Power Grid Reliability: Identification of Probable Downed Lines from Phasor Measurements

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Overview

- A robust and reliable electric power grid is essential for society to function
- The network is complex, unpredictable, and only partially observable
- Fast detection of line outages can improve operations
- New Phasor Measurement Units (PMUs) are being installed in the U.S. grid
- We identify downed lines by estimating topology probabilities using PMU data

The DOE aims to improve reliability and resiliency of the power grid.

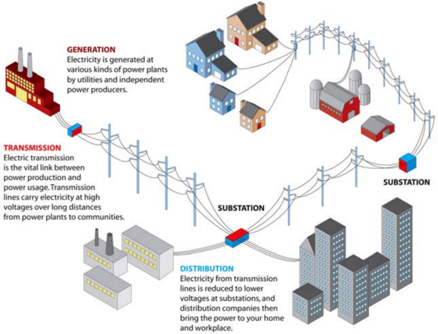


Image Credit: U.S. DOE (2006)

Nimble use of PMU data will facilitate rapid response to abnormal grid conditions.

Conventional State Estimation

- Estimate voltages and flows given topology and current measurements
- Iterate nonlinear least squares with greedy topology search
  - estimate state **x** by NLS
  - poor fit indicates a topology error
  - modify topology and repeat
- Assume single best fit is correct
- Works well in practice, with a few notable failures
  - Incorrect topology contributed to 2003 northeast blackout

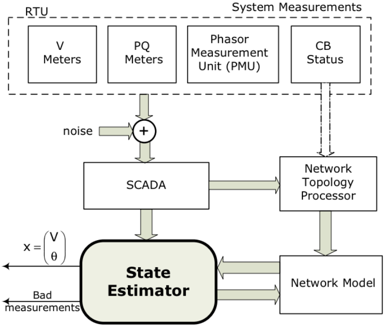
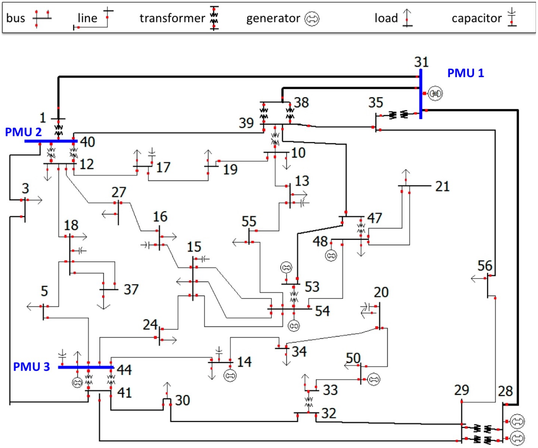


Image Credit: Slobodan Pajić (2007)

Ultimately, resilient grid operation requires real time topology estimation, with quantified uncertainty.

Power Flow Solver and Measurements



- Computational model used by state estimator
  - implements electrical laws
  - uses physical properties of grid components
- **Inputs:** Grid topology and node quantities (complex valued voltage or power load)
- **Outputs:** power flowing on the lines
- **Measurements:** PMUs measure complex voltage phasors on some buses

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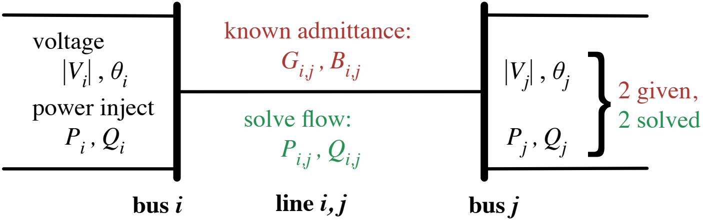
Power Flows Satisfy Nonlinear AC Equations

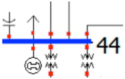


Lossless flow equations for real ($P_i, P_{i,j}$) and reactive ($Q_i, Q_{i,j}$) power:

$$
\begin{aligned}
P_i &= \sum_{j=1}^{N} |V_i||V_j|[G_{i,j}\cos(\theta_i - \theta_j) + B_{i,j}\sin(\theta_i - \theta_j)] = \sum_{j=1}^{N} P_{i,j} \\
Q_i &= \sum_{j=1}^{N} |V_i||V_j|[G_{i,j}\sin(\theta_i - \theta_j) - B_{i,j}\cos(\theta_i - \theta_j)] = \sum_{j=1}^{N} Q_{i,j}
\end{aligned}
$$

$(i = 1, \ldots, N)$

Inputs and Computed Values for Different Bus Types



Power inject at a bus may be composed of several parts.

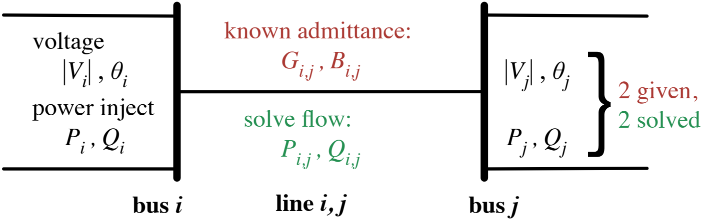
$$P_i = P_i^{\text{load}} + P_i^{\text{gen}} \qquad \text{and} \qquad Q_i = Q_i^{\text{load}} + Q_i^{\text{gen}} + Q_i^{\text{cap}}$$

Bus Includes	P_i^{load}	Q_i^{load}	P_i^{gen}	Q_i^{gen}	Q_i^{cap}	θ_i	$\lvert V_i \rvert$
Slack Generator	in	in	out	out	in	in	in
Regular Generator	in	in	in	out	in	out	in
No Generator	in	in	0	0	in	out	out

Priors on loads are independent Gaussian with known moments.

$$P_i^{\text{load}} \sim N[\mu_i, (0.2\mu_i)^2], \qquad Q_i^{\text{load}} \sim N[\nu_i, (0.2\nu_i)^2]$$

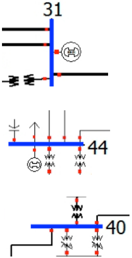
PMU Measurements



PMUs on some buses measure voltage angle and magnitude. Assume independent Gaussian.

$$A_i \mid \theta_i \sim N[\theta_i, 0.01^2], \qquad \text{(degrees)}$$

$$M_i \mid V_i \sim N[|V_i|, (0.001|V_i|)^2] \quad \text{(volts)}$$



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Model Bank Topology Estimation

Singh, et al. consider estimating the correct topology from a finite collection of possibilities

- a bank of models contains all important network configurations
- estimate probabilities for each model in the bank by combining
  - system measurements
  - prior information about loads
  - power flow model
- Did not implement Bayes rule correctly

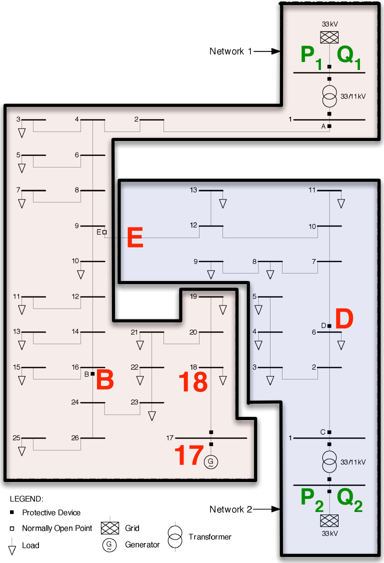

Image Credit: Modified from Singh, Manitsas, Pal, Strbac (2010)

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Model Bank Topology Estimation

We compute probabilities on topologies from a vector of PMU measurements

- sample loads from priors
- propagate priors to PMU measurements for each possible topology by solving power flow equations
- approximate resultant distributions of PMU measurements
- apply Baye's rule

Algorithm is fast enough to be used in real time

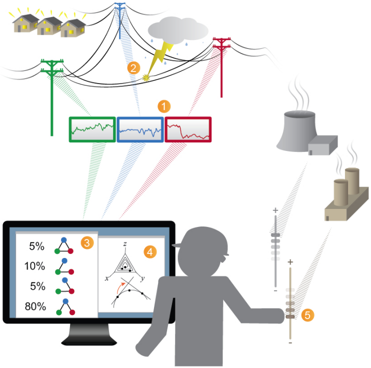

Image shows topology estimates and risk-optimal power generation

Statistical Model Set-Up

- Model bank of grid topologies (states)

$$\mathcal{S} = \{s_1, \ldots, s_m\}$$

- Prior

$$\Pr(\text{state is } s) = \pi(s)$$

- PMU measurements $\mathbf{y}$

$$\mathbf{y} \mid s \sim g(\mathbf{y} \mid s)$$

- Posterior state estimate

$$\Pr(\text{state is } s \mid \mathbf{y}) \propto \pi(s) g(\mathbf{y} \mid s)$$

Q: How to get $g(\mathbf{y} \mid s)$ for $s = s_1, \ldots, s_m$?
A: Simulate $[\mathbf{y} \mid s]$ and fit log-spline density Gaussian copulae (LSD-GC).

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Log-spline Density (LSD)

A scalar random variable $y \sim$ LSD if its density is

$$f(y; \mathbf{b}, \mathbf{c}) \propto \exp\left[b_0 y + \sum_{i=1}^{k} b_i (y - c_i)_+^3\right]$$

where

$$(\cdot)_+ \equiv \max(0, \cdot),$$

$\mathbf{c} = (c_1, \ldots, c_k)$ are the knots, and

$\mathbf{b} = (b_0, \ldots, b_k)$ are coefficients with k degrees of freedom

and $\log f$ is constrained to be linear outside of range($\mathbf{c}$). $F \equiv$ CDF of f.

Convenient parametric form; easy to store, reuse, and evaluate.

Linked Kooperberg's (2009) C++ implementation to Matlab for analysis. Fits $\mathbf{b}$, $\mathbf{c}$ with automatic choice of k.

Gaussian Copula (GC)

For a p-vector $\mathbf{y}$ of PMU measurements, transform each element to standard Gaussian.

$$\mathbf{z}(\mathbf{y}) = \left(\Phi^{-1}(F[y_1; \mathbf{b}_1, \mathbf{c}_1]), \ldots, \Phi^{-1}(F[y_p; \mathbf{b}_p, \mathbf{c}_p])\right)$$

Model dependencies through

$$\mathbf{z}(\mathbf{y}) \sim \mathrm{N}_p(\mathbf{0}, \boldsymbol{\Sigma})$$

where $\boldsymbol{\Sigma}$ is a correlation matrix.

Parameters for the joint distribution of $\mathbf{y}$ are $\boldsymbol{\theta} \equiv (\mathbf{b}_1, \mathbf{c}_1 \ldots, \mathbf{b}_p, \mathbf{c}_p, \boldsymbol{\Sigma})$ and the LSD-GC density is

$$g(\mathbf{y}; \boldsymbol{\theta}) = \phi_p(\mathbf{z}(\mathbf{y}); \mathbf{0}, \boldsymbol{\Sigma}) \prod_{i=1}^{p} \left| \frac{f(y_i; \mathbf{b}_i, \mathbf{c}_i)}{\phi_1(z_i(y_i))} \right|$$

where ϕ_p is the MVN density and ϕ_1 is the standard normal density.

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Offline Precomputation

for all $s \in \mathcal{S}$ **do**
  for $j = 1, \ldots, J$ **do**
    draw random loads from priors: $\left\{P_i^{\text{load}}, Q_i^{\text{load}}\right\}$
    solve power flow equations for s
    extract voltages from PMU buses: $\{\theta_i, |V_i|\}$
    draw noisy measurements: $\{A_i \mid \theta_i, M_i \mid V_i\}$
    form $\mathbf{y}_j$ from measurements: $\{A_i - A_{i_0}, M_i\}$ (i_0=reference bus)
  end for
  fit LSD-GC parameters $\boldsymbol{\theta}_s$ to $\mathbf{Y}_s = \{\mathbf{y}_1, \ldots, \mathbf{y}_J\}$
end for

$\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_m$ parameterize distributions $[\mathbf{y} \mid s_1], \ldots, [\mathbf{y} \mid s_m]$

Online State Estimation

State probabilities are calculated quickly as

$$\Pr(\text{state is } s \mid \mathbf{y}) = \frac{\pi(s) g(\mathbf{y}; \boldsymbol{\theta}_s)}{\sum_{i=1}^{m} \pi(s_i) g(\mathbf{y}; \boldsymbol{\theta}_{s_i})}$$

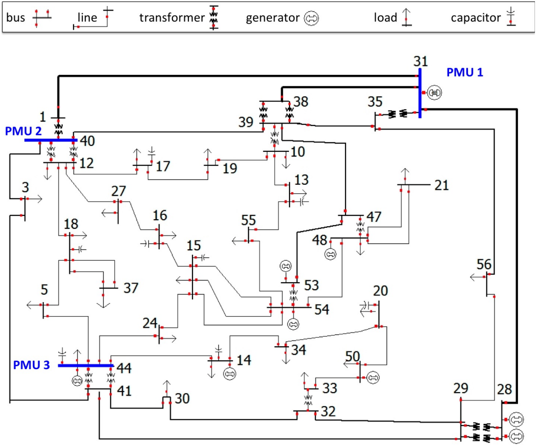
Distribution System Example

- System studied by Tate and Overbye (2008, 2009)
- 37 buses, 57 lines
- Three PMUs as in T&O
- Model bank $\mathcal{S}$ of 58 states
  - normal
  - each single line down
- Simulated $J = 4600$ measurements sets

  $$\mathbf{Y} = (\mathbf{y}_1, \ldots, \mathbf{y}_{4600})$$

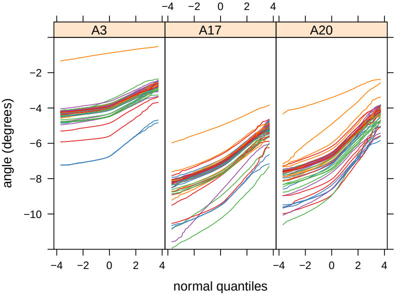
  for each state

  $$\mathbf{Y}_1, \ldots, \mathbf{Y}_{58}$$

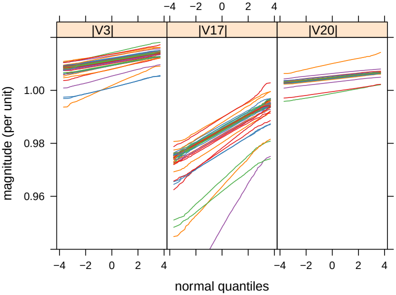


Normal Plots of Measurements $\mathbf{Y}_s$ at Three Buses

Each line assumes one of the 58 topologies, s.



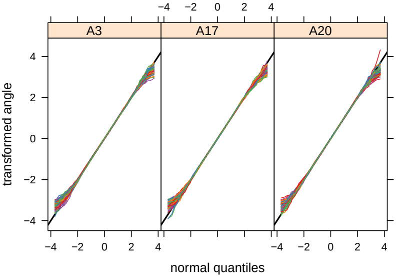
Voltage Angles, A_i



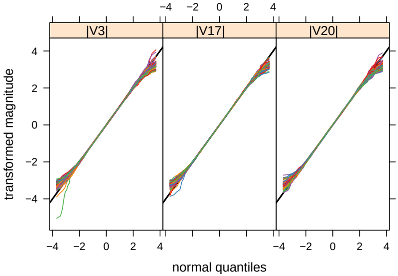
Voltage Magnitudes, M_i

Normal Plots of $\mathbf{z}_s(\mathbf{Y}_s)$, the LSD-Transform to Gaussian

Transform to N(0,1) for each measurement and topological state.



Transformed Angles, A_i



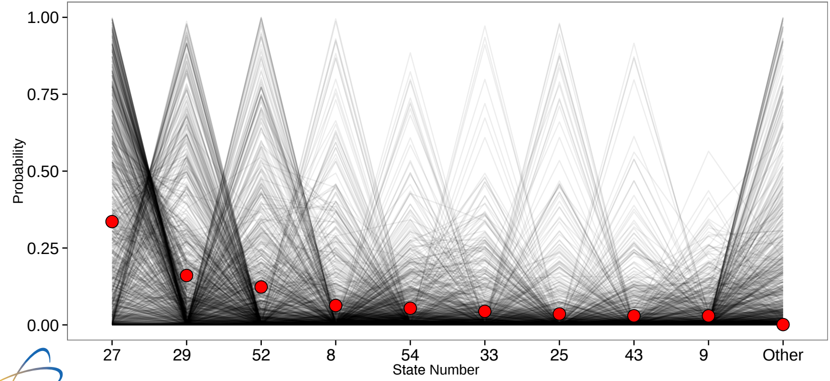
Transformed Magnitudes, M_i

Classification of Topological State

For each topology:

1. Generate 1000 simulated observations, $\mathbf{y}_1, \ldots, \mathbf{y}_{1000}$
2. Compute $\mathbf{p}(\mathbf{y}_i) \equiv [\Pr(s = 1 \mid \mathbf{y}_i), \ldots, \Pr(s = 58 \mid \mathbf{y}_i)]$

Example for state $s = 27$



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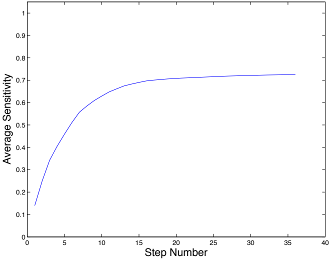
Next Step: Greedy PMU Placement

Start with PMU at reference bus, 31

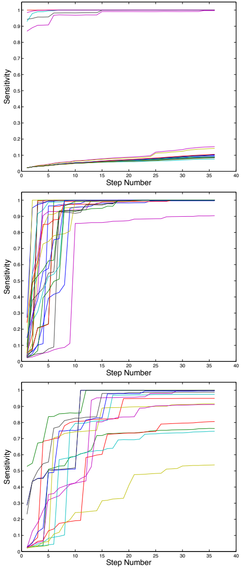
Stepwise add PMU to bus that most improves Pr(correct classification)

Average Pr(correct classification) vs. step.



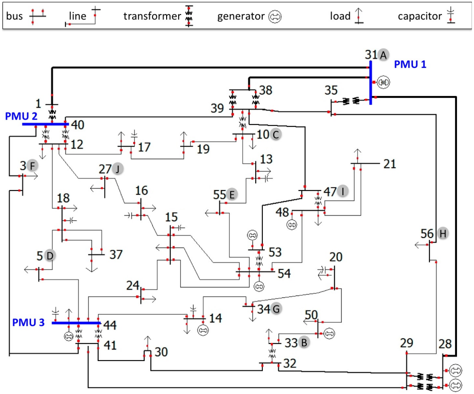
Diminishing returns at ∼10 PMUs

Split by state

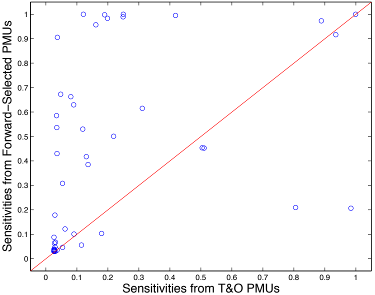


Greedy PMU Placement Results

First 10 PMUs placed greedily are marked A,. . . , J



Average Pr(correct) for {A,B,C} Vs. {31,40,44}



Plot shows 58 states

Greedy placement improves sensitivities substantially.

Summary

- Pre-computation to estimate joint distribution of PMU measurements for each single-line down.
  - simulate data sets for each state
  - fit marginal log-spline densities and transform to N(0,1)
  - fit Gaussian copula to capture dependencies
- Provides fast state estimation

$$\Pr(\text{state is } s \mid \mathbf{y}) = \frac{\pi(s)g(\mathbf{y};\boldsymbol{\theta}_s)}{\sum_{i=1}^{m}\pi(s_i)g(\mathbf{y};\boldsymbol{\theta}_{s_i})}$$

- Method suitable for greedy PMU placement
- Could extend to more optimal placement

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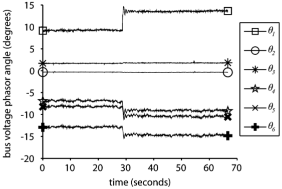
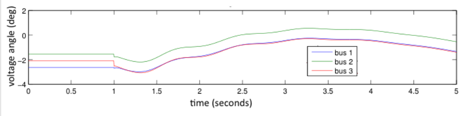
Future Work

Detect and classify outages using *dynamic* PMU traces within a few seconds





- Promising initial work with elastic net on 58 scenarios (Bayesian classifier)
- Cross-validated confusion matrix for 3 PMUs on 58 topological states using a 2.5 second data horizon