Planning Fatigue Tests for Polymer Composites

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Outline

- Introduction
- Test Plan Setting & Model
- Optimum and Compromise Test Plans
- Test Plan Assessment
- Conclusion & Areas for Future Research

Motivation

- Cyclic constant amplitude testing is the most common procedure used to assess tensile fatigue behavior in polymer composites.
- The current standards for constant amplitude testing use balanced designs for determining fatigue properties.
- A common property of interest is the lifetime by which a particular proportion of product will fail at a specified use condition.
- Optimum plans based on a model from the fatigue literature could provide more accurate estimates of this property.

Notation

- Let s be the number of stress levels with k_i samples allocated to stress levels $i = 1, 2, \ldots, s$. The total sample size is $k = \sum_{i=1}^{s} k_i$.
- The ultimate tensile strength of the material is denoted by σ_u. The maximum tensile stress for level i is denoted by σ_i.
- Let T_m be the censoring time for a single stress level.
- The design points for the experiment are represented by $q_i = \sigma_i / \sigma_u$.
- Let $\pi_i = k_i / k$ be the proportion of the total sample size allocated to level i.
- The decision variables for the test plan are $\boldsymbol{\eta} = (q_1, \cdots, q_s, \pi_1, \cdots, \pi_s)'$.

Test Plan Model

- The data are denoted by $\{T_{ij}, d_{ij}\}, j = 1, \ldots, k_i, i = 1, \ldots, s$, where d_{ij} is the censoring indicator.

- A log-location-scale family is used to describe the lifetime distribution (e.g. lognormal and Weibull distributions)

$$f(t_{ij}) = \frac{1}{\nu t}\phi\left(\frac{\log(t_{ij}) - \mu(\sigma_i)}{\nu}\right)$$
$$F(t) = \Phi\left(\frac{\log(t_{ij}) - \mu(\sigma_i)}{\nu}\right)$$

- The location function is based on the model proposed by Epaarachchi and Clausen (2003)

$$\mu(\sigma_i) = \frac{1}{B}\log\left\{\left(\frac{B}{A}\right) f^B \left(\frac{\sigma_u}{\sigma_i} - 1\right)\left(\frac{\sigma_u}{\sigma_i}\right)^{\gamma(\alpha)-1}[1 - \psi(R)]^{-\gamma(\alpha)} + 1\right\},$$

- The unknown parameters are $\boldsymbol{\theta} = (A, B, \nu)'$

Test Plan Model (cont'd)

- The unknown parameter A represents "external" effects on the fatigue process.
- The unknown parameter B represents material-specific properties.
- The parameter ψ is a function of the ratio $R = \sigma_{\min}/\sigma_{\max}$,

$$\psi(R) = \begin{cases} R & -\infty < R < 1 \\ \frac{1}{R} & 1 < R < \infty \end{cases}.$$

- The parameter γ is a function of α, the smallest angle between the testing direction and the fiber direction,

$$\gamma(\alpha) = 1.6 - \psi|\sin(\alpha)|.$$

- Parameter f is the frequency of the cyclic testing procedure.

Optimality Criterion

- We seek the following for the optimal design:

$$
\begin{aligned}
& \min_{\boldsymbol{\eta}} \operatorname{AVar}\left\{\log\left[\widehat{T}_p(\sigma_{\text{use}})\right]\right\} \\
\text{subject to} \quad & q_i \in [q_L, q_U], i = 1, \ldots, s, \\
& \pi_i \in [0, 1], i = 1, \ldots, s, \\
& \pi_i k \in \mathbb{N}, \\
& \text{and } \sum_i \pi_i = 1.
\end{aligned}
$$

where σ_{use} is the use stress level and $\log\left[\widehat{T}_p(\sigma_{\text{use}})\right]$ is the maximum likelihood estimator (MLE) of the p-th quantile of the log-lifetime distribution at σ_{use}.

Design Types

- Three types of designs will be considered:

Traditional - Plan used to obtain motivating data. Consists of three design points ($q_1 = 0.35, q_2 = 0.50, q3 = 0.75$) and equal allocation to each level ($\pi_i = 1/3, i = 1, 2, 3$).

Optimal - Satisfies the optimality criterion with no further constraints.

Compromise - Satisfies the optimality criterion subject to constraints on the minimum number of design points and the minimum allocation to each level.

Test Plan Specifications

- Due to proprietary sensitivity reasons, the motivating data has been rescaled and relabeled.
- The planning values for the model are the MLEs of the parameters based on the rescaled data.
- The planning range for the q_i is the same as in the traditional plan, $[0.35, 0.75]$.
- A minimum distance (0.10 or 0.05) between any two design points and a minimum sample size ($\pi_{\text{min}} = 0.10$) were enforced for the compromise plans.
- The specific optimization criterion is

$$\min_{\boldsymbol{\eta}} \mathrm{AVar}\left\{\log\left[\widehat{T}_{0.05}(0.10\sigma_u)\right]\right\}.$$

Examples of Optimum Plans

k	AVar	Stress Level		Allocation		Sample Size	
		q_1	q_2	π_1	π_2	k_1	k_2
9	0.232	0.35	0.75	0.667	0.333	6	3
10	0.211	0.35	0.75	0.700	0.300	7	3
11	0.190	0.35	0.75	0.636	0.364	7	4
12	0.174	0.35	0.75	0.667	0.333	8	4
13	0.161	0.35	0.75	0.615	0.385	8	5
14	0.149	0.35	0.75	0.643	0.357	9	5
15	0.139	0.35	0.75	0.667	0.333	10	5
16	0.131	0.35	0.75	0.625	0.375	10	6
17	0.123	0.35	0.75	0.647	0.353	11	6
18	0.116	0.35	0.75	0.667	0.333	12	6
19	0.110	0.35	0.75	0.632	0.368	12	7
20	0.104	0.35	0.75	0.650	0.350	13	7
21	0.099	0.35	0.75	0.667	0.333	14	7
22	0.095	0.35	0.75	0.636	0.364	14	8
23	0.091	0.35	0.75	0.652	0.348	15	8
24	0.087	0.35	0.75	0.667	0.333	16	8
25	0.084	0.35	0.75	0.640	0.360	16	9
26	0.080	0.35	0.75	0.654	0.346	17	9
27	0.077	0.35	0.75	0.667	0.333	18	9
28	0.075	0.35	0.75	0.643	0.357	18	10
29	0.072	0.35	0.75	0.655	0.345	19	10
30	0.070	0.35	0.75	0.667	0.333	20	10

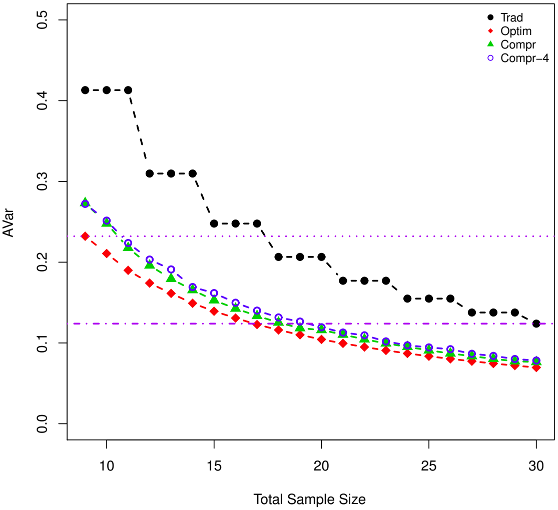
Examples of Compromise Plans

k	AVar	Stress Level			Allocation			Sample Size		
		q_1	q_2	q_3	π_1	π_2	π_3	k_1	k_2	k_3
9	0.273	0.35	0.65	0.75	0.556	0.222	0.222	5	2	2
10	0.248	0.35	0.65	0.75	0.500	0.200	0.300	5	2	3
11	0.218	0.35	0.65	0.75	0.545	0.182	0.273	6	2	3
12	0.196	0.35	0.65	0.75	0.583	0.167	0.250	7	2	3
13	0.179	0.35	0.65	0.75	0.615	0.154	0.231	8	2	3
14	0.165	0.35	0.65	0.75	0.571	0.143	0.286	8	2	4
15	0.153	0.35	0.65	0.75	0.600	0.133	0.267	9	2	4
16	0.143	0.35	0.65	0.75	0.625	0.125	0.250	10	2	4
17	0.134	0.35	0.65	0.75	0.588	0.118	0.294	10	2	5
18	0.125	0.35	0.65	0.75	0.611	0.111	0.278	11	2	5
19	0.118	0.35	0.65	0.75	0.632	0.105	0.263	12	2	5
20	0.116	0.35	0.65	0.75	0.600	0.150	0.250	12	3	5
21	0.110	0.35	0.65	0.75	0.571	0.143	0.286	12	3	6
22	0.104	0.35	0.65	0.75	0.591	0.136	0.273	13	3	6
23	0.099	0.35	0.65	0.75	0.609	0.130	0.261	14	3	6
24	0.095	0.35	0.65	0.75	0.583	0.125	0.292	14	3	7
25	0.091	0.35	0.65	0.75	0.600	0.120	0.280	15	3	7
26	0.087	0.35	0.65	0.75	0.615	0.115	0.269	16	3	7
27	0.084	0.35	0.65	0.75	0.593	0.111	0.296	16	3	8
28	0.080	0.35	0.65	0.75	0.607	0.107	0.286	17	3	8
29	0.077	0.35	0.65	0.75	0.621	0.103	0.276	18	3	8
30	0.076	0.35	0.65	0.75	0.600	0.133	0.267	18	4	8

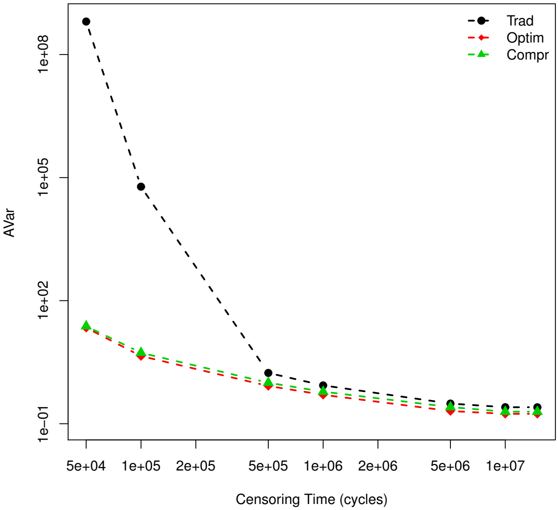
Assessment Plan

- The test plans are assessed in three ways:
  1. Assess the effect of the design parameters (total sample size, censoring time, etc.).
  2. Assess the sensitivity to the model parameters and distribution choice.
  3. Perform simulation studies to compare exact and large-sample variances at selected design parameters.
- The default settings are $k = 12$ samples, $T_M = 150{,}000$ cycles, and a lognormal lifetime distribution.

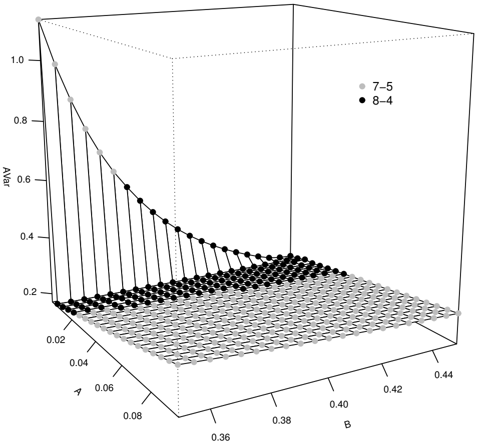
Total Sample Size & Stress Levels



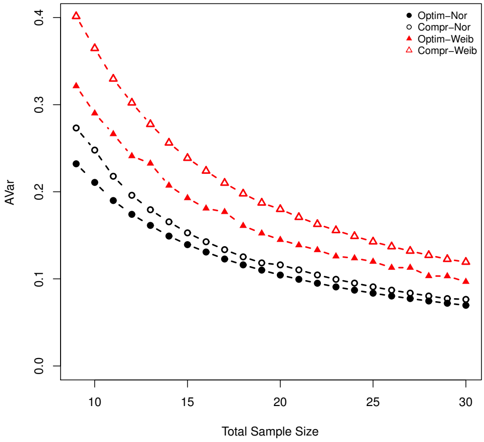
Censoring Time



Parameter Effects (Optimal Designs)



Distribution Effect



- Compromise - Lognormal: 35-65-75 ; Weibull: 35-45-75

Simulation Results

Table : Lognormal

k	True Var		Asymp Var	
	Optim	Comp	Optim	Comp
9	0.237	0.273	0.232	0.273
12	0.175	0.195	0.174	0.196
15	0.138	0.152	0.139	0.153
18	0.114	0.120	0.116	0.125
21	0.102	0.114	0.099	0.110
24	0.089	0.096	0.087	0.095
27	0.078	0.085	0.077	0.084
30	0.070	0.075	0.070	0.076

Table : Weibull

k	True Var		Asymp Var	
	Optim	Comp	Optim	Comp
9	0.370	0.411	0.321	0.401
12	0.273	0.293	0.241	0.302
15	0.217	0.233	0.198	0.239
18	0.179	0.191	0.162	0.198
21	0.156	0.157	0.143	0.171
24	0.130	0.141	0.124	0.149
27	0.116	0.125	0.109	0.132
30	0.108	0.114	0.096	0.119

- 10,000 runs per sample size

Conclusions

- Both optimum and compromise plans are more efficient than the traditional plan.
- Minor deviations in the value of B do not affect the designs as strongly as deviations in the value of A.
- The choice of lifetime distribution will have an impact on test plan configuration and optimum criterion.

Areas for Future Research

- Extend the criterion to include other possibilities (e.g. parameter estimate variance, expected lifetime, etc.)
- Consider criterion consisting of multiple stress levels
- Derive optimum designs based on models for block and spectral loading schemes.

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