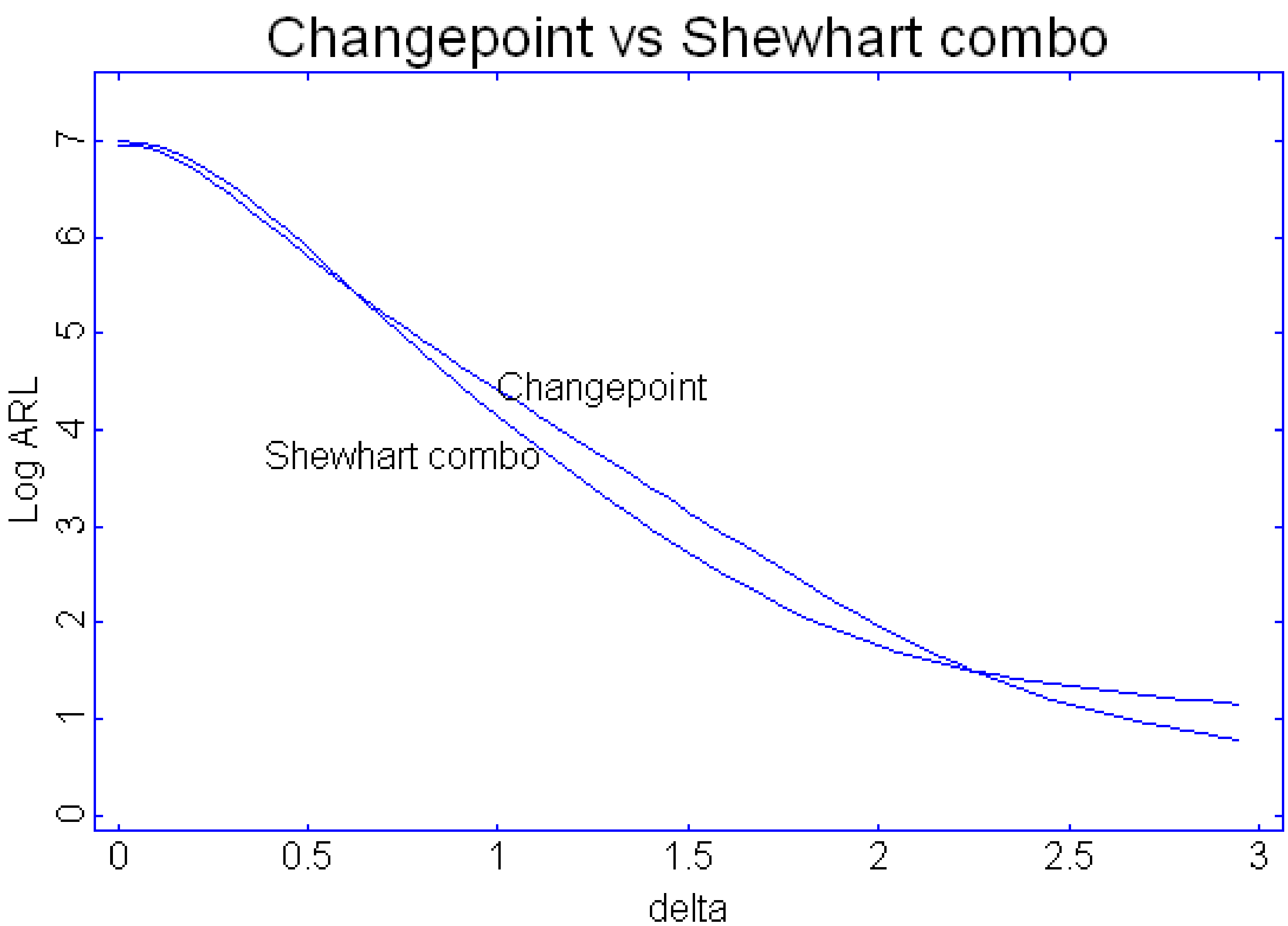
- Changepoint ARL uniformly below two-location-cusum pair.
- Performance enhancement appreciable for moderate shifts.

RIP Shewhart? Not so fast

- Is Shewhart dead? By no means.
- Another scenario – we get 5 observations per day, as before.
- But Monday values are displaced relative to Tuesday – Friday.

Combo charts -- Shewhart vs scale




Changepoint vs Shewhart combo
Log ARL
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
3
delta
Changepoint
Shewhart combo

Comparison

- Augmenting location cusum with Shewhart handily beats using scale cusum.
- Changepoint is more even match. Shewhart wins between 0.6 and 2.2 σ, with changepoint winning beyond.

Final overview

- Shewhart remains attractive for *transient* special causes.
- Location/scale cusum pair remains attractive for *persistent* special causes.
- Changepoint formulation shows promise in both settings.